## CSE 326 Lecture 8: Getting to know AVL Trees

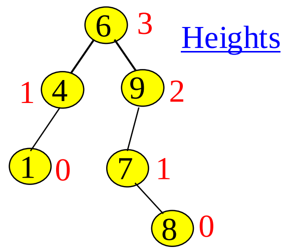
- Today's Topics:
  - Balanced Search Trees
    - AVL Trees and Rotations
    - Splay trees
- Covered in Chapter 4 of the text



## Recall from Last Time: AVL Trees

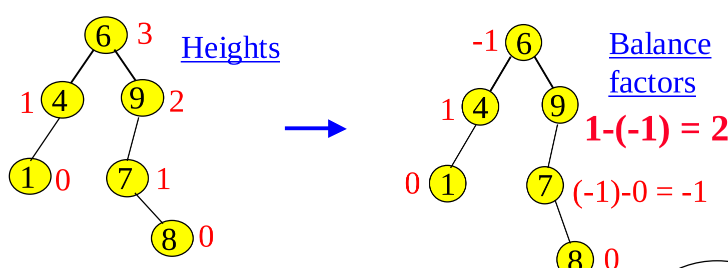
- AVL trees are height-balanced binary search trees
- Balance factor of a node = height(left subtree) - height(right subtree)
- An AVL tree can only have balance factors +1, 0, or –1 at every node
  - Height of an empty subtree is defined to be -1
- Implementation: Store current heights in each node and calculate balance factors when needed from subtrees' root nodes.

# Is this tree AVL?



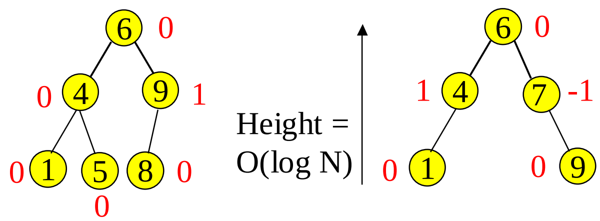



# Is this tree AVL?




## Why AVL?

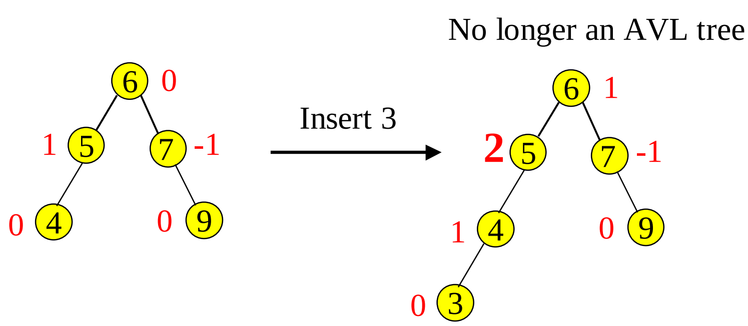
- Can prove: Height of an AVL tree of N nodes is always O(log N) (see previous lecture and textbook)
- Run time for accessing any node is therefore O(log N)




- One problem: Insert/Remove may upset AVL balance



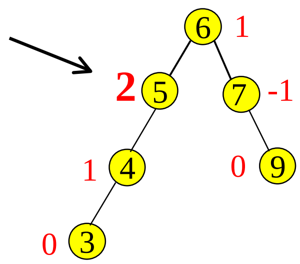
## Insert Example




Problem: Insert may cause balance factor to become 2 or –2 for some node on the path from insertion point to root node

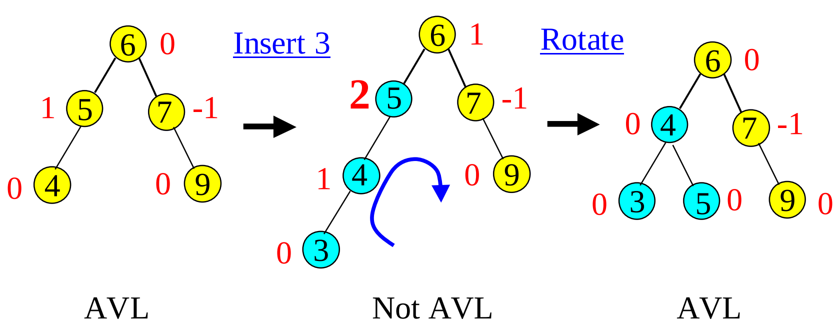
## Restoring Balance

- Idea: After Inserting the new node,
  1. Back up to root updating heights along the access path
  2. If Balance Factor = 2 or –2, adjust tree by rotation around deepest such node.

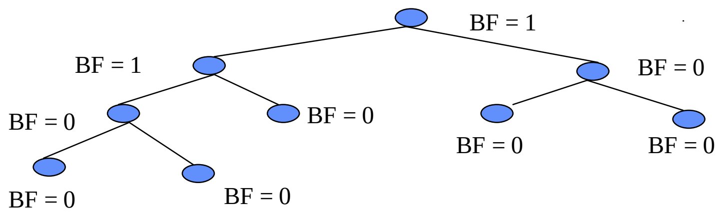





## Rotating to restore Balance: A Simple Example



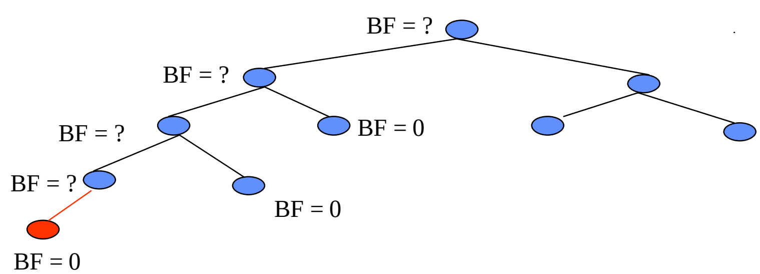
## Various Cases of Insertion




Tree before insertion
(BF = Balance Factor)

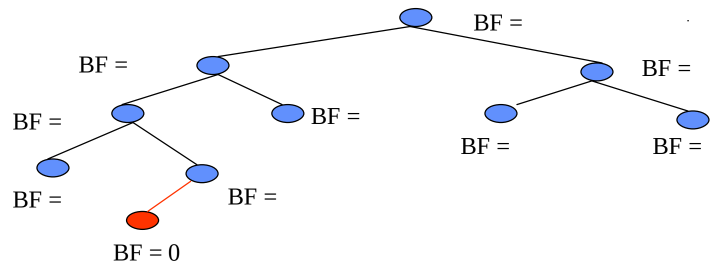


## "Outside" Case




Tree after insertion

## “Inside” Case




Tree after insertion



## Insertions in AVL Trees

Let the node that needs rebalancing be $\alpha$.

There are 4 cases:

Outside Cases (require single rotation) :

1. Insertion into left subtree of left child of $\alpha$.
2. Insertion into right subtree of right child of $\alpha$.

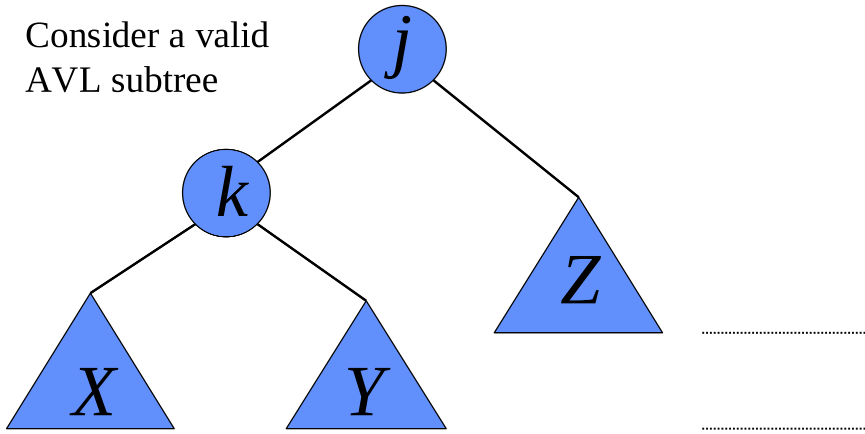
Inside Cases (require double rotation) :

3. Insertion into right subtree of left child of $\alpha$.
4. Insertion into left subtree of right child of $\alpha$.

Rebalancing is performed through four separate rotation algorithms.

## Insertions in AVL Trees: Outside Case

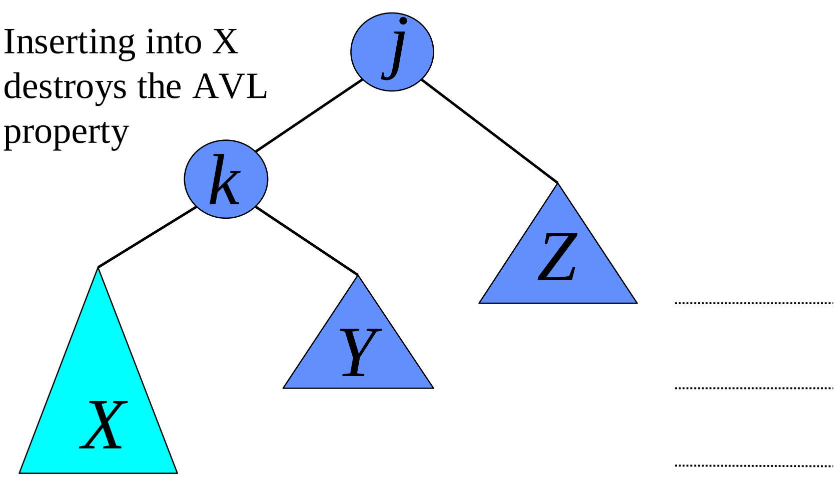
Consider a valid AVL subtree



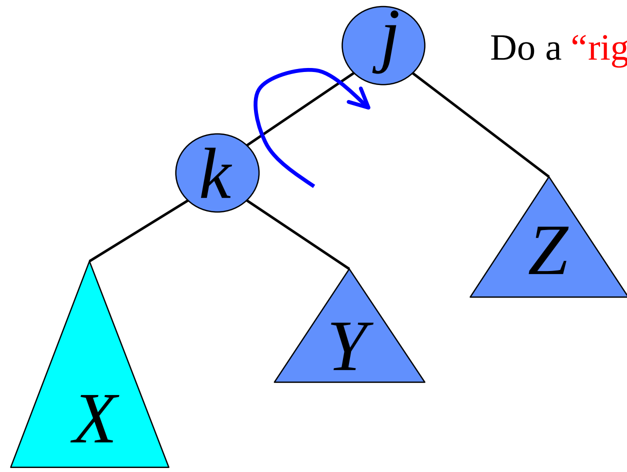



## Insertions in AVL Trees: Outside Case

Inserting into X destroys the AVL property



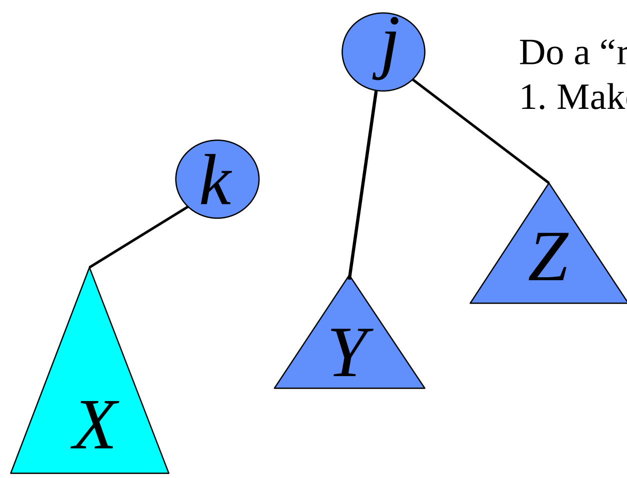
## Insertions in AVL Trees: Outside Case




Do a "right rotation"



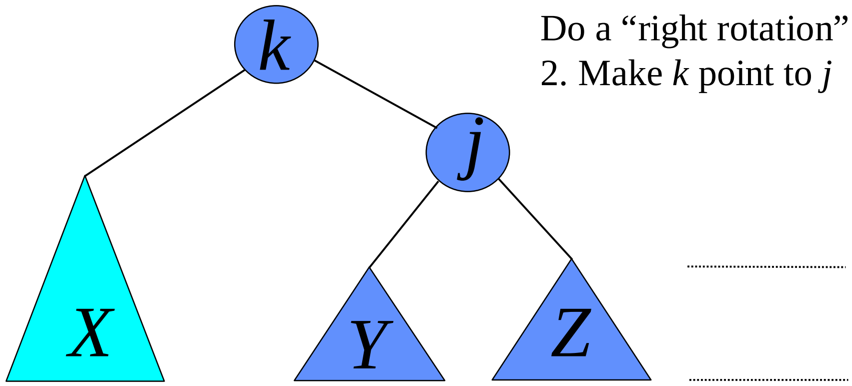
## Insertions in AVL Trees: Outside Case




Do a "right rotation":
1. Make *j* point to *Y*

## Insertions in AVL Trees: Outside Case

Do a “right rotation”:
2. Make *k* point to *j*

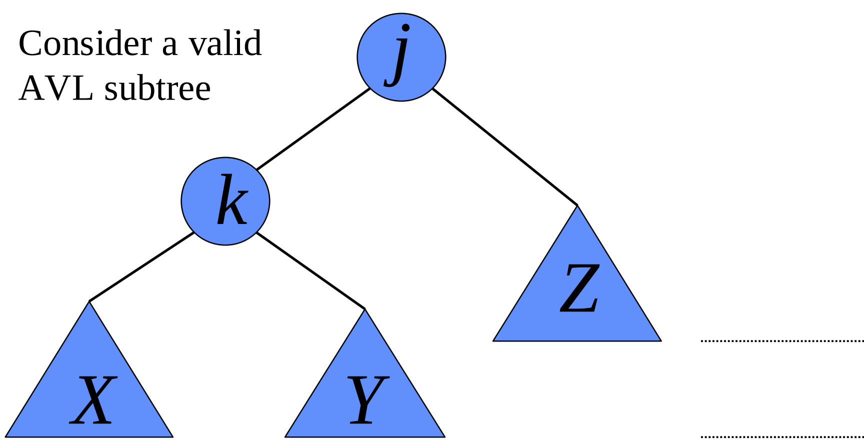



AVL property has been restored!
(“Left rotation” is mirror symmetric)



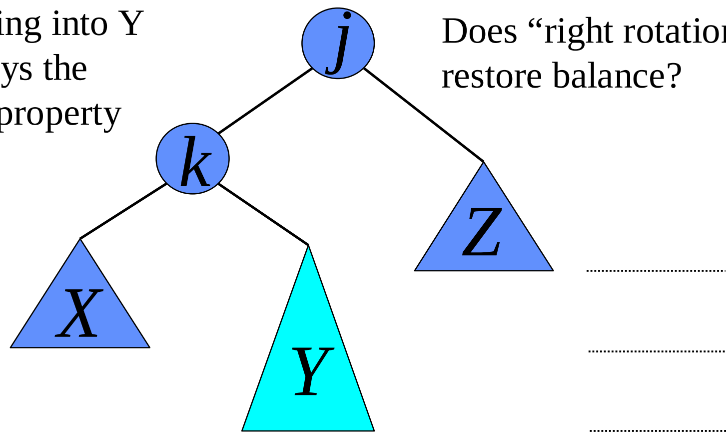
## Insertions in AVL Trees: Inside Case

Consider a valid
AVL subtree



## Insertions in AVL Trees: Inside Case

Inserting into Y destroys the AVL property

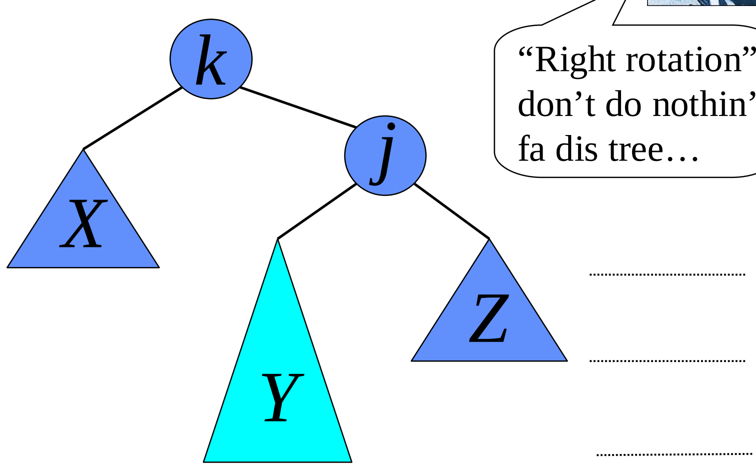



Does “right rotation” restore balance?



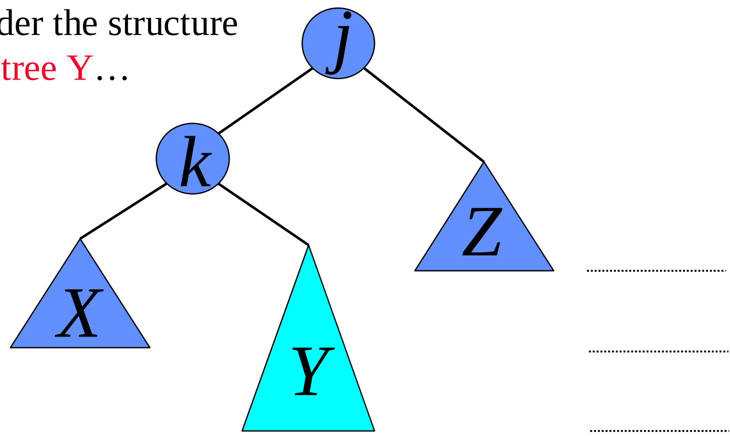
## Insertions in AVL Trees: Inside Case

“Right rotation” don’t do nothin’ fa dis tree…



## Insertions: Inside Case Take 2

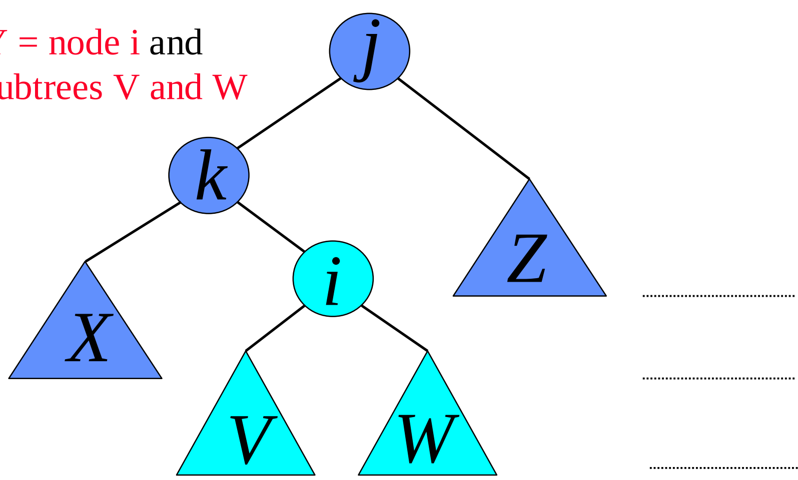
Consider the structure of subtree Y…



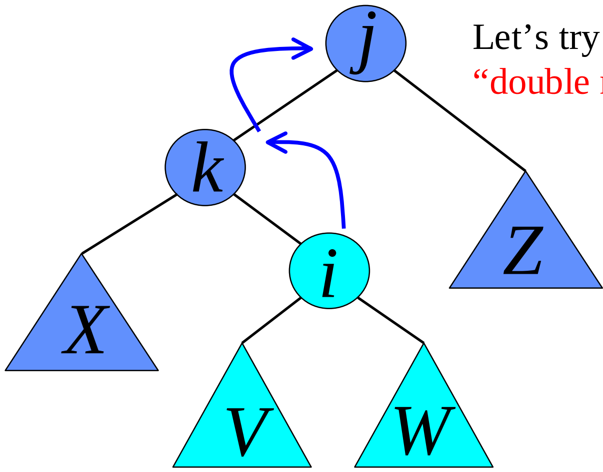



## Insertions in AVL Trees: Inside Case

Y = node i and subtrees V and W



## Insertions in AVL Trees: Inside Case

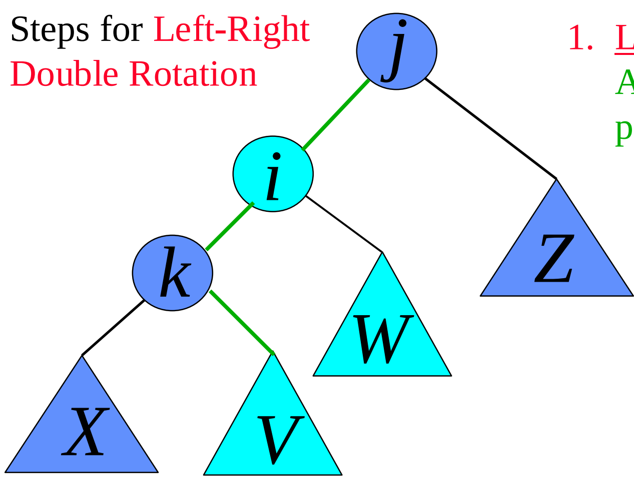



Let's try a left-right "double rotation" . . .



## Insertions in AVL Trees: Inside Case

Steps for Left-Right Double Rotation

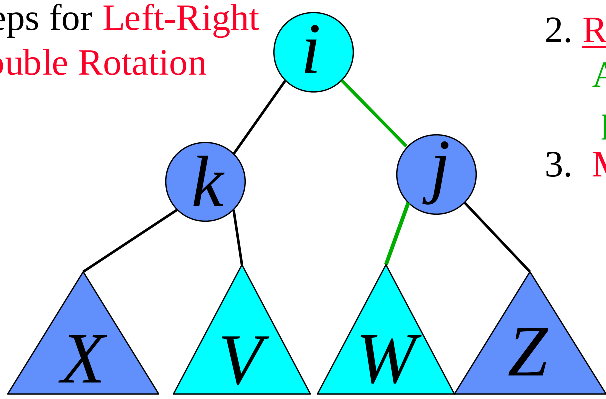



1. Left Rotation: Adjust relevant pointers…

## Insertions in AVL Trees: Inside Case

Steps for Left-Right Double Rotation

2. Right Rotation: Adjust relevant pointers
3. Make ***i*** the root



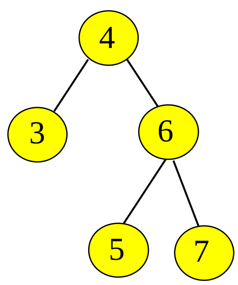

*Balance has been restored!*

(Right-left case is mirror-symmetric)



## AVL Tree On Board Exercise

- Insert 8, 1, 0, 2 in that order into following AVL tree:



## Pros and Cons of AVL Trees

Arguments for AVL trees:

1. Search is O(log N) since AVL trees are always balanced.
2. The height balancing adds no more than a constant factor to the speed of insertion. (Why?)

Arguments against using AVL trees:

1. Difficult to program & debug; more space for height info.
2. Asymptotically faster but can be slow in practice.
3. Most large searches are done in database systems on disk and use other structures (e.g. B-trees).
4. May be OK to have O(N) for a single operation if total run time for many consecutive operations is fast…

(enter **Splay Trees**)



## Splay Trees

Splay trees are tree structures that:

1. Are not perfectly balanced all the time
2. Allow actual Find operations to balance the tree so that future operations may run faster

Based on the heuristic:
If X is accessed once, it is likely to be accessed again.

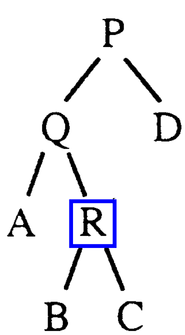
- After node X is accessed, perform "splaying" operations to bring X up to the root of the tree.

- Do this in a way that leaves the tree more balanced as a whole.



## Splaying: A Motivating Example

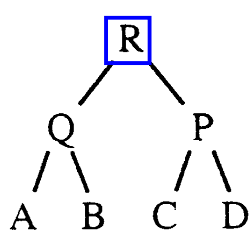
Initial tree




After Find(R)

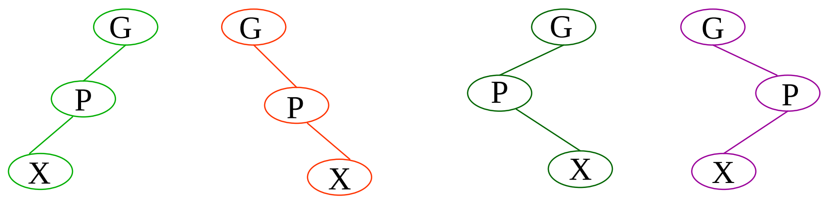
Splay Idea: Get R up to the root using rotations

After splaying with R



## Splay Tree Terminology

- Let X be a non-root node with ≥ 2 ancestors.
- Let P be its parent node.
- Let G be its grandparent node.






## Splay Tree Operations

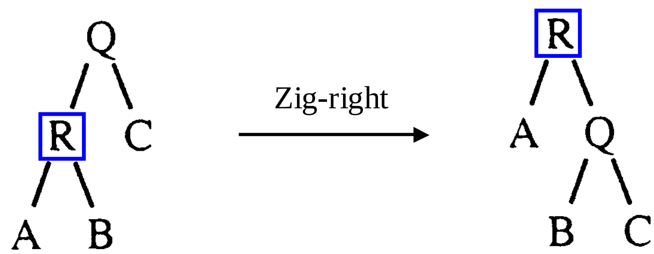
1. Nodes must contain a parent pointer.

| element | left | right | parent |
|---|---|---|---|

2. When X is accessed, apply one of six rotation operations:
   - Single Rotations (X has a P but no G)
     - zig-left, zig-right
   - Double Rotations (X has both a P and a G)
     - zig-zig-left, zig-zig-right
     - zig-zag-left, zig-zag-right

## Splay Trees: Zig operation

- "Zig" is just a single rotation, as in an AVL tree
- Suppose R was the node that was accessed (e.g. using Find)

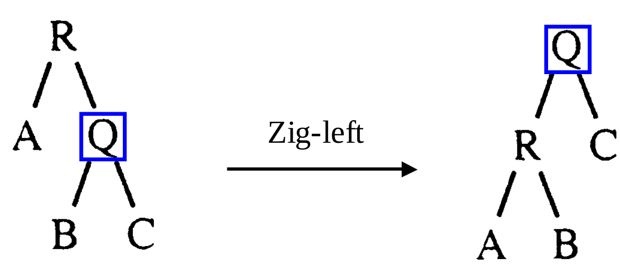



- Zig-right moves R to the top    can access R faster next time



## Splay Trees: Zig operation

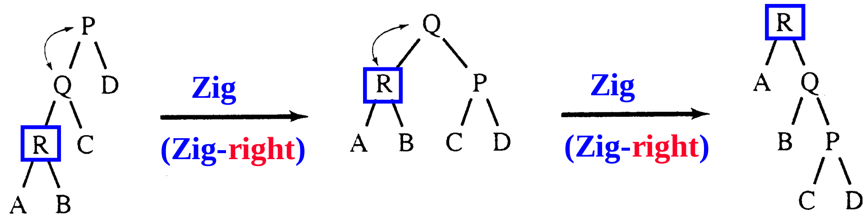
- Suppose Q is accessed (e.g. using Find)




- Zig-left moves Q to the top

## Splay Trees: Zig-Zig operation

- "Zig-Zig" consists of <u>two single rotations</u> of the <u>same type</u> (assume R is the node that was accessed):

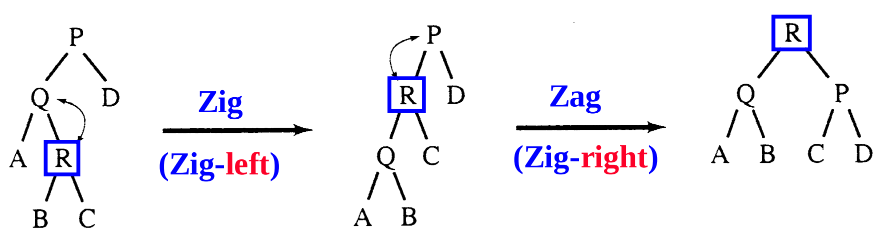



- Again, due to "zig-zig" splaying, R has bubbled to the top!
- <u>Note: Parent-Grandparent rotated first</u>.



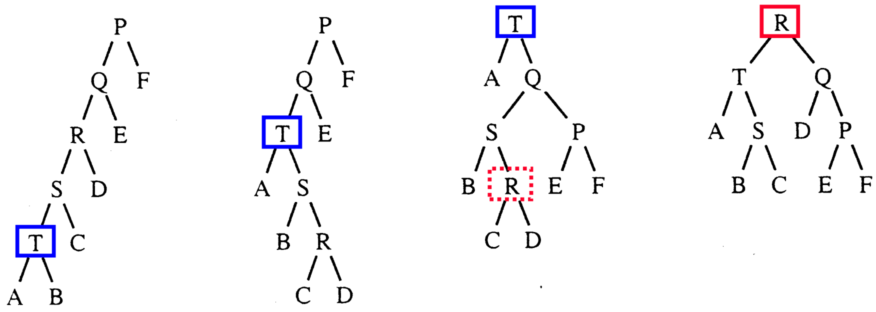
## Splay Trees: Zig-Zag operation

- "Zig-Zag" consists of <u>two rotations</u> of the <u>opposite type</u> (assume R is the node that was accessed):




- "Zig-Zag" splaying also causes R to move to the top.

## Splay Trees: Example




(a) (b) (c) (d)

**Restructuring a tree with splaying after accessing *T* (a–c) and then *R* (c–d).**



## Splay Trees: Do-It-Yourself Exercise

- Insert the keys 1, 2, …, 7 in that order into an empty splay tree.
- What happens when you access "7"?

## Analysis of Splay Trees: Amortization

Examples suggest that splaying causes tree to get balanced.
The actual analysis is rather advanced and is in Chapter 11.

Result of Analysis: Any sequence of M operations on a splay tree of size N takes O(M log N) time.

So, the amortized running time for one operation is O(log N).

This guarantees that even if the depths of some nodes get very large, you cannot get a long sequence of O(N) searches because each search operation causes a rebalance.
Without splaying, total time could be O(MN).



Next Class:

Beyond Binary Trees: B-trees

To Do:

Finish Chapter 4 and Start Chapter 6

Homework # 2