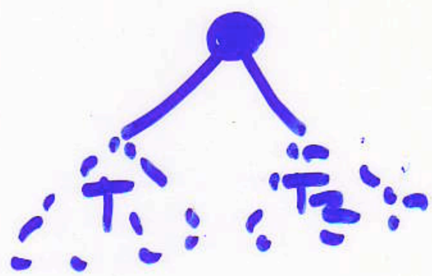
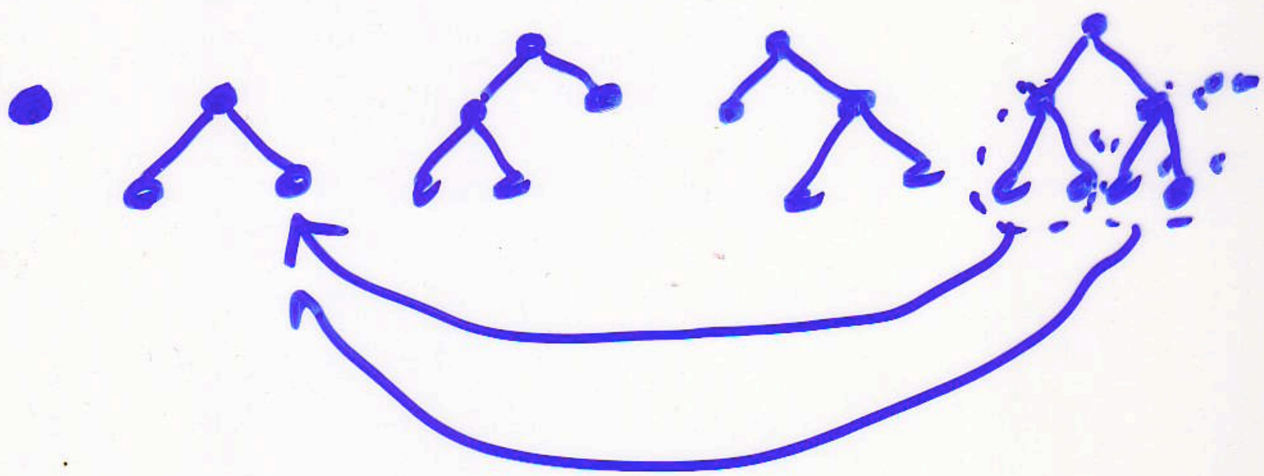
Full binary Trees

is either

1) a node by itself •

or 2) a node joined to two other full binary trees





$n(T) = 1$ if T is •

$= 1 + n(T_1) + n(T_2)$

$h(T) = 0$ if T is •

$= 1 + \max(h(T_1), h(T_2))$

$$n(T) \le 2^{h(T)+1} - 1$$

if $T =$ •

$n(T) = 1$

$h(T) = 0$

$2^{0+1} - 1 = 1$ ✓

$T =$ 

$$n(T) = 1 + n(T_1) + n(T_2)$$

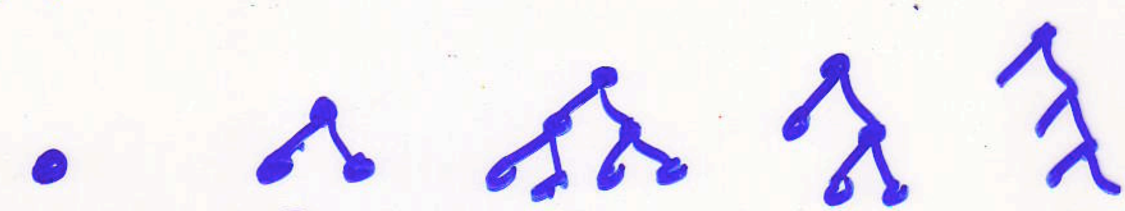
$$\le 1 + 2^{h(T_1)+1} - 1 + 2^{h(T_2)+1} - 1$$

$$\le 2 \cdot 2^{\max(h(T_1), h(T_2))+1} - 1$$

$$2^{\underline{\max(h(T_1), h(T_2))+2}} - 1$$

$$= 2^{h(T)+1} - 1$$

$$n(T) \geq 2h(T) + 1$$



n	1	3	7	5	7
h	0	1	2	2	3
$2h+1$	1	3	5	5	7

<u>Basis</u>:

$T = \bullet$ $\quad n=1,\ h=0 \quad 1 \geq 2\cdot h + 1$ ✓

<u>Ind</u>:

$n(T) = 1 + n(T_1) + n(T_2)$

$T =$ 

$T_1 \quad T_2$

at least one of $T_1\ T_2$ has height $= h-1$

T has height $h > 0$

The other has height $0 \leq k \leq h-1$

$$\begin{aligned} n(T) &= 1 + n(T_1) + n(T_2) \\ &\geq 1 + (2(h-1)+1) + (2\cdot 0 + 1) \\ &= 2h + 1 \end{aligned}$$

A well-formed formula (WFF)

$p \vee q \wedge \neg r \quad (p \vee (q \wedge (\neg r)))$

$p \wedge \vee q \neg$

Defn

1. $\underline{T}$, $\underline{F}$, $\underline{p}$ for some propositional variable p are WFF

2. if E & F are WFF's then so are

(a) $\underline{(\neg E)}$

(b) $\underline{(E \vee F)}$

(c) $\underline{(E \wedge F)}$

In any WFF E, # of left parens − # of right parens = 0

Basis

E = T or F or p

each has no parens so L − R = 0

Ind.

case 1 E = (¬G)

\# of left in (E) = 1 + # of left in G

.... right ... = 1 + ... right in G

by Ind #left(G) − #right(G) = 0

$$\frac{1 + 1\text{left}(G) - (1 + \#\text{right}(G)) = 0}{\#\text{left}(E) - \#\text{right}(E) = 0}$$

case 2 E = (G ∨ H)

similar

case 3 E = (G ∧ H)

similar

if x is a prefix of a WFF E

then $\#\text{left}(x) - \#\text{right}(x) \geq 0$

$$(T \wedge (\neg(\neg(p \vee T)) \wedge F))$$

$($

$(T$

$(T \wedge$

$(T \wedge ($

$\vdots$

prefixes