CSE 321 Discrete Structures

January 8, 2010

Lecture 03

Logical Proofs and Predicate Calculus

Logical Proofs

- We will discuss natural deduction in class

- Please use the *Natural Deduction* handout

Predicate Calculus

- *Predicate* or *Propositional Function*
  - A function that returns a truth value
- "x is a cat"
- "x is prime"
- "student x has taken course y"
- "$x > y$"
- "$x + y = z$"

Quantifiers

- $\forall x\, P(x)$: $P(x)$ is true for every x in the domain
- $\exists x\, P(x)$: There is an x in the domain for which $P(x)$ is true

Statements with quantifiers

- $\exists x\ \text{Even}(x)$
- $\forall x\ \text{Odd}(x)$
- $\forall x\ (\text{Even}(x) \lor \text{Odd}(x))$
- $\exists x\ (\text{Even}(x) \land \text{Odd}(x))$
- $\forall x\ \text{Greater}(x+1, x)$
- $\exists x\ (\text{Even}(x) \land \text{Prime}(x))$

Domain:
Positive Integers

Even(x)
Odd(x)
Prime(x)
Greater(x,y)
Equal(x,y)

Statements with quantifiers

- $\forall x \exists y$ Greater (y, x)
- $\forall x \exists y$ Greater (x, y)
- $\forall x \exists y$ (Greater$(y, x) \wedge$ Prime(y))
- $\forall x$ (Prime$(x) \rightarrow$ (Equal$(x, 2) \vee$ Odd(x)))
- $\exists x \exists y$(Equal$(x, y + 2) \wedge$ Prime$(x) \wedge$ Prime(y))

Domain:
Positive Integers

Even(x)
Odd(x)
Prime(x)
Greater(x,y)
Equal(x,y)

Statements with quantifiers

- "There is an odd prime"

Domain:
Positive Integers

- "If x is greater than two, x is not an even prime"
- $\forall x \forall y \forall z\ ((\text{Equal}(z, x+y) \wedge \text{Odd}(x) \wedge \text{Odd}(y)) \rightarrow \text{Even}(z))$
- "There exists an odd integer that is the sum of two primes"

Even(x)
Odd(x)
Prime(x)
Greater(x,y)
Equal(x,y)

Goldbach's Conjecture

- Every even integer greater than two can be expressed as the sum of two primes

Domain:
Positive Integers

Even(x)
Odd(x)
Prime(x)
Greater(x,y)
Equal(x,y)