Assignment #6
Due: Friday, Nov 21, 2008

## Reading Assignment: Sipser 2.2,2.3

1. Convert the following CFG into an equivalent CFG in Chomsky normal form:

$$\begin{array}{ccc} A & \to & BAB|B|\varepsilon \\ B & \to & 11|\varepsilon \end{array}$$

2. Find a pushdown automaton which recognizes the language

$$\{a^m b^n | n \le m \le 2n, m, n \ge 0\}$$

You may give your answer as a state diagram (do not use the shorthand we used in class for pushing multiple symbols onto the stack in your transition function.) You need not turn in a proof of correctness, though it would be good reassurance for yourself to do such a proof.

3. (a) Convert the following CFG into a PDA

$$S \to (S)|[0S]|SS|\varepsilon$$

For your answer you may give a state diagram, but you expand out all of the states and not use the shorthand we used in class for pushing multiple symbols onto the stack.

- (b) Now, for the PDA you have constructed, show a sequence of configurations (state and stack) which would cause your PDA to accept ()[0[0()[0]]].
- 4. Give a CFG for the language recognized by the following pushdown automata:

$$\begin{array}{c}
1,\varepsilon \to 1 & 2,1 \to \varepsilon & 3,\varepsilon \to \varepsilon \\
\hline
q_1 & \xrightarrow{\varepsilon,\varepsilon \to \$} & q_2 & \xrightarrow{\varepsilon,\varepsilon \to \varepsilon} & q_3 & \xrightarrow{\varepsilon,\$ \to \varepsilon} & q_4
\end{array}$$

$$\downarrow^{\varepsilon,\varepsilon \to \varepsilon} \qquad \downarrow^{\varepsilon,\varepsilon \to \varepsilon} \qquad \downarrow^{q_5} \xrightarrow{\varepsilon,\varepsilon \to \varepsilon} & q_6 & \xrightarrow{\varepsilon,\$ \to \varepsilon} & q_7$$

$$\downarrow^{2,\varepsilon \to \varepsilon} \qquad 3,1 \to \varepsilon$$

You may do this in any manner you like (i.e. you do not have to follow the method used in the book to convert PDAs to CFGs.)

- 5. For any language A, let  $PREFIX(A) = \{x | xy \in A \text{ for some string } y\}$ . Show that the class of context free languages is closed under the PREFIX operation. Assume you are working over a fixed alphabet  $\Sigma$ .
- 6. Extra Credit For the glory, not for the points! Prove that any grammar for the language  $A = \{a^i b^j c^k | i = j \text{ or } j = k\}$  must be ambiguous.