Data Structures: Practice Midterm

(closed book, closed notes)

Answer each question in the space provided.
Ask if you need additional blank sheets.
Write your name and student ID on each sheet.
Do the easy questions first!

Total: 100 points, 5 questions. Time: 50 minutes.

1. **Mathematical Background** (a, b, and c: 5, 10, and 10 points)

   a. Complete the following definition:
   f(N) is O(g(N)) if there are positive constants c and N_0 such that

   ____

   b. Show that 373N+100 is O(N) (by selecting appropriate constants c and N_0).

   c. Suppose T(N) is the running time of the following recursive function for input integer N. Which *two* of the following statements are true and why?

   (i) T(N) is $O(2^N)$
   (ii) T(N) is $\Omega(\log N)$
   (iii) T(N) is $\Theta(N)$
   (iv) T(N) is $o(2^N)$

```
int FunWith(N){
if (N == 0) return 1;    /* 1 */
else return FunWith(N-1) + FunWith(N-1);} /* 2 */
```

2. **Trees and Stacks** (a, b, and c: 5, 5, and 10 points)

   a. Draw the final tree that results from inserting the integers 5, 2, 4, 3, 9, 12, 6 (in that order) into an empty binary search tree with no balance conditions.

   b. What is the sequence of elements that results from a preorder traversal of your tree in part (a)?

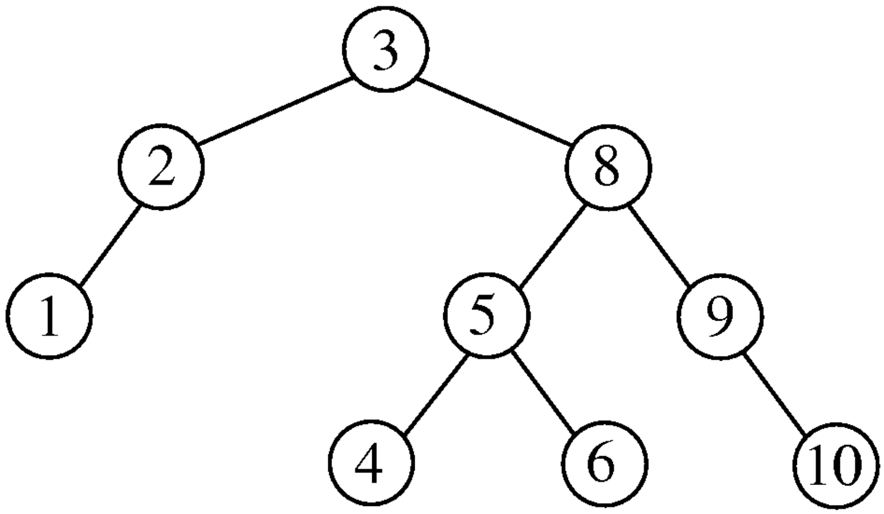
   c. Fill in the blanks in the following routine for preorder traversal of a binary tree using a stack. Choose one of the following to fill in each blank: `pop(S), pop(T), pop(T -> Left), pop(T -> Right), push(T -> Left, S), push(T -> Right, S), push(T,S)`

```
void Stack_Preorder (Tree T, Stack S) {
if (T == NULL) return; else ______________________;
while (!isempty(S)) {
   T = _______________________;
   print_element(T -> Element);
   if  (T -> Right !=  NULL) ________________________;
   if  (T -> Left !=  NULL) ________________________;
   }
}
```

3. **Binary Search Trees** (a and b: 5 and 10 points)

   a. What is the worst case running time for the Find operation on a tree of N nodes when you use: (i) an unbalanced binary search tree, (ii) an AVL tree, and (iii) a splay tree? Select one of the following for each: O(1), O(log N), O($\sqrt{N}$), O(N), O(N log N) (choose the best possible upper bound).

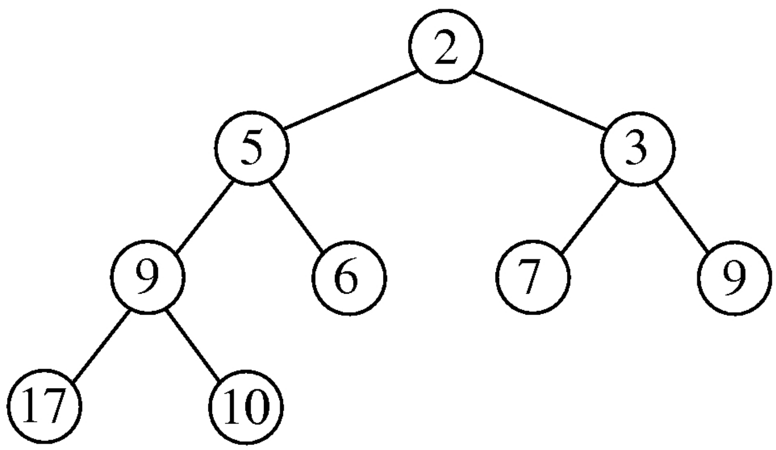
b. Draw the tree that results from inserting 11 followed by 7 into the following AVL tree:



4. **Binary Heaps and Binomial Queues** (a, b, and c: 5, 10, and 10 points)

   a. What are the two properties that make a binary tree a binary heap?

   b. Draw the binary heap that results from deleting the minimum and then inserting 4 into the following binary heap:



c. Draw the binomial queue that results from inserting the integers 1, 2, 3, 4, 5, 6, 7 (in that order) into an empty binomial queue and then deleting the minimum.

5. **Hashing** (a, b, and c: 5 points each)

Consider the hash function Hash(X) = X mod 10 and the ordered input sequence of keys 51, 23, 73, 99, 44, 79, 89, 38. Draw the result of inserting these keys in that order into a hash table of size 10 (cells indexed by 0, 1, …, 9) for each of the following collision resolution strategies:

a. separate chaining;
b. open addressing with linear probing, where $F(i) = i$;
c. open addressing with quadratic probing, where $F(i) = i^2$.