

Reading Assignment: Sipser 2.2,2.3

1. Find a pushdown automaton which recognizes the language

$$\{a^m b^n \mid n \leq m \leq 2n, m, n \geq 0\}$$

For the transition function, you may give a state diagram (do not use the shorthand we used in class for pushing multiple symbols onto the stack in your transition function.) You need not turn in a proof of correctness, though it would be good reassurance for yourself to do such a proof.

2. (a) Convert the following CFG into a PDA

$$S \rightarrow (S) \mid [0S] \mid SS \mid \varepsilon$$

For your answer you may give a state diagram, but you expand out all of the states and not use the shorthand we used in class for pushing multiple symbols onto the stack.

- (b) Now, for the PDA you have constructed, show a sequence of configurations (state and stack) which would cause your PDA to accept $() [0 [0 () [0]]]$.
3. For any language A , let $PREFIX(A) = \{x \mid xy \in A \text{ for some string } y\}$. Show that the class of context free languages is closed under the $PREFIX$ operation. Assume you are working over a fixed alphabet Σ .
4. Convert the PDA in Figure 2.19, Page 114 in Sipser's text (Fig 2.8, Page 106 in First edition) to a CFG.
5. **Extra Credit** For the glory, not for the points! Prove that any grammar for the language $A = \{a^i b^j c^k \mid i = j \text{ or } j = k\}$ must be ambiguous.
6. **Extra Credit** For the glory, not for the points! For a PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$, we say a string $a \in \Gamma^*$ is a possible stack of M if there is some input and some choice of moves of M such that a appears on the stack during its computation. Prove that the language $L \subseteq \Gamma^*$ of all possible stacks of M is regular.