



Suppose  $\mathbb{R}^+$  are countable

...  $[0, 1)$  ...

0	0. <u>0</u> 0 0 0
1	0. 2 <u>1</u> 2 1 2 1 2 1 ...
2	0. 1 2 <u>3</u> 4 1 2 3 4 5 6 ...
3	0. 3 1 4 <u>1</u> 5 9 2 6 5 3 5 7 7 7 2 7 ...
4	0. 2 7 1 8 <u>9</u> 9 9 9 9 9 9 9 4 ...
5	0. 2 7 1 9 <u>0</u> 0 5 0 0 0 ...

0. 1 2 4 2 8 1 ...

Let  $d_{ij}$  be  $j^{\text{th}}$  digit of  $i^{\text{th}}$  row

For digit  $d$ , let  $\bar{d}$  be any other digit, excluding  $\{9, 0\}$

Let  $x$  be the real whose  $i^{\text{th}}$  digit is  $\bar{d}_{ii}$ .

Then  $x$  is not in the table since it differs from  $i^{\text{th}}$  row at  $i^{\text{th}}$  digit (at least)

[and avoids the nuisance  $.499... = .5000... = .5$  issue] 13-4

$$x = 0.\overline{499} \dots$$

$$10x = 4.99\dots$$

$$10x - x$$

$$\begin{array}{r} 4.99\dots \\ - 0.499\dots \\ \hline 4.5 \end{array}$$

$$9x = 4.5 \quad \checkmark \quad 000$$

$$x = \frac{4.5}{9} = \frac{9}{14} = \frac{1}{2} = .5$$

Fact. if  $A \subseteq B$  &  $B$  is countable then so is  $A$ .

The set of computable functions from  $\mathbb{N} \rightarrow \{0, 1\}$  is countable.

Set of valid java  $\subseteq \Sigma^*$

$$B = \{ f \mid f: \mathbb{N} \rightarrow \{0, 1\} \}$$

Suppose  $B$  is enumerable

	0	1	2	3	...
$f_0$	0	1	0	1	0
$f_1$	0	0	0	0	0
$f_2$	1	1	0	1	1
$f_3$	1	0	1	0	0
$d$	1	1	1	0	1

$d(i) = 1 - f_i(i)$

$d: \mathbb{N} \rightarrow \{0, 1\}$  so  $d \in B$

but  $d$  differs from  $f_i$  at  $i$

$$d(i) = 1 - f_i(i) \neq f_i(i)$$

$$d \neq f_i$$

Let finite alphabet  $\Sigma = \{a, b, c\}$

$\Sigma^*$  set of finite strings over  $\Sigma$

$\lambda$   
~~a~~  
~~b~~  
~~c~~  
~~aa~~  
~~ab~~  
~~ac~~  
~~ba~~  
~~bb~~  
~~bc~~  
~~ca~~  
~~cb~~  
~~cc~~  
~~aaa~~  
~~aab~~  
~~aac~~  
~~aba~~  
~~abb~~  
~~abc~~  
~~aca~~  
~~acb~~  
~~ccb~~  
~~cba~~  
~~cbb~~  
~~ccc~~  
~~...~~

aa  
 ab  
 ac  
 ba  
 bb  
 bc  
 ca  
 cb  
 cc  
 aaa  
 aab  
 aac

0  
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$\Sigma^*$  is countable

$$\forall x \in \Sigma^* \quad f(x) = \begin{cases} 0 & \text{if } x = \lambda \\ r \cdot f(y) + g(z) & \text{if } x = yz \text{ where } z \in \Sigma, y \in \Sigma^* \end{cases}$$

$$f: \Sigma^* \rightarrow \mathbb{N}$$

$$r = |\Sigma|$$

$g: \Sigma \rightarrow \{1, 2, \dots, r\}$   
 is a bijection

Exercise:  
 Prove  $f$  is  
 a bijection 14-2