

Finite State Automaton (FSA)

5 pieces

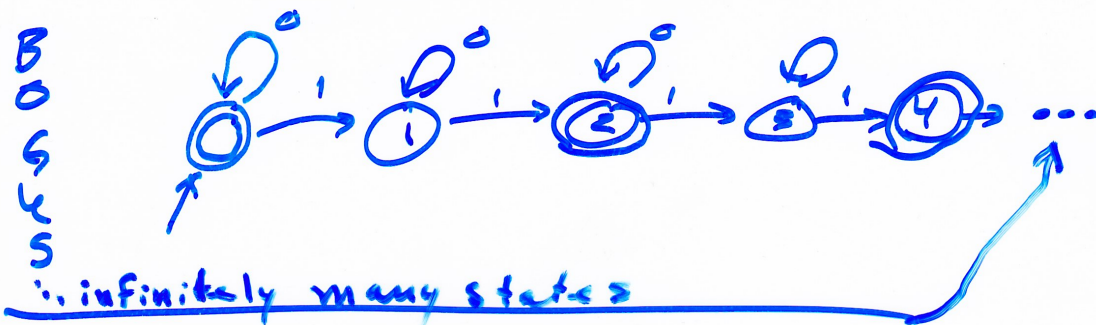
- States
- Alphabet
- Transitions
- Start
- Final or Accept

An Example

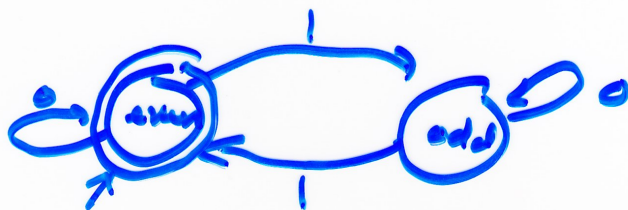
$$\Sigma = \{0, 1\}$$

$$L = \{ w \in \Sigma^* \mid \# \text{ of } 1\text{'s in } w \text{ is even} \}$$

The "obvious" algorithm: first count the 1's, then decide whether the count is even:



It works, but is not a finite state machine. This is:



Formal definition:

A finite state machine

$$M = (Q, \Sigma, \delta, q_0, F)$$

where

Q is a ^{finite} set (states)

$q_0 \in Q$ start state

Σ is a finite set (alphabet)

$F \subseteq Q$ Final states
Accepting states

$\delta : Q \times \Sigma \rightarrow Q$ transition
function
function

Formal version of the above example (done 2 different ways):

$$M_{\text{parity}} = (Q, \Sigma, \delta, q_0, F)$$

where

$$Q = \{\text{even}, \text{odd}\}$$

$$\Sigma = \{0, 1\}$$

$$q_0 = \text{even} \quad (\text{one element})$$

$$F = \{\text{even}\} \quad (\text{a set containing one element})$$

$$\delta(q, a):$$

q	a	
	0	1
even	even	odd
odd	odd	even

Even more succinctly
if we let $Q = \{0, 1\}$ also

$$\text{then } \delta(q, a) = (q+a) \bmod 2$$

$\forall q \in Q$
 $\forall a \in \Sigma$
2.9

Another example:

$$\Sigma = \{a, b\}$$

$$L = \{w \mid 3^{\text{rd}} \text{ letter from the right end of } w \text{ is 'a'}\}$$

aab \bar{Y}
 bab N
 abb \bar{Y}
 aaa \bar{Y}
 ab $) N$
 a $) N$
 ϵ $) N$

