CSE 321 Winter 2010
Homework #3

Due Friday, January 30th at the start of lecture. Bring to class to turn in.

1. (10 Points) Section 4.1, problem 6. Give two solutions:
   - Using induction.
   - Using telescoping.
2. (10 Points) Section 4.1, problem 14. Use induction.
3. (10 Points) Section 4.1, problem 16. Use telescoping.
4. (10 Points) Section 4.1, problem 30. Use induction.
5. (10 Points) Section 4.3, problem 4.
6. (10 Points) Section 4.3, problem 8. Your recursive definitions should not use n, other than as an index in a_n or a_{n-1} (or $a_{n-2}, \ldots$). For example, for the sequence $a_n = n(n+1)$, the recursive definition $a_n = a_{n-1} + 2n$ is not acceptable because of the expression $2n$: you need to find an expression like $a_n = a_{n-1} \cdot a_{n-5}^2 + 3$ (not the real answer).
7. (10 Points) Section 4.3, problem 12. Use induction.
8. (10 Points) Section 4.3, problem 38.
9. (10 Points) Section 4.3, problem 40.
10. (10 Points) Section 4.3, problem 44.

Extra credit (10 points) The game of Nim is played as follows. There are n matches, placed in k rows. Two players take turn, and each player may take one ore more matches from a single row. The player who takes the last match wins. (See lecture notes for a discussion of the game of Nim for two rows only.)

Let a_i be the number of matches in row i: thus, $n = \sum_{i=1,k} a_i$. Express a_i in binary, and compute the exclusive-or of all values a_i:

$$S = a_1 \oplus a_2 \oplus \ldots \oplus a_k$$

Prove by strong induction on n the following statement: if $S = 0$ then the second player has a winning strategy, and if $S \neq 0$, then the first player has a winning strategy.

Example 1. Consider the Nim game with two rows, each with 14 matches: $a_1 = a_2 = 14 = 1110_2$, and $S = 1110 \oplus 1110 = 0000$. Player 2 has a winning strategy: we discussed this in class.

Example 2. Consider the Nim game with three rows, with 1, 3, 5. matches:

$$\begin{aligned} a_1 &= 1_2 \\ a_2 &= 11_2 \\ a_3 &= 101_2 \\ S &= a_1 \oplus a_2 \oplus a_3 = 001 \oplus 011 \oplus 101 = 111 \end{aligned}$$

Player 1 has a winning strategy.