Given a regular language L

& a string x how hard is it to decide

- $x \in L$ ?
- $L = \phi$ ?
- $L = \Sigma^*$ ?

A key issue: how is L (in general an infinite thing) "given" as input to our program? Some options

1. DFA
2. NFA
3. Reg. Exp.
4. A Java program

   ⋮

Does it matter?

| | $x \in L$? | $L = \emptyset$? | $L = \Sigma^*$? |
|---|---|---|---|
| DFA | Yes $O(n)$ | Yes $O(n)$ | Yes $O(n)$ |
| NFA | Yes $2^n$ | | |
| RegExp | Yes $2^n$ | | |
| Java program | ? | ? | ? |
| RegExp extend with ($\neg$) | Yes $\underbrace{2^{2^{\cdot^{\cdot}}}}_{\text{any fixed}} < T < \underbrace{2^{2^{\cdot^{\cdot}}}}_{n}$ | | |

$\neg(a \cup b)^*$

$\sim 10^{20{,}000}$

$2 \quad 2^2 = 4 \quad 2^{2^2} = 16 \quad 2^{2^{2^2}} = 2^{16} = 65000 \quad 2^{2^{2^{2^2}}} = 2^{65000}$

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# How much can we compute?

Visualize a fast small computer, say:

One petaflop ($10^{15}$ ops $\sec^{-1}$)

Femtometer ($10^{-15}$) in diameter (~ size of a neutron)

Buy a few: say enough to pack the visible universe

Radius of visible universe:

$10^{10}$ light years x $\pi$ x $10^7$ s/year x 3 x $10^8$ m/s = $10^{26}$ m

Volume: $(10^{26})^3 = 10^{78}$ $m^3$

# processors: $10^{78}/(10^{-15})^3 = 10^{123}$ (.1 yotta-googles)

Let it run for a little while, say $10^{10}$ years

$10^{10}$ yr x $\pi$ x $10^7$ s/yr x $10^{15}$ ops/s x $10^{123}$ processors

**= $10^{155}$ ops since the dawn of time**

(somewhat optimistically)

$$2,\ 2^2 = 4,\ 2^{2^2} = 2^4 = 16,\ \ 2^{2^{2^2}} = 2^{16} = 65536,\ \ 2^{2^{2^{2^2}}} \approx 10^{20000}$$

# Context-free languages

$\Sigma = \{ a, +, *, (, ) \}$

Rules

$E \to P + E$

$E \to P * E$

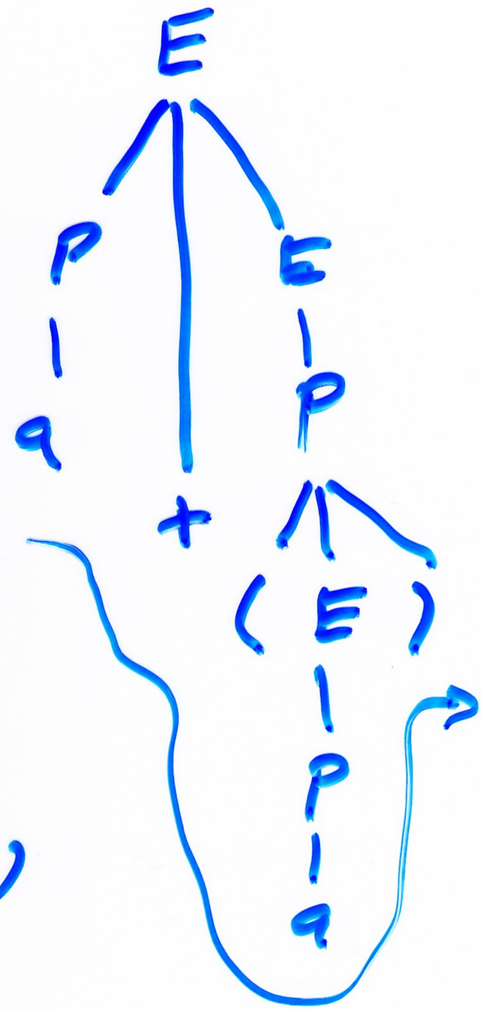
$E \to P$

$P \to (E)$

$P \to a$

a

(a)




a + (a)

A CFG $G = (V, \Sigma, R, S)$

$V$ a finite set ("variables")

$V \cap \Sigma = \emptyset$

$S \in V$

$R$ is a finite subset of $V \times (V \cup \Sigma)^*$

$\rightarrow$ in rules

$\Rightarrow$ "yeilds"

relation on strings in $(V \cup \Sigma)^*$

$\alpha A \beta \Rightarrow \alpha \gamma \beta$

if $A \rightarrow \gamma$ is a rule

for all $\alpha, \beta \in (V \cup \Sigma)^*$

$\Rightarrow^*$ "derives"

$\alpha \Rightarrow^* \beta$ ~~means~~ $\exists\ \alpha_1\ \alpha_2 \cdots \alpha_k$

$\alpha \Rightarrow \alpha_1 \Rightarrow x_2 \Rightarrow \alpha_3 \cdots \alpha_k \Rightarrow \beta$

$L(G) = \{ w \in \Sigma^* \mid S \Rightarrow^* w \}$