Independence

Flip coin   roll die

H   6

(i,j)

2×6

H	1	1/12
T	1	⋮
H	2	⋮
T	2	
H	6	1/12

$E: (H + 6)$

$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

$P(H \& 6) = P(H) \cdot P(6)$

Defn E & F are indep. iff

$$P(E \cap F) = P(E) \cdot P(F)$$

Flip

if H   roll fair die

T   roll loaded die allways 6

(i,j)

2×6

H	6	1/2
T	1	1/12
T	2	1/12
⋮		
T	6	1/12

$P(H) = \frac{1}{2}$

$P(6) = \frac{7}{12}$

$P(H \& 6) = \frac{1}{2}$

$\neq \frac{1}{2} \cdot \frac{7}{12}$

3 bits uniform

E 1st bit is 1

F # of 1's is even

$P(E) = 1/2$

$P(F) = 4/8$ 000, 110, 101, 011

$P(E \cap F) = 2/8 = 1/2 \cdot 1/2$

yes - indp.

25-2

Bernoulli Trials

"Trial" success / failure

p q $q = 1 - p$

$2/3$ $1/3$

7 coin flips, what is prob of 4 heads?

$$\binom{7}{4} p^4 q^3$$

$$\binom{7}{4} \begin{cases} HHHHTTT & p^4 q^3 \\ HHTTHHT & \underline{p^4 q^3} \quad p \cdot p \cdot q \cdot q \cdot p \cdot p \cdot q \\ \vdots & \end{cases}$$

$$\frac{7\,6\,5\,4}{4\,3\,2\,1} \cdot \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^3 = \frac{35 \cdot 16}{3^7}$$

$$\approx .25$$

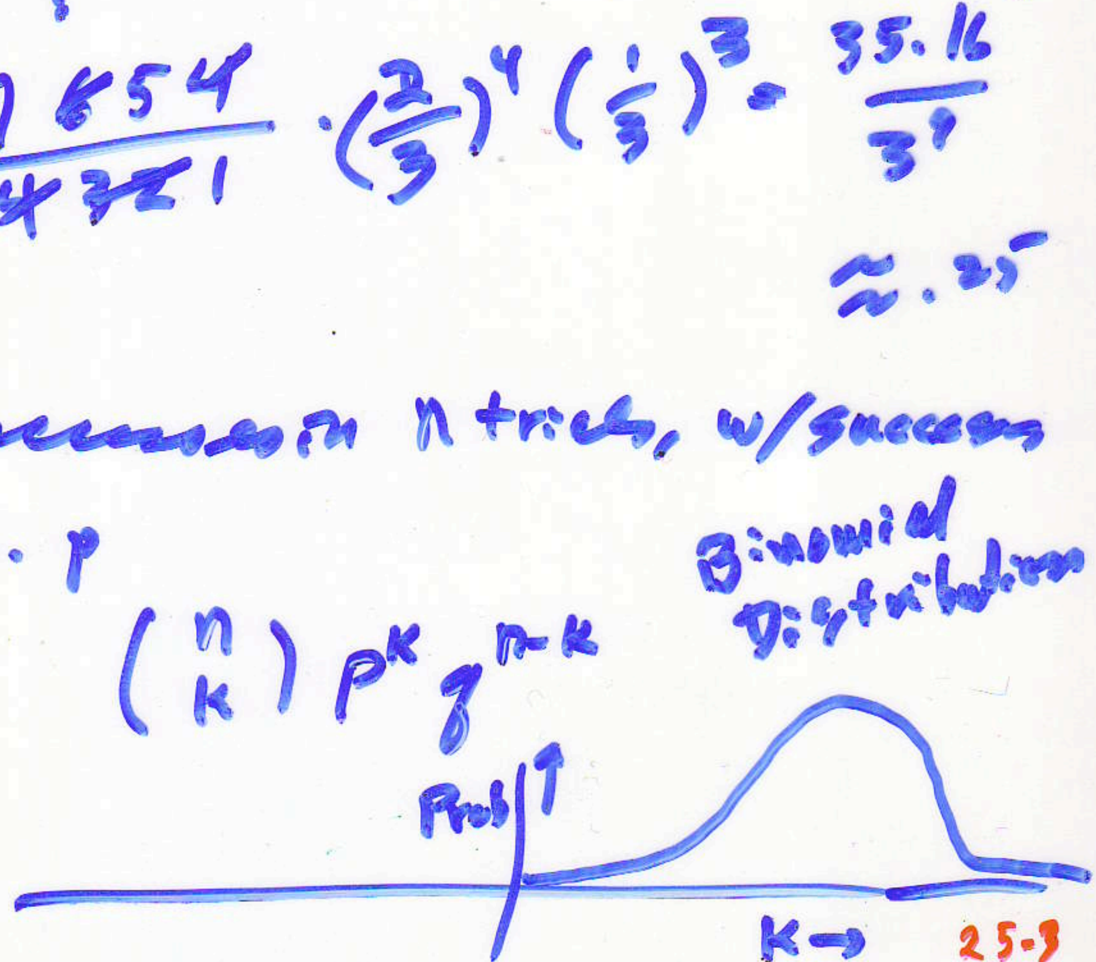
K successes in n trials, w/ success prob. p

$$\binom{n}{k} p^k q^{n-k}$$

Binomial Distribution

$$\sum_i \binom{n}{i} p^i q^{n-i}$$

$$= (p+q)^n = 1$$



2 coins, fair

HH $1/4$

HT $1/4$) $1/2$

TH $1/4$

TT $1/4$

Law of Total Probability
.... Alternatives

$$P(E) = P(E \mid A)\cdot P(A) + P(E \mid \bar{A})\cdot P(\bar{A})$$

"

$$\frac{P(E\cap A)}{\cancel{P(A)}}\cdot \cancel{P(A)} + \frac{P(E\cap \bar{A})}{\cancel{P(\bar{A})}}\cdot \cancel{P(\bar{A})}$$

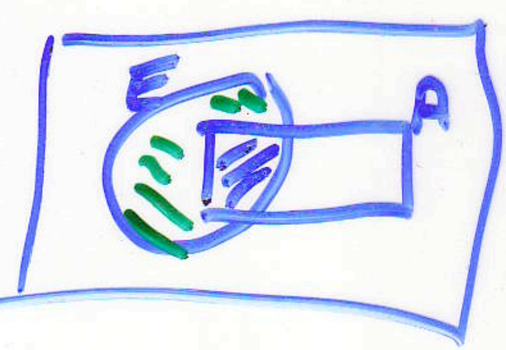
A & $\bar{A}$ disjoint

$E\cap A$ & $E\cap \bar{A}$ "

$$= P((E\cap A)\cup(E\cap \bar{A}))$$

$(E\cap A)\cup E\cap \bar{A}$

$= E$



$\to P(E)$

20% of women

1% of men

nCG 1/3 women 2/3 men

$.20 \times \frac{1}{3} + \frac{.01}{100} \cdot \frac{2}{3}$

$\frac{22}{300}$

Bayes Thm

$$P(B|A) = P(A|B)\frac{P(B)}{P(A)}$$

$$P(D|T) = P(T|D)\cdot\frac{P(D)}{P(T)}$$