# PDA → CFG

I. WLOG, PDA:

a) has only 1 final state

b) accepts only when stack empty

c) all transitions either push or pop, never both/neither.

II. $\forall\, p, q \in Q$

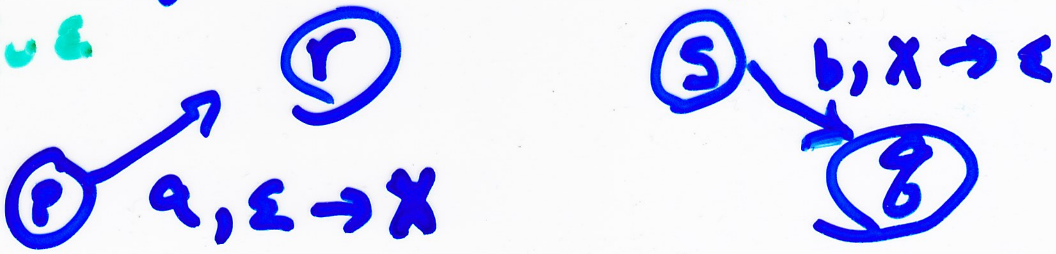
nonterminal $A_{pq}$

$A_{pp} \to \varepsilon \qquad \forall\, p \in Q$

$A_{pq} \to A_{pr} A_{rq} \quad \forall\, p,q,r \in Q$

$\forall\, a, b \in \Sigma \cup \varepsilon$

$X \in \Gamma$




$A_{pq} \to a\, A_{rs}\, b$

start $= A_{start,\,final}$

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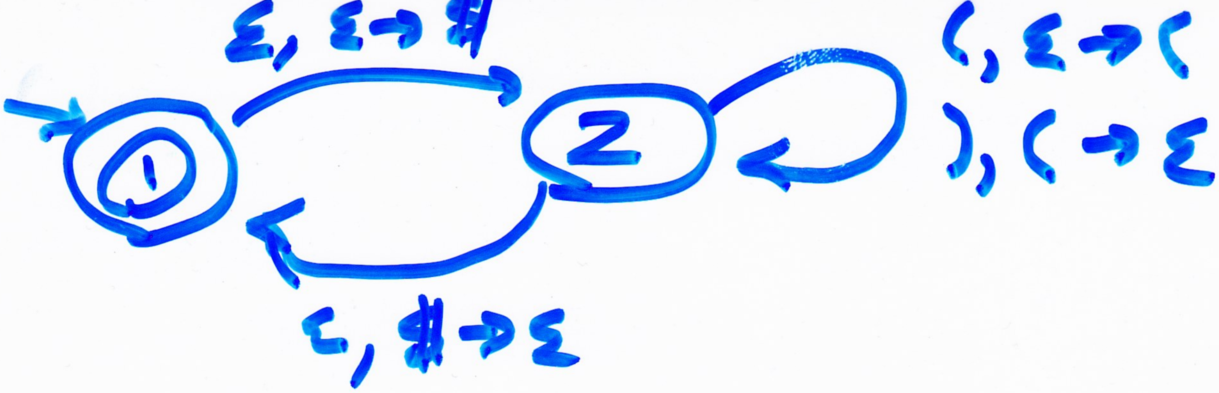



| p | a | x | r | s | b | q |
|---|---|---|---|---|---|---|
| 1 | ε | $ | 2 | 2 | ε | 1 |
| 2 | ( | ( | 2 | 2 | ) | 2 |

$A_{11} \to \varepsilon A_{22} \varepsilon$

$A_{22} \to ( A_{22} )$

$A_{11} \to \varepsilon$

$A_{22} \to \varepsilon$

$A_{11} \to A_{11} A_{11} \mid A_{12} A_{21}$

$A_{12} \to A_{11} A_{12} \mid A_{12} A_{22}$

$A_{21} \to A_{21} A_{11} \mid A_{22} A_{21}$

$A_{22} \to A_{21} A_{12} \mid A_{22} A_{22}$

NB: G can be simplified. Eg. remove $A_{12}$, $A_{21}$ & all rules using them since, eg. there is no $x \in \Sigma^*$ st. $A_{21} \Rightarrow^* x$. This is just what we want in the construction, since there is no $x$ st. $[2, \varepsilon, x] \vdash^* [1, \varepsilon, \varepsilon]$

**Claim** $\forall x \in \Sigma^* \quad A_{pq} \Rightarrow^* x$ $\forall p,q \in Q$

iff $[p, \varepsilon, x] \vdash^* [q, \varepsilon, \varepsilon]$

**Cor** $L(G) = L(M)$

Since $L(G) = \{x \mid A_{init,final} \Rightarrow^* x\}$

defn.

$= \{x \mid [init, \varepsilon, x] \vdash^* [final, \varepsilon, \varepsilon]\}$

by claim

$= L(M)$

defn.

claim ($\Longrightarrow$) induct on deriv length

basis

$A_{pq} \overset{0}{\Rightarrow} x$ : impossible; nothing to prove

$A_{pq} \overset{1}{\Rightarrow} x$ : must be $x=\varepsilon$, $p=q$

$$[p, \varepsilon, \varepsilon] \vdash^* [q, \varepsilon, \varepsilon]$$

Ind

$\Rightarrow^{k+1}$ either (i) $A_{pq} \to A_{pr} A_{rq} \Rightarrow^k x$

(ii) $\underbrace{A_{pq} \to a A_{rs}}_{*} b \Rightarrow^k x$

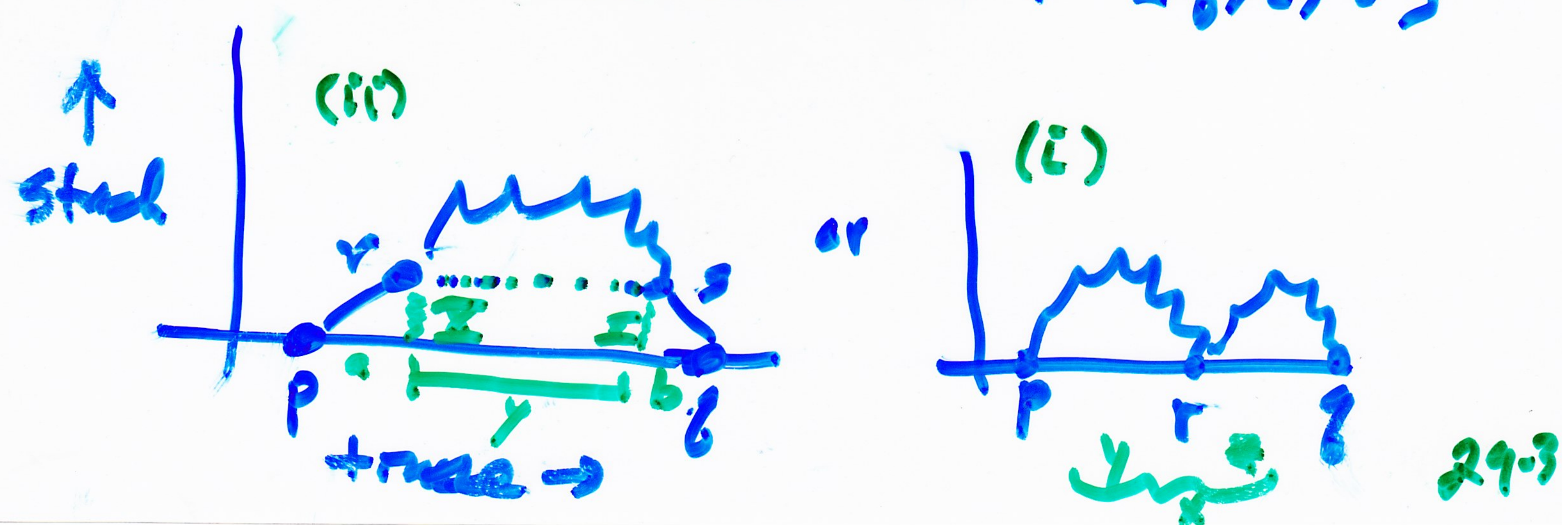
case (ii):

$$x = ayb \;\&\; A_{rs} \Rightarrow^k y$$

by Ind $[r, \varepsilon, y] \vdash^* [s, \varepsilon, \varepsilon]$

Since $\circledast$ in G

$$[p, \varepsilon, ayb] \vdash [r, X, yb] \vdash^* [s, X, b] \vdash [q, \varepsilon, \varepsilon]$$



$\Leftarrow$ direction of claim is similar, by induction on # of steps in $\vdash^*$

basis: 0 steps, use $\varepsilon$ rule in G

ind: $k+1 > 0$ steps, then

Stack either is (case i)

or is not (case ii) empty

at some intermediate step.

In case i, I.H. & construction

give $A_{pq} \to A_{pr} A_{rq}$ etc.

In case ii, $A_{pq} \to a A_{rs} b$ etc.

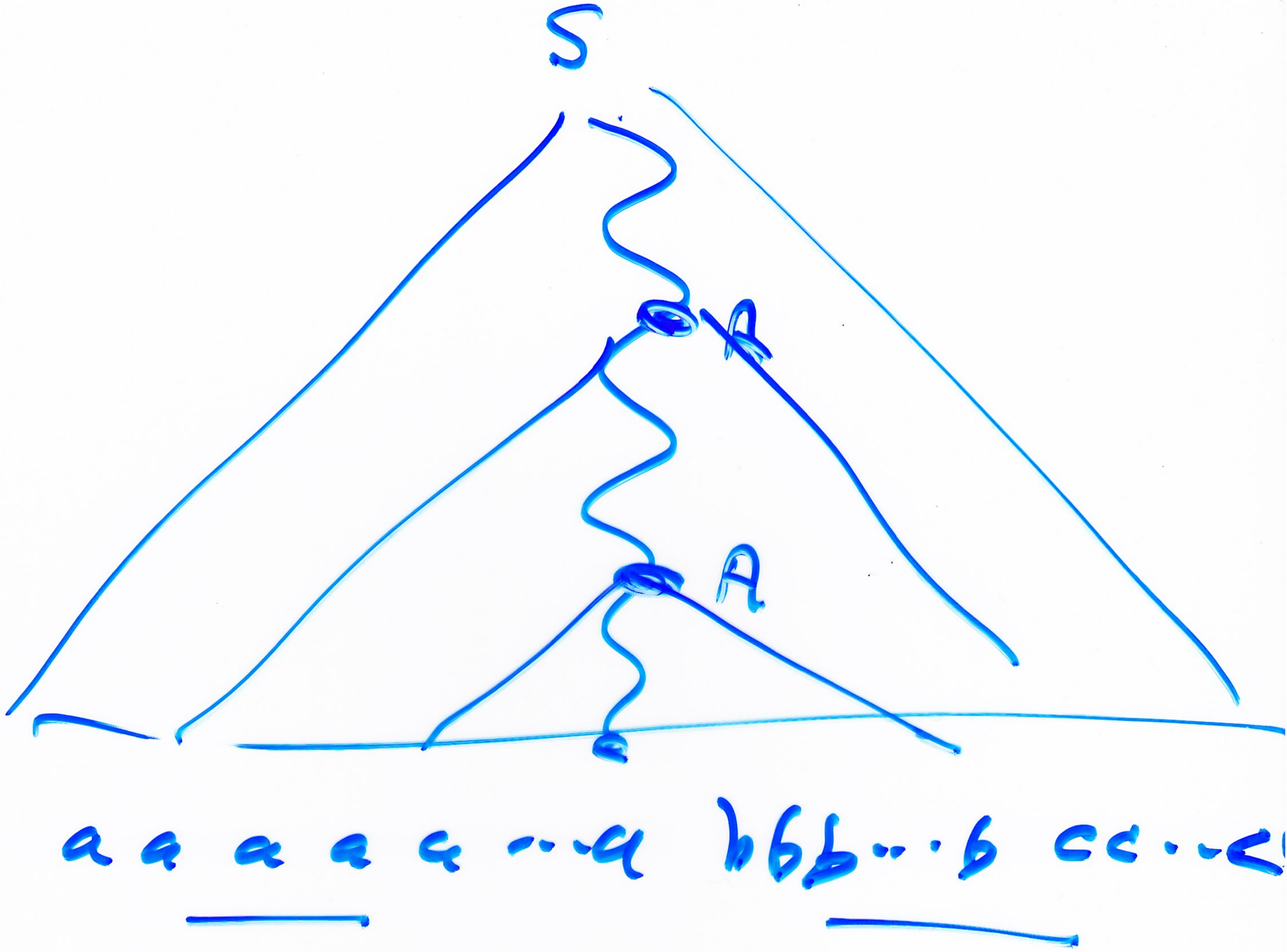
This construction & proof are just like the text's version, so more details there.

$$\{a^i b^j c^k \mid i=j \text{ or } i=k\}$$

$$\{a^n b^n c^n \mid n \geq 0\}$$

$$\{ww^R \mid w \in \{a,b\}^*\}$$

$$\{ww \mid w \in \{a,b\}^*\}$$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a a a a a ... a b b b ... b c c ... c