CSE 326 Lecture 16: All sorts of sorts

- What's on our plate today?
  - Sorting Algorithms: The Best of the Fastest…
    - Heapsort
    - Mergesort
    - Quicksort
- Covered in Chapter 7 of the textbook

Review of Sorting Algorithms

- "Simple" sorts
  - Bubblesort, Selection Sort, and Insertion Sort
  - Run Time = $O(N^2)$
- Insertion Sort: O(N) if elements already sorted
- Shellsort
  - Works by running Insertion sort on subsets of elements over several passes
  - $O(N^{1.5})$ using Hibbard's increment sequence

Review of Sorting Algorithms

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Canya beat $O(\mathbf{N}^{1.5})$ usin' a Binary Search Tree to sort?

Using Binary Search Trees for Sorting

- Can we beat $O(\mathbf{N}^{1.5})$ using a BST to sort N elements?
  - Yes!!
  - Insert each element into an initially empty BST
  - Do an In-Order traversal to get sorted output
- Running time = ?

Using Binary Search Trees for Sorting

- Can we beat $O(N^{1.5})$ using a BST to sort N elements?
  - Yes!!
  - Insert each element into an initially empty BST
  - Do an In-Order traversal to get sorted output
- Running time = N Inserts, each takes O(log N) time, plus O(N) for In-Order traversal = **O(N log N)** = $o(N^{1.5})$
- Any Drawbacks?

Using Binary Search Trees for Sorting

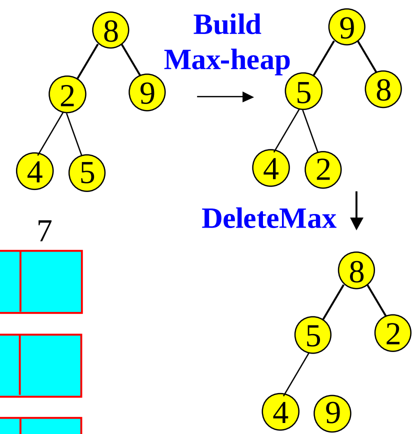
- Drawback: Uses Extra Space
  - Need to allocate space for tree nodes and pointers
  - O(N) extra space needed, not *in place* sorting

Waittaminute…what if the tree is complete, and we use an array representation – can we sort in place?

Recall your favorite data structure with the initials B. H.

Using a Binary Heap for Sorting

- Main Idea:
  - Build a max-heap
  - Do N DeleteMax operations and store each Max element in the unused end of array



0	1	2	3	4	5	6	7
8	2	9	4	5			

0	1	2	3	4	5	6	7
9	5	8	4	2			

0	1	2	3	4	5	6	7
8	5	2	4	9			

Largest element in correct place

Heapsort: Analysis

- Heapsort is in-place…is it also stable?
- Running time = time needed for building max-heap + time for N DeleteMax operations = ?

Heapsort: Analysis

- Running time = time to build max-heap + time for N DeleteMax operations = O(N) + N O(log N) = **O(N log N)**
- Can also show that running time is Ω(N log N) for some inputs, so *worst case* is **Θ(N log N)**
- *Average case* running time is also O(N log N) (see text for proof)

How about a "Divide and Conquer" strategy?

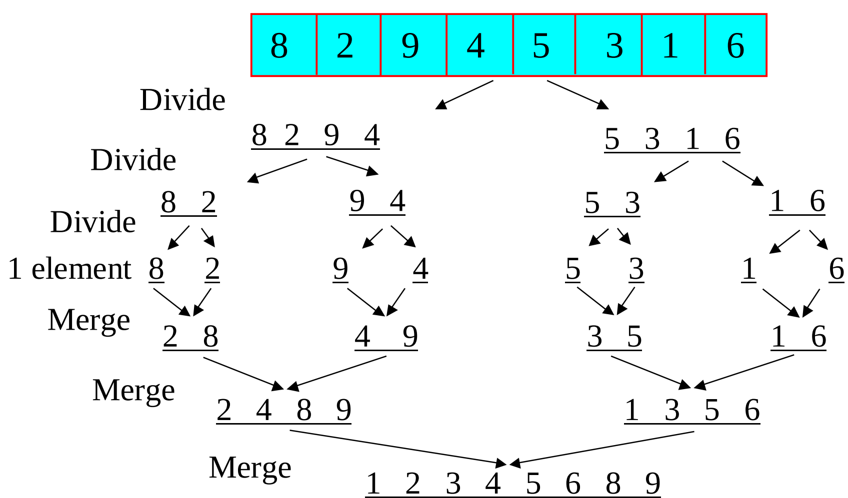
- Very important strategy in computer science:
  1. Divide problem into smaller parts
  2. Independently solve the parts
  3. Combine these solutions to get overall solution

How about a "Divide and Conquer" strategy?

- **Idea**: Divide array into two halves, *recursively* sort left and right halves, then *merge* two halves
  - Known as **Mergesort**
- Example: Mergesort this input array:

0	1	2	3	4	5	6	7
8	2	9	4	5	3	1	6

Mergesort Example



Mergesort Analysis

- Mergesort divides array in half and calls itself on the two halves. After returning, it merges both halves using a temporary array (see textbook for code).
- Is Mergesort stable? In-place?
- Let T(N) be the running time for an array of N elements
- Recurrence relation for run time = ?

Mergesort Analysis

- Let T(N) be the running time for an array of N elements
- Mergesort divides array in half and calls itself on the two halves. After returning, it merges both halves using a temporary array (see textbook for code).
- Each recursive call takes T(N/2) and merging takes O(N)
- Therefore, the recurrence relation for T(N) is:
  - T(1) = O(1) (Base case: 1 element array = constant time)
  - T(N) = 2T(N/2) + N
- What is T(N) as a *big-oh function* of N?

Squeezing the big-oh out of our recurrence…

- Can solve the recurrence by expanding the terms:
  T(N) = 2*T(N/2) + N
  - T(N/2) = 2*T(N/4) + N/2, T(N/4) = … etc. Therefore:
  - T(N) = 2*[2*T(N/4) + N/2] + N
    = 2^2*T(N/2^2) + 2*N
    = 2^2[2*T(N/8) + N/4] + 2*N
    = 2^3*T(N/2^3) + 3*N
    … (recall that $2^{\log N} = N$)
    = $2^{\log N}$*T(N/$2^{\log N}$) + (log N)*N
    = N * T(1) + N log N
    = N * O(1) + N log N = O(N log N)
  - T(N) = O(N log N)

Being Quick without taking up Space…

- Mergesort requires temporary array for merging = O(N) extra space – can we do in place sorting without extra space?
  - Want a divide and conquer strategy that does not use the O(N) extra space
- Enter…"**Quicksort**":
  Idea:
  Partition the array such that Elements in left sub-array < elements in right sub-array.
  Recursively sort left and right sub-arrays

How do we partition the array?

- Choose an element from the array as the pivot
- Move all elements < pivot into left sub-array and all elements > pivot into right sub-array
  - If element = pivot, can be handled in several ways

7   18   2   15   9   11

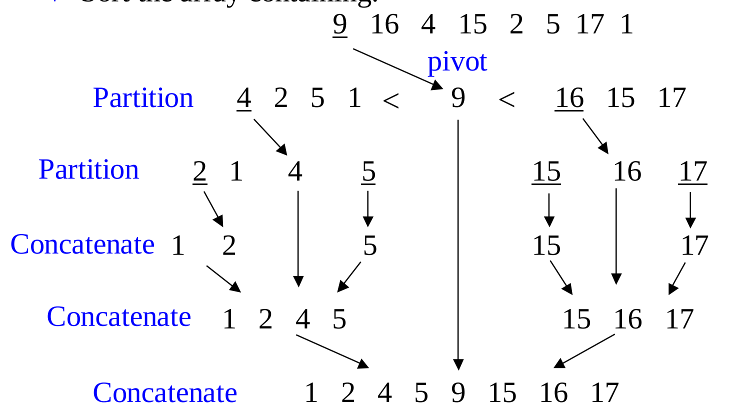
  - Suppose pivot = 7
  - Left subarray = 2   Right sub-array = 18   15   9   11

Now we are ready to Quicksort

- **Quicksort Algorithm**:
  1. Partition array into left and right sub-arrays such that: Elements in left sub-array < elements in right sub-array
  2. Recursively sort left and right sub-arrays
  3. Concatenate left and right sub-arrays with pivot in middle
- **How to Partition the Array**:
  1. Choose an element from the array as the pivot
  2. Move all elements < pivot into left sub-array and all elements > pivot into right sub-array
- Pivot? One choice: use first element in array

Quicksort Example

- Sort the array containing:



Partitioning In Place

- Hmmm…seems like we need an *extra array* for partitioning and concatenating left/right sub-arrays
  - No!
- Algorithm for In Place Partitioning:
  1. Swap pivot with last element: swap A[pivot] and A[N-1]
  2. Set pointers i and j to beginning and end of array
  3. Increment i until you hit an element A[i] > pivot
  4. Decrement j until you hit an element A[j] < pivot
  5. Swap A[i] and A[j]
  6. Repeat until i and j cross (i exceeds or equals j)
  7. Swap pivot and A[i]
- Example: Partition in place:
  9 16 4 15 2 5 17 1 (pivot = A[0] = 9)

The Pivotal Role of Pivots

- How do we pick the pivot for each partition?
  - Pivot choice can make a big difference in run time
- First Idea: Pick the *first* element in (sub-)array as pivot
  - What if it is the smallest or largest?
  - What if the array is sorted? How many recursive calls does quicksort make?

<u>2</u> 4 6 8 9

2 <u>4</u> 6 8 9

Choosing the Right Pivot

- 2nd Idea: Pick a *random* element
  - Gets rid of asymmetry in left/right sizes
  - But…requires calls to pseudo-random number generator – expensive/error-prone
- Third idea: Pick *median* (N/2th largest element)
  - Ideal but hard to compute without sorting!
  - Compromise: Pick median of three elements

9 16 4 <u>15</u> 2

2 4 9 <u>15</u> 16

Median-of-Three Pivot

- Find the median of the first, middle and last element

  2 4 9 15 16 → 9

  5 4 2 15 16 → 5

- Takes only O(1) time and not error-prone like the pseudo-random pivot choice
- Less chance of poor performance as compared to looking at only 1 element
- For sorted inputs, splits array nicely in half each recursion
  - Good performance

Quicksort Performance Analysis

- Best Case Performance: Algorithm always chooses best pivot and keeps splitting sub-arrays in half at each recursion
  - T(0) = T(1) = O(1) (constant time if 0 or 1 element)
  - For N > 1, 2 recursive calls + linear time for partitioning
  - Recurrence Relation for T(N) = ?
  - Big-Oh function for T(N) = ?

Quicksort Performance Analysis

- Best Case Performance: Algorithm always chooses best pivot and keeps splitting sub-arrays in half at each recursion
  - T(0) = T(1) = O(1)   (constant time if 0 or 1 element)
  - For N > 1, 2 recursive calls + linear time for partitioning
  - T(N) = 2T(N/2) + O(N)   (Same as Mergesort)
  - T(N) = O(N log N)
- Worst Case Performance: What is the worst case?

Quicksort Performance Analysis

- Worst Case Performance: Algorithm keeps picking the worst pivot – one sub-array empty at each recursion
  - T(0) = T(1) = O(1)
  - Recurrence relation for T(N) = ?
  - Big-Oh function for T(N) = ?

Quicksort Performance Analysis

- Worst Case Performance: Algorithm keeps picking the worst pivot – one sub-array empty at each recursion
  - ➩ $T(0) = T(1) = O(1)$
  - ➩ $T(N) = T(N-1) + O(N)$

    $= T(N-2) + O(N-1) + O(N) = \ldots$

    $= T(0) + O(1) + \ldots + O(N)$
  - ➩ $T(N) = O(N^2)$
- Fortunately, *average case performance* is O(N log N) (see text for proof)

Can We Sort Any Faster?

- Heapsort, Mergesort, and Quicksort all run in O(N log N) best case running time
- Can we do any better?
- Can Joey Sortiepants from Hackersville, USA come up with an O(N) sorting algorithm?

Questions to ponder over the Weekend

How fast can one sort?

Can I find time to read Chapter 7?

What was the meaning of the midterm?

What is the meaning of life? (extra credit)

Have a great weekend!