CSE 321: Discrete Structures
Assignment #2
January 15, 2009
Due: Thursday, January 22, in Section

Reading Assignment: Sections 1.5 – 1.7 and handout on logic.

Problems:

1. Determine the truth value of $\exists x \forall y (x \leq y^2)$ when the universe of discourse is the
   (a) positiver reals
   (b) non-negative reals
   (c) positive integers
   (d) non-negative integers
2. Let $P(x)$, $Q(x)$, and $R(x)$ be the statements "x is a clear explanation", "x is satisfactory" and "x is an excuse", respectively. Suppose that the domain for x consists of all English text. Express each of these statements using quantifiers, logical connectives, and $P(x)$, $Q(x)$, and $R(x)$.
   (a) All clear explanations are satisfactory.
   (b) Some excuses are unsatisfactory.
   (c) Some excuses are not clear explanations.

   Does (c) follow from (a) and (b)? Explain why or why not.
3. Prove or disprove the claim that the proposition $\forall x(P(x) \rightarrow Q(x))$ is logically equivalent to $\forall x P(x) \rightarrow \forall x Q(x)$.
4. 6th edition: Section 1.4, Exercise 12, parts a) through g). (same in 5th edition)
5. 6th edition: Section 1.5, Exercise 6. (5th edition: Section 1.5, Exercise 4.)
6. 6th edition: Section 1.5, Exercise 16. (5th edition: Section 1.5, Exercise 12.)
7. Prove or disprove: $n^2 + 3n + 1$ is always prime for every integer $n > 0$.
8. Prove that for any integer n, if $3n + 2$ is even, then n is even.

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9. **Extra Credit:** We define a new logical operator '$|$' as follows: $p|q$ is true when either p or q or both are false, and it is false when p and q are both true. Show that $\{|\}$ is a functionally complete collection of logical operators. (For the meaning of functionally complete, see the discussion in Section 1.2 of Rosen. In the 6th edition, it precedes exercise 43, while in the 5th edition it precedes exercise 37.)

10. **Extra Credit:** Prove that if m and n are integers and mn is even, then m is even or n is even.