CSE 321: Discrete Structures
Assignment #5
February 4, 2009
Due: Wednesday, February 11, in class

Reading Assignment: Sections 4.1 – 4.3 and 5.1.

Problems:

1. Which amounts of money can be formed using just dimes and quarters? Prove your answer using a form of mathematical induction.

2. Assume that a chocolate bar consists of n squares arranged in a rectangular pattern. Any chocolate bar can be broken into two pieces along a horizontal or vertical line separating the squares (e.g. a 2×4 chocolate bar can be broken in 4 different ways: along the one horizontal line, or along one of the three vertical lines). Assuming that only one piece can be broken at a time, determine how many breaks you must successively make to break the bar into n separate squares. Use strong induction to prove your answer.

3. Section 4.2, exercise 12.

4. Give a recursive definition of

   (a) the set of odd positive integers,

   (b) the set of positive integer powers of 3,

   (c) the set of positive integers congruent to 4 modulo 5.

5. Let f_n be the n-th Fibonacci number. Prove that $f_1^2 + f_2^2 + \ldots + f_n^2 = f_n f_{n+1}$ whenever n is a positive integer.

6. Structural induction: Show that the set S defined by $1 \in S$ and $s + t \in S$ whenever $s \in S$ and $t \in S$ is the set of positive integers.

   (Hint: You are showing that two sets A and B are equal, which requires showing that both $A \subseteq B$ and $B \subseteq A$.)