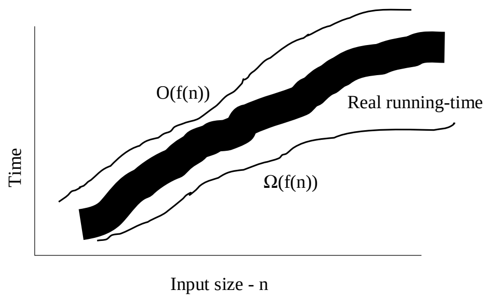
CSE 326 Quiz Section: Analyzing Analysis

April 18, 2002

"Plain" Algorithmic Analysis

- Algorithm *Foo* is O(f(n))
  - For all inputs, *Foo* takes at most cf(n) steps.
  - Upper-bound is f(n)
- Algorithm *Foo* is Ω(f(n))
  - For all inputs, *Foo* takes at least cf(n) steps.
  - Lower-bound is f(n)
- Algorithm *Foo* is Θ(f(n))
  - For all inputs, *Foo* takes approximately cf(n) steps.
  - Lower-bound and upper-bound are both f(n)

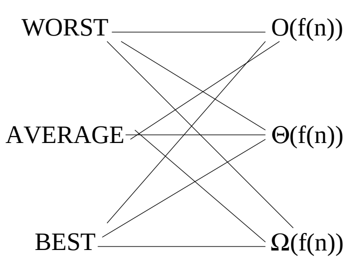
Visual Representation: Plain



Worst, Best, Average… Oh My!

WORST	O(f(n))
AVERAGE	Θ(f(n))
BEST	Ω(f(n))

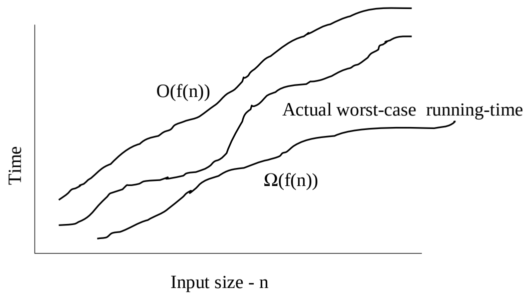
Worst, Best, Average… Oh My!



Worst-case Analysis

- Idea: What is the most work algorithm *Foo* will ever have to do?
- What the bounds mean for worst-case analysis:
  - O(f(n)): For any input, *Foo* takes at most cf(n) time
  - Ω(f(n)): There exists an input of length n such that *Foo* takes at least cf(n) time
  - Θ(f(n)): There exists an input of length n such that *Foo* takes at least cf(n) time and no other input of length n takes more than df(n) time.

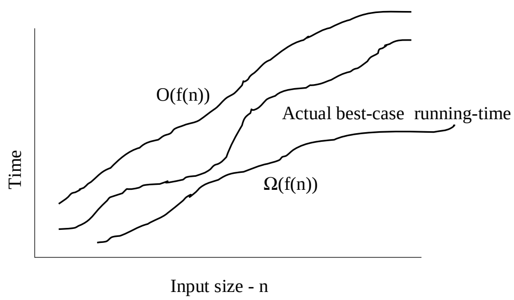
Visual Representation: Worst



Best-case Analysis

- Idea: What is the least work algorithm *Foo* will ever have to do?
- What the bounds mean for worst-case analysis:
  - O(f(n)): There exists an input of length n such that *Foo* takes at most cf(n) time
  - Ω(f(n)): For any input, *Foo* takes at least cf(n) time
  - Θ(f(n)): There exists an input of length n such that *Foo* takes at most cf(n) time and no other input of length n takes less than df(n) time.

Visual Representation: Best



Average-case Analysis

- The book calls this "expected analysis"
- IDEA: On average, how much work will *Foo* do?
- The method: for a fixed input size n
  compute T(n) for all inputs
  take the average of all these T(n)
- O(f(n)) and Ω(f(n)) act just like mathematical upper and lower bounds.

Real-World Expected Analysis

- In a purely mathematical sense, which is more likely of an input to sort?

1,2,4,6,8,7,11,10,12,13

or

3,7,5,1,9,12,8,6,4,10

- What about more likely in the real world?

Real-World Expected Analysis

IDEA: Reflect the actual probability of inputs in the cost analysis

1. Fix input size n
2. For each input i of size n, assign a probability of it occurring
3. Compute

$$\sum_{\text{input } i} P(i) \cdot (\text{Time cost of } i)$$

Expected Analysis

- Expected analysis usually refers to analyzing the performance of a randomized algorithm
- Randomized algorithms involve random choices in their operations, meaning the amount of time spent for one input can vary from run to run
- Idea for the analysis:
  - average time for a randomized algorithm over different random seeds for any input

Now into some reality

	Sorted Linked List	Unsorted Array	Sorted Array
Find	O(n)	O(n)	O(log n)
Insert	O(n)	O(1) or O(n)?	O(n)
Delete w/ Find	O(n)	O(n)	O(n)
Delete w/o Find	O(1)	O(1)	O(n)

Stretchy Array Implementation

Best case insert = O(1)

Worst case insert = O(n)

```
int * data;
int maxsize, end;

insert(e){
  if (end == maxsize){
     temp = new int[2*maxsize];
     for (i=0;i<maxsize;i++)
        temp[i]=data[i];
     delete data;
     data = temp;
     maxsize = 2*maxsize;
  }
  data[++end] = e;
}
```

Inserting in an Unsorted Array

- Inserting is usually O(1) time
- Stretching the array takes O(n) time
- Does inserting always take linear time?

Amortized Analysis

- Consider any sequence of operations applied to a data structure
  - *your worst enemy could choose the sequence!*
- Some operations may be fast, others slow
- Goal: show that the average time per operation is still good

$$\frac{\text{total time for n operations}}{n}$$

Stretchy Array Amortized Analysis

- Consider sequence of n operations
  insert(3); insert(19); insert(2); …
- What is the max number of stretches?
- What is the total time?
  - let's say a regular insert takes time *a*, and stretching an array contain *k* elements takes time *bk*.
- Amortized time =

Stretchy Array Amortized Analysis

- Consider sequence of n operations
  insert(3); insert(19); insert(2); …
- What is the max number of stretches? log n
- What is the total time?
  - let's say a regular insert takes time *a*, and stretching an array contain *k* elements takes time *bk*.

$$an + b(1 + 2 + 4 + 8 + ... + n) = an + b\sum_{i=0}^{\log n} 2^i$$

- Amortized time =

Stretchy Array Amortized Analysis

- Consider sequence of n operations
  insert(3); insert(19); insert(2); …
- What is the max number of stretches? log n
- What is the total time?
  - let's say a regular insert takes time *a*, and stretching an array contain *k* elements takes time *bk*.

$$an + b(1 + 2 + 4 + 8 + ... + n) = an + b\sum_{i=o}^{\log n} 2^i$$

$$= an + b(2n - 1)$$

- Amortized time = $(an+b(2n-1))/n$ = O(1)