CSE 321: Discrete Structures
Assignment #7
February 20, 2009
Due: Wednesday, February 25, in class

Reading Assignment: Sections 6.1 – 6.4.

Justify your answers to the following problems. You do not need to provide explicit numbers for permutations and combinations (for example, no need to expand C(3,4)).

Problems:

1. What is the coefficient of a^4b^6 in $(a^2 + b)^8$?
2. Prove the Binomial Theorem using mathematical induction.
3. A fair coin is flipped n times. What is the probability that all the heads occur at the start of the sequence?
4. Which is more likely: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled?
5. In the following problems, you are given a 5-card hand from a randomly shuffled deck of 52 cards.
   (a) Given that you have at least one ace, what is the probability that you have another ace?
   (b) Given that you have the ace of diamonds, what is the probability that you have another ace?
   (c) Given that you have a red ace (diamonds and hearts are red), what's the probability that you have another ace?
   (d) Suppose you select a random card from your hand and its an ace. Whats the probability that you have another ace?
6. Assume that the probability a child is a boy is 0.51 and that the sexes of children born into a family are independent. What is the probability that a family of five children has
   (a) exactly three boys?
   (b) at least one boy?
   (c) all children of the same sex?
7. **Extra credit**: The 120 seats on an Alaska Airlines flight were completely booked, with each of the 120 passengers having different assigned seats. The passengers entered the plane one-by-one. Unfortunately, the first passenger, Joe Isclumsy, couldn't read his boarding pass

because he spilled coffee on it and sat in a (uniformly) random seat. Each subsequent passenger sat in their assigned seat if it was available when they entered and sat in a (uniformly) random empty seat otherwise. What is the probability that the last passenger sat in their assigned seat?