CSE 321 Discrete Structures

February 22nd, 2010

Lecture 19: Probability Theory

Discrete Probability

Experiment: Procedure that yields an outcome

Sample space: Set of all possible outcomes

Event: subset of the sample space

S a sample space of equally likely outcomes,
E an event, the probability of E, $p(E) = |E|/|S|$

Example: Dice

Events:

What are their probabilities ?

Roll a 6-6

Roll 1-6

Roll two odd numbers

The sum of the two numbers is 7

Example: Poker

Probability of 4 of a kind

Combinations of Events

E^C is the complement of E

$P(E^C) = 1 - P(E)$

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Probability Concepts

- Probability Distribution
- Conditional Probability
- Independence
- Bernoulli Trials / Binomial Distribution
- Random Variable

Discrete Probability Theory

- Set S
- Probability distribution $p : S \rightarrow [0,1]$
  - For $s \in S$, $0 \leq p(s) \leq 1$
  - $\Sigma_{s\in S}\, p(s) = 1$

- Event E, $E \subseteq S$
- $p(E) = \Sigma_{s\in E}\, p(s)$

Conditional Probability

Let E and F be events with p(F) > 0. The conditional probability of E given F, defined by p(E | F), is defined as:

$$p(E \mid F) = \frac{p(E \cap F)}{p(F)}$$

Independence

The events E and F are independent if and only if
$p(E \cap F) = p(E)p(F)$

E and F are independent if and only if
$p(E \mid F) = p(E)$

Are these independent?

- Flip a coin three times
  - E: the first coin is a head
  - F: the second coin is a head
- Roll two dice
  - E: the sum of the two dice is 5
  - F: the first die is a 1
- Roll two dice
  - E: the sum of the two dice is 7
  - F: the first die is a 1
- Deal two five card poker hands
  - E: hand one has four of a kind
  - F: hand two has four of a kind