Discrete Structures

Logic

Chapter 1, Sections 1.1–1.4

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Outline

◇ Propositional Logic

◇ Propositional Equivalences

◇ First-order Logic

Propositional Logic

Let p and q be propositions.

◇ **Negation** $\neg p$ The statement "It is not the case that p." is true, whenever p is false and is false otherwise.

◇ **Conjunction** $p \wedge q$ The statement "p and q" is true when both p and q are true and is false otherwise.

◇ **Disjunction** $p \vee q$ The statement "p or q" is false when both p and q are false and is true otherwise.

◇ **Exclusive or** $p \oplus q$ The *exclusive or* of p and q is true when exactly one of p and q is true and is false otherwise.

Proposition?

- There is life on Mars.
- Today is Friday.
- $2 + 2 = 4$
- Bayern Munich is the best soccer team ever!
- $x + 2 = 5$
- Why are we taking this class?
- This statement is false.
- This statement is true.

Propositional Logic

Let p and q be propositions.

◊ **Implication** $p \to q$ The *implication* $p \to q$ is false when p is true and q is false and is true otherwise. p is called the hypothesis (antecedent, premise) and q is called the conclusion (consequence).

- "if p, then q" "p implies q" "p only if q"
  "p is sufficient for q" "q is necessary for p"
- $q \to p$ is called the converse of $p \to q$
- $\neg q \to \neg p$ is called the contrapositive of $p \to q$

◊ **Biconditional** $p \leftrightarrow q$ The *biconditional* $p \leftrightarrow q$ is true whenever p and q have the same truth values and is false otherwise.

Translating English Sentences

♢ You can access the Internet from campus only if you are a computer science major or you are not a freshman.

♢ You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.

Logical Equivalences

- $\diamondsuit$ **Tautology** A compound statement that is always true.
- $\diamondsuit$ **Contradiction** A compound statement that is always false.
- $\diamondsuit$ **Contingency** A compound statement that is neither a tautology nor a contradiction.
- $\diamondsuit$ **Logical equivalence** $p \equiv q$ Propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology.

Tautologies?

$\diamondsuit$ I don't jump off the Empire State Building implies if I jump off the Empire State Building then I float safely to the ground.

$\diamondsuit$ ((Smoke $\wedge$ Heat) $\rightarrow$ Fire) $\equiv$ ((Smoke $\rightarrow$ $Fire$) $\vee$ (Heat $\rightarrow$ Fire))

Logical Equivalences

$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

First-order Logic

◇ **Universal quantifier $\forall$:** The *universal quantification* of $P(x)$ is the proposition "$P(x)$ is true for all values of x in the universe of discourse."

◇ **Existential quantifier $\exists$:** The *existential quantification* of $P(x)$ is the proposition "There exists an element x in the universe of discourse such that $P(x)$ is true."