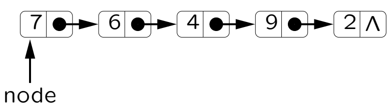
2: Asymptotic Analysis

CSE326 Spring 2002

April 1, 2002

Linked List Search




```
bool ListFind(int k, Node *node)
{



}
```


Analyzing Algorithms

```
bool ListFind(int k, Node *node)
{
  while (node) {
    if (node->key == k)
      return true;
    n = n->next;
  }

  return false;
}
```

- Best case:
- Worst case:
- Most of the time:

Array Search

7	6	4	9	2	1	3	6	7	5

↑

array; n=10

```
bool ArrayFind(int k, int *array, int n)
{



}
```


Analyzing Algorithms

```
bool ArrayFind(int k,
               int *key_array,
               int n)
{
  for (int i = 0; i < n; i++)
   if (key_array[i] == k)
    return true;
  return false;
}
```

- Best case:
- Worst case:
- Most of the time:

Binary Search

7	6	4	9	2	1	3	6	7	5

```
bool BinarySearch(int k, int *array, int high, int low=0)
{
   assert(low >= 0);

   if (low >= high)
     return false;

   int mid = (high-low)/2;
   if (k == array[mid])
     return true;
   else if (k < array[mid])
     return BinarySearch(k, array, mid, low);

   // k > array[mid]
   return BinarySearch(k, array, high, mid+1);
}
```

Analyzing Algorithms

```
bool BinarySearch(int k,
                  int *array,
                  int high,
                  int low=0)
{
  if (low >= high)
    return false;

  int mid = (high-low)/2;
  if (k == array[mid])
    return true;
  else if (k < array[mid])
    return BinarySearch(k, array, mid, low);

  // k > array[mid]
  return BinarySearch(k, array, high, mid+1);
}
```

- Best case:
- Worst case:

Solving the Recurrence Relation

Analysis Summary

	List	Array (linear search)	Binary
Best Case			
Worst Case			
Most of the Time			

Which algorithm is best?

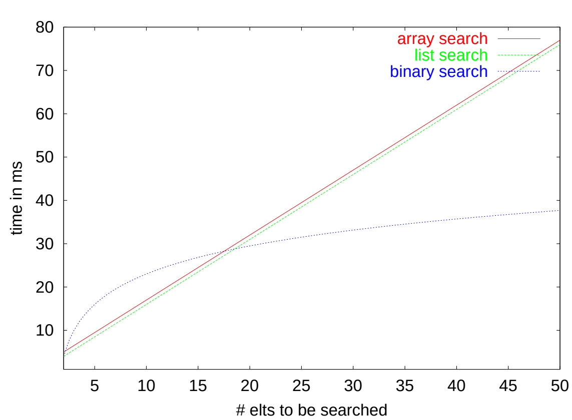
Criteria for chosing an algorithm:

- Speed of execution
- Ease of coding
- Preprocessing required

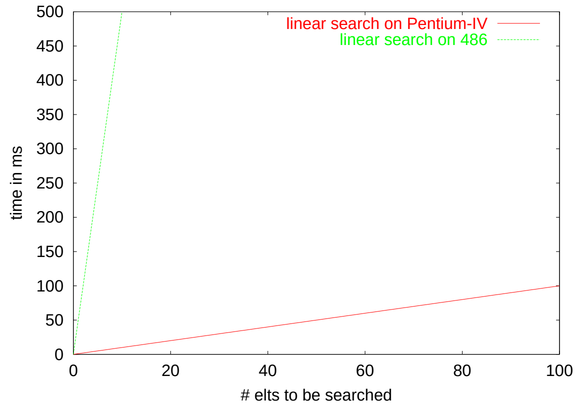
Speed is usually most important—easiest to quantify.

Best was to compare speeds is to measure and then graph.

The Need for Speed

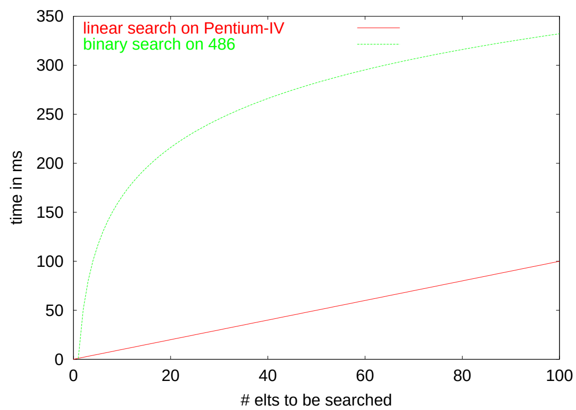


Fast Computer vs. Slow Computer



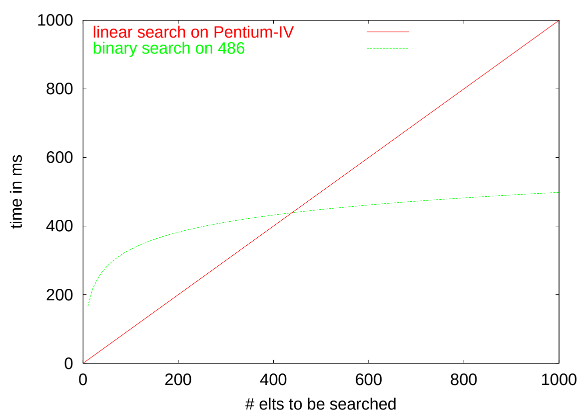
With same algorithm, the faster computer always wins.

Fast Computer vs. Smart Programmer I



Linear search beats binary search?

Fast Computer vs. Smart Programmer II



Binary search always wins—eventually.

Asymptotic Analysis

- Asymptotic analysis looks at the *order* of the running-time of an algorithm.
  - What happens when the input gets *large*?
  - Ignore *effects of different machines*.
- Linear search is $O(n)$ (whether on a list or an array!).
- Binary search is $O(\log n)$.

Order Notation: Intuition

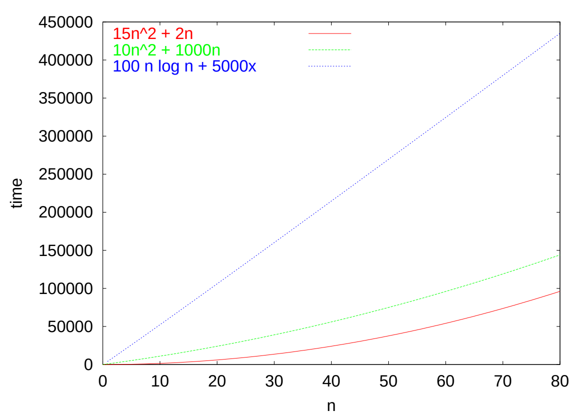
Suppose we have 3 algorithms running at the following speeds:

- $100n \log n + 5000n$
- $10n^2 + 1000n$
- $15n^2 + 2n$

Which algorithm is fastest?

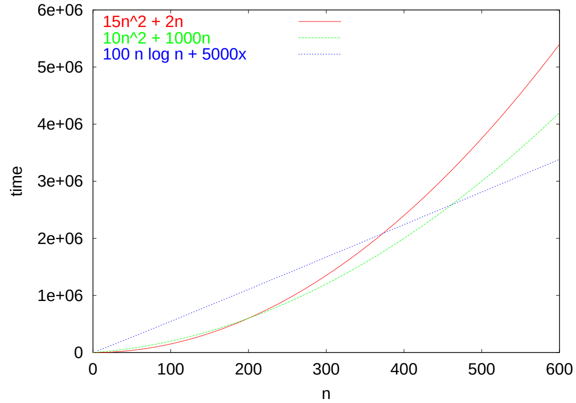
$\iff$ *Which function grows slowest?*

Order Notation: Intuition



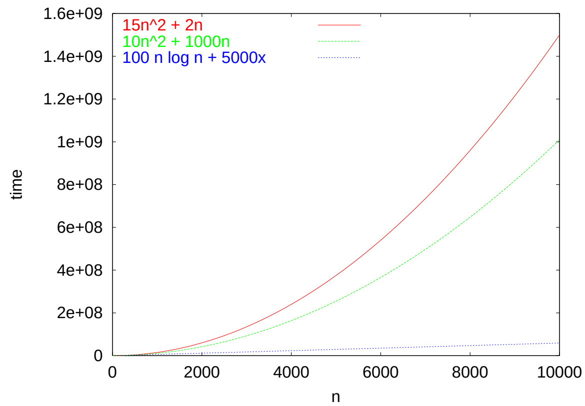
$15b^2 + 2n \prec 10n^2 + 1000n \prec 100n \log n + 5000n$?

Order Notation: Intuition



$100n \log n + 5000n \prec 10n^2 + 1000n \prec 15n^2 + 2n$

Order Notation: Intuition



$O(n \log n)$ vs. $O(n^2)$ is what matters

Order Notation

Precise Definition

- $O(f(n))$ is a *set of functions*
- $g(n) \in O(f(n))$ when

  There exist c and n_0 such that

$$g(n) \leq c \cdot f(n), \text{ for all } n \geq n_0.$$

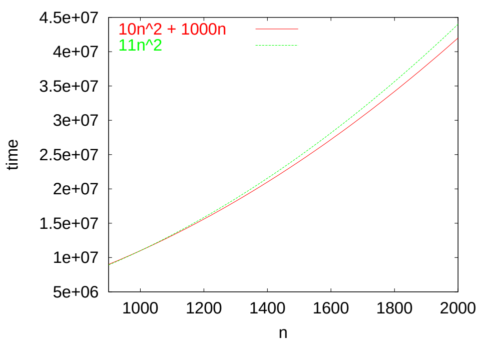
Order Notation: Example

- $10n^2 + 1000n \leq 1 \cdot (15n^2 + 2n)$ for $n \geq 250$,

  $\Rightarrow 10n^2 + 1000n \in O(15n^2 + 2n)$

- $10n^2 + 1000n$ vs. n^2?

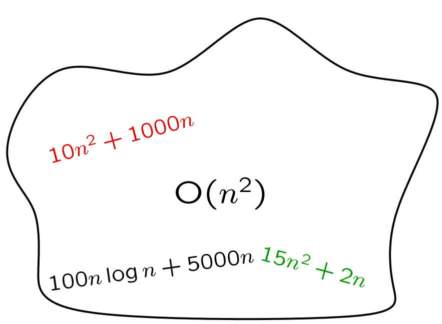
Order Notation: Definition



$10n^2 + 1000n \leq 11 \cdot n^2$ for $n \geq 1200$,

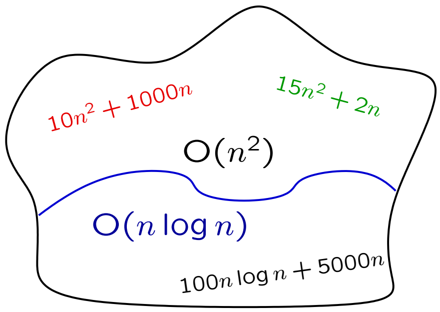
$\Rightarrow 10n^2 + 1000n \in O(n^2)$

Order Notation



$O(15n^2 + 2n) = O(n^2)$

Order Notation



$$O(n \log n) \subset O(n^2)$$

Hierarchy of Orders

↑ bigger / smaller ↓

$O(2^n)$	
$O(n^k)$	(for constant k)
$O(n^3)$	
$O(n^2)$	
$O(n^{1+\epsilon})$	(for constant $\epsilon > 0$)
$O(n \log n)$	
$O(n^\epsilon)$	(for constant $\epsilon > 0$)
$O(\log^k n)$	(for constant k)
$O(\log n)$	
$O(\log \log n)$	
$O(1)$	

$\Omega(\cdot)$,$\Theta(\cdot)$

- $O(f(n))$ is all functions asymptotically *less than or equal* to $f(n)$.
  "Big Oh"
  - $4n^2 \in O(n^2)$
  - $\log n \in O(n^2)$
  - $n^3 \notin O(n^2)$
- $\Omega(f(n))$ is all functions asymptotically *greater than or equal* to $f(n)$.
  "Big Omega"
  - $4n^2 \in \Omega(n^2)$
  - $\log n \notin \Omega(n^2)$
  - $n^3 \in \Omega(n^2)$
- $\Theta(f(n))$ is all functions asymptotically *equal* to $f(n)$
  "Big Theta"
  - $4n^2 \in \Theta(n^2)$
  - $\log n \notin \Theta(n^2)$
  - $n^3 \notin \Theta(n^2)$

Menagerie of Symbols

Asymptotic Notation	Mathematics Relation
O	$\le$
Ω	$\ge$
Θ	$=$
o	$<$
ω	$>$

Rules of Thumb

- Polynomials

  Take term with highest power, drop coefficient.

$$45n^4 + 20n^2 + 45n + 60 \in O(n^4)$$
$$34n^2 + 16n^{0.1} + 784 \log n + 2 \in O(n^2)$$

- Logorithms

  Take log-term with highest power, drop coefficients and powers in argument.

$$50 \log n^{200} \in O(\log n)$$
$$32 \log^2 n + \log n^4 + \log \log n^2 \in O(\log^2 n)$$