

# Section 1: Code Reasoning

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# Today's Goals

- Review of code reasoning
- Practice forward and backward reasoning on straight-line and if-statement code
- Practice identifying the strongest assertion

# Before we begin . . .

- “=” vs. “==”
- Read the lecture notes



# Reasoning About Code

- Two purposes
  - Prove our code is correct
  - Understand why code is correct
- Forward reasoning: determine what follows from initial conditions
- Backward reasoning: determine sufficient conditions to obtain a certain result



# Worksheet

- Problems 1 through 4
- 15 Minutes – get as far as you can
- You can collaborate with other students
- Grab a TA if you feel stuck



# Forward Reasoning

$\{x \geq 0, y \geq 0\}$

$y = 16;$

$\{x \geq 0, y = 16\}$

$x = x + y$

$\{x \geq 16, y = 16\}$

$x = \text{sqrt}(x)$

$\{x \geq 4, y = 16\}$

$y = y - x$

$\{x \geq 4, y \leq 12\}$

# Forward Reasoning

```
{true}
```

```
if (x > 0) {
```

```
    {x > 0}
```

```
    abs = x
```

```
    {x > 0, abs = x}
```

```
}
```

```
else {
```

```
    {x <= 0}
```

```
    abs = -x
```

```
    {x <= 0, abs = -x}
```

```
}
```

```
{x > 0, abs = x OR x <= 0, abs = -x}
```

```
{abs = |x|}
```

# Backward Reasoning

$$\{x + 3b - 4 > 0\}$$

$$a = x + b;$$

$$\{a + 2b - 4 > 0\}$$

$$c = 2b - 4$$

$$\{a + c > 0\}$$

$$x = a + c$$

$$\{x > 0\}$$



# Backward Reasoning

```
{y > 15 || (y <= 5 && y + z > 17)}
```

```
if (y > 5) {  
    {y > 15}  
    x = y + 2  
    {x > 17}
```

```
}
```

```
else {  
    {y + z > 17}  
    x = y + z;  
    {x > 17}
```

```
}
```

```
{x > 17}
```

# Implication

- Hoare triples are just an extension of logical implication
  - Hoare triple:  $\{P\} S \{Q\}$
  - $P \rightarrow Q$  after statement  $S$
- Everything implies true
- False implies everything

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

# Weaker vs. Stronger

- If  $P1 \rightarrow P2$ , then
  - P1 is stronger than P2
  - P2 is weaker than P1
- Weaker statements are more general
- Stronger statements are more restrictive

# Worksheet

- Problem 5



# Worksheet

- “I attend quiz sections.” “I attend quiz sections on Thursdays.”
- “ $y > 23$ ” “ $y \geq 23$ ”
- “ $y = 23$ ” “ $y \geq 23$ ”
- “ $y < 0.00023$ ” “ $y < 0.23$ ”
- “ $y$  is prime” “ $y \leq 17$ ”

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# Weakest Precondition

- The most lenient assumptions such that a postcondition will be satisfied
- If  $P^*$  is the weakest precondition for  $\{P\} S \{Q\}$ , then  $P \rightarrow P^*$  for all  $P$  that make the Hoare triple valid
- Notation:  $WP = wp(S, Q)$

# Weakest Precondition

`wp (x = y*y, x > 4)`



# Weakest Precondition

`wp (x = y*y, x > 4)`

`|y| > 2`



# Weakest Precondition

`wp (x = y*y, x > 4)`

`|y| > 2`

`wp (y = x+1; z = y-3, z = 10)`

# Weakest Precondition

`wp (x = y*y, x > 4)`

`|y| > 2`

`wp (y = x+1; z = y-3, z = 10)`

`wp (y = x+1, wp (z = y-3, z = 10))`

`wp (y = x+1, y-3 = 10)`

`wp (y = x+1, y = 13)`

`x = 12`

# Questions

