CSE 321 Discrete Structures

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Lecture 04

Predicate Calculus

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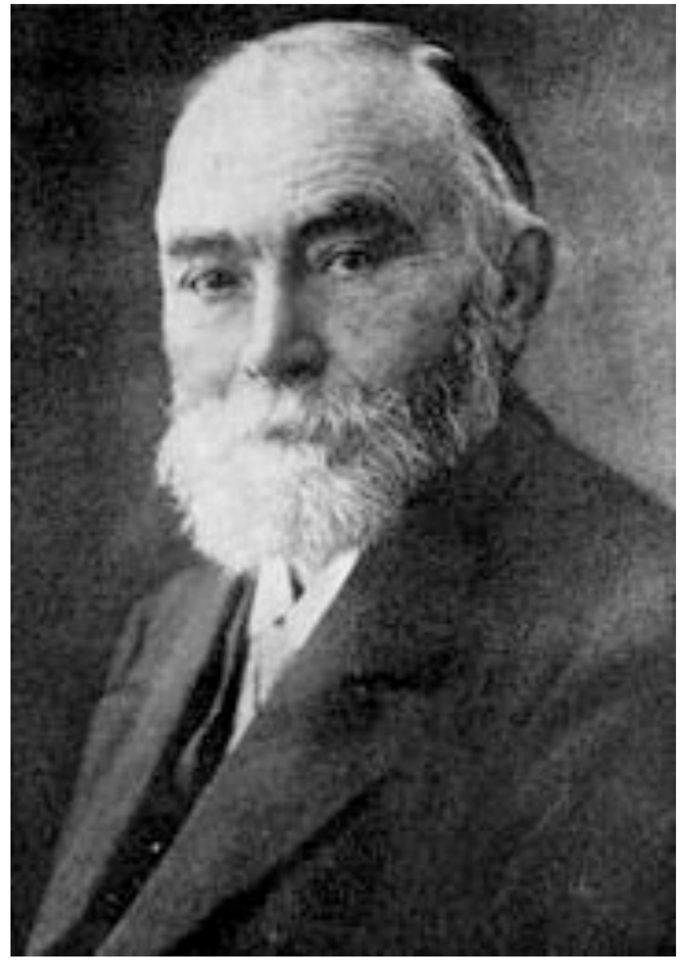
Adds variables to propositional calculus

Friedrich Ludwig Gottlob Frege



Nested Quantifiers

- Iteration over multiple variables
- Nested loops
- Details
  - Use distinct variables
    - $\forall x(\exists y(P(x,y) \rightarrow \forall x\, Q(y, x)))$
  - Variable name doesn't matter
    - $\forall x \exists y\, P(x, y) \equiv \forall a \exists b\, P(a, b)$
  - Positions of quantifiers can change (but order is important)
    - $\forall x\, (Q(x) \wedge \exists y\, P(x, y)) \equiv \forall x \exists y\, (Q(x) \wedge P(x, y))$

Quantification with two variables

Expression	When true	When false
∀ x ∀ y P(x,y)		
∃ x ∃ y P(x,y)		
∀ x ∃ y P(x, y)		
∃ x ∀ y P(x, y)		

Quantification with two variables

Expression	When true	When false
∀ x ∀ y P(x,y)	Forall values a,b, P(a,b) is true. Example: ∀ x ∀ y $x^2+y^2 \geq xy$	There exists a,b s.t. P(a,b) is false. Eg: ∀ x ∀ y. (x+y)=xy: false for x=3, y=4
∃ x ∃ y P(x,y)	There exists a,b s.t. P(a,b) is true. eg: ∃ x ∃ y . (x+y)=xy. Take x=2, y=2	When ∀ x ∀ y ¬P(x,y) is true. Example: ∃ x ∃ y. $x^2+y^2 < xy$
∀ x ∃ y P(x, y)	For all x we can find y. We can wait to see x before showing y. Example: ∀ x ∃ y. y > x+1	There is some x where we can't find a y s.t. P(x,y). Example (over integers):∀ x ∃ y. x=2*y.
∃ x ∀ y P(x, y)	There is some y s.t. forall x. P(x,y). E.g. ∃ y ∀ x. x*y=y (What is y???)	Forall x we can find y s.t. P(x,y) is false.

The Beers-Drinker-Bar Example

- Frequents(d,a) -- drinker d frequents bar a
- Serves(a,b) – bar a servers beer b
- Likes(d,b) – drinker d likes beer b
- Example:

Likes

D	B
Alex	Miller
Alex	Bud
Alex	Sam
Bob	Bud

Seves

A	Beer
Alibi Room	Bud
Alibi Room	Miller
Dahlia Lounge	Miller
Kells Irish	Bud

Freq

D	A
Alex	Alibi Room
Alex	Dahlia Lounge
Bob	Dahlia Lounge
Bob	Kells Irish

The drinkers-bars-beers example

Likes(drinker, beer)
Frequents(drinker, bar)
Serves(bar, beer)

All drinkers frequent <u>some</u> bar that serves <u>some</u> beer they like.

All drinkers frequent <u>only</u> bars that serves <u>some</u> beer they like.

Some drinkers frequent <u>some</u> bar that serves <u>only</u> beers they like.

Some drinkers frequent <u>only</u> bars that serves <u>only</u> beer they like.

The drinkers-bars-beers example

Likes(drinker, beer)
Frequents(drinker, bar)
Serves(bar, beer)

All drinkers frequent <u>some</u> bar that serves <u>some</u> beer they like.

$\forall x.\ \exists y.\ \exists z.\ \text{Frequents}(x, y) \wedge \text{Serves}(y,z) \wedge \text{Likes}(x,z)$

All drinkers frequent <u>only</u> bars that serves <u>some</u> beer they like.

$\forall x.\ \forall y.\ \text{Frequents}(x, y) \Rightarrow (\exists z.\ \text{Serves}(y,z) \wedge \text{Likes}(x,z))$

Some drinkers frequent <u>some</u> bar that serves <u>only</u> beers they like.

$\exists x.\ \exists y.\ \text{Frequents}(x, y) \wedge \forall z.(\text{Serves}(y,z) \Rightarrow \text{Likes}(x,z))$

Some drinkers frequent <u>only</u> bars that serves <u>only</u> beer they like.

$\exists x.\ \forall y.\ \text{Frequents}(x, y) \Rightarrow \forall z.(\text{Serves}(y,z) \Rightarrow \text{Likes}(x,z))$