Warmup

A programmer's roommate tells her, "Would you mind going to the store and picking up a loaf of bread? Also, if they have eggs, get a dozen."

The programmer returns with 12 loaves of bread.

Section 3:
HW4, ADTs, and more

Justin Bare and Deric Pang

with material from Vinod Rathnam, Alex Mariakakis,Krysta Yousoufian, Mike Ernst, Kellen Donohue

Agenda

- Announcements
  - HW3: due tonight at 11pm
  - HW4: due Thursday January 28
- Polynomial arithmetic
- Abstract data types (ADT)
- Representation invariants (RI)
- Abstraction Functions
- Further information found in **Calendar/info** & **docs/handouts** link on website

HW4: Polynomial Graphing Calculator

- **Problem 0:** Write pseudocode algorithms for polynomial operations
- **Problem 1:** Answer questions about RatNum
- **Problem 2:** Implement RatTerm
- **Problem 3:** Implement RatPoly
- **Problem 4:** Implement RatPolyStack
- **Problem 5:** Try out the calculator

Rat Objects

- RatNum
  - ADT for a Rational Number
  - Has NaN
- RatTerm
  - Single polynomial term
  - Coefficient (RatNum) & degree
- RatPoly
  - Sum of RatTerms
- RatPolyStack
  - Ordered collection of RatPolys

Polynomial Addition

$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$

Polynomial Addition

$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

$$5x^4 + 4x^3 - x^2 + 5$$

$$+ \quad 3x^5 - 2x^3 + x - 5$$

Polynomial Addition

$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrrrr} & 5x^4 & + & 4x^3 & - & x^2 & 0x & + & 5 \\ + & 3x^5 & 0x^4 & - & 2x^3 & 0x^2 & + & x & - 5 \end{array}$$

Polynomial Addition

$$(5x^4 + 4x^3 - x^2 + 5) + (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrr} & 5x^4 & + 4x^3 & - x^2 & 0x & + 5 \\ + & 3x^5 & 0x^4 & - 2x^3 & 0x^2 + x & - 5 \\ \hline & 3x^5 & + 5x^4 + 2x^3 & - x^2 + x & + 0 \end{array}$$

Polynomial Subtraction

$$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$$

$$5x^4 + 4x^3 - x^2 + 5$$

$$- \quad 3x^5 - 2x^3 + x - 5$$

Polynomial Subtraction

$$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$$

$$5x^4 + 4x^3 - x^2 + 0x + 5$$

$$- \quad 3x^5 + 0x^4 - 2x^3 + 0x^2 + x - 5$$

Polynomial Subtraction

$$(5x^4 + 4x^3 - x^2 + 5) - (3x^5 - 2x^3 + x - 5)$$

$$\begin{array}{rrrrrrrrr} & 5x^4 & + & 4x^3 & - & x^2 & 0x & + & 5 \\ - & 3x^5 & 0x^4 & - & 2x^3 & 0x^2 & + x & - & 5 \\ \hline -3x^5 & + & 5x^4 & + & 6x^3 & - & x^2 & - x & + 10 \end{array}$$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$4x^3 - x^2 + 5$

$*$ $x - 5$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$4x^3 - x^2 + 5$

$*$ $x - 5$

$-20x^3 + 5x^2 - 25$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$4x^3 - x^2 + 5$

* $x - 5$

$-20x^3 + 5x^2 - 25$

$4x^4 - x^3 + 5x$

Polynomial Multiplication

$(4x^3 - x^2 + 5) * (x - 5)$

$$
\begin{array}{rl}
 & 4x^3 - x^2 + 5 \\
* & x - 5 \\
\hline
 & -20x^3 + 5x^2 \quad - 25 \\
+ & 4x^4 \quad -x^3 \quad + 5x \\
\hline
 & \mathbf{4x^4 - 21x^3 + 5x^2 + 5x - 25}
\end{array}
$$

Polynomial Division

$(5x^6 + 4x^4 - x^3 + 5) / (x^3 - 2x - 5)$

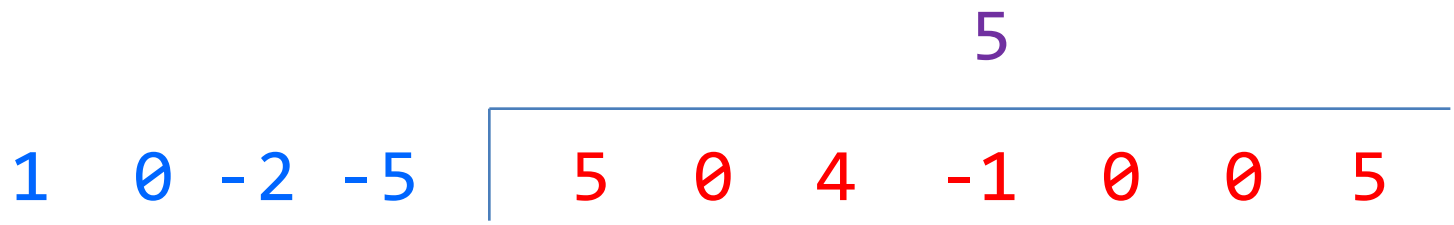
Polynomial Division

$(5x^6 + 4x^4 - x^3 + 5) \;/\; (x^3 - 2x - 5)$

$x^3 - 2x - 5 \;\overline{)\; 5x^6 + 4x^4 - x^3 + 5}$

Polynomial Division

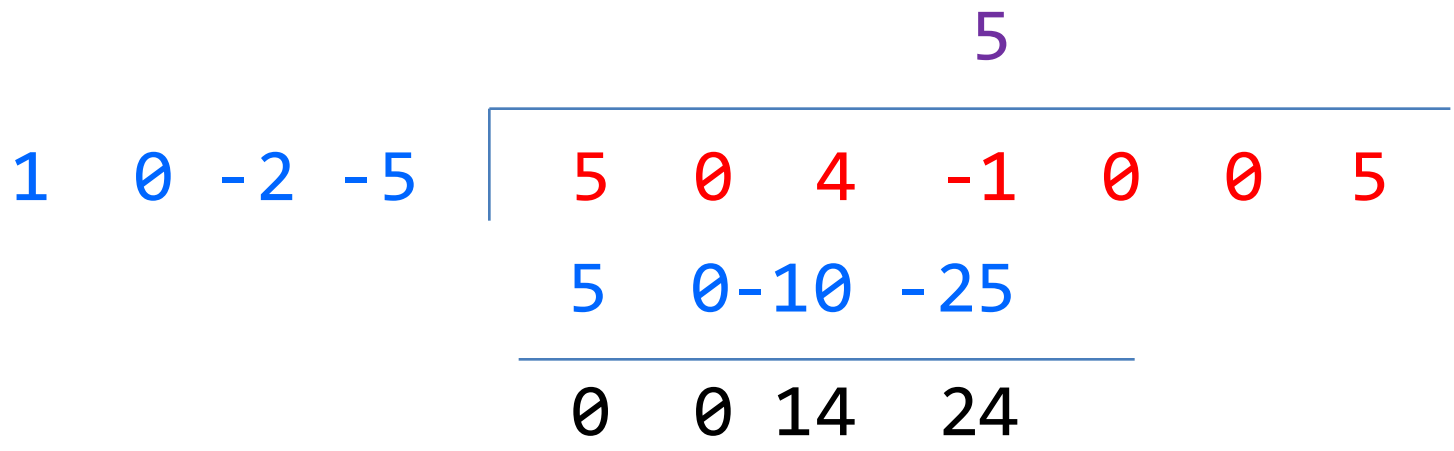
Polynomial Division



Polynomial Division

				5			
1 0 -2 -5	5	0	4	-1	0	0	5
	5	0 -10	0	-25			

Polynomial Division



Polynomial Division

```
                            5
                  ______________________________
1  0 -2 -5       |  5   0   4   -1   0   0   5
                    5   0-10  -25
                    ________________
                    0   0 14   24
                          14   24 0
```

Polynomial Division

```
                                  5   0
                   ___________________________________
1  0 -2 -5  |   5   0   4   -1   0   0   5
                5   0-10 -25
                ________________
                0   0 14   24
                      14   24 0
```

Polynomial Division

```
                          5  0
           _________________________________
1  0 -2 -5 |  5  0  4  -1  0  0  5
              5  0-10 -25
              ---------------
              0  0 14  24
                   14  24 0
                   14  24 0  0
```

Polynomial Division

```
                        5   0 14
              ______________________________
1  0 -2 -5   |   5   0   4   -1   0   0   5
                 5   0-10  -25
                 ____________________
                 0   0 14   24
                       14   24 0
                       14   24 0   0
```

Polynomial Division

```
                              5   0  14
                ------------------------------
1  0 -2 -5  |   5   0   4   -1   0   0   5
                5   0-10 -25
                ---------------
                0   0  14   24
                       14   24  0
                       14   24  0   0
                       14   0 -28 -70
```

Polynomial Division

```
                    5   0 14
           ___________________________
1  0 -2 -5 |  5   0   4   -1   0   0   5
              5   0-10 -25
              ----------------
              0   0 14   24
                    14   24 0
                    14   24 0   0
                    14   0 -28 -70
                    ---------------------
                    0   24   28   70
```

Polynomial Division

```
                   5  0 14
              ______________________________
1  0 -2 -5   |  5  0  4  -1  0  0  5
                5  0-10 -25
                ____________________
                0  0 14  24
                     14  24 0
                     14  24 0  0
                     14  0 -28 -70
                    ___________________________
                     0  24  28  70
                        24  28  70 5
```

Polynomial Division

```
                     5  0 14 24
1  0 -2 -5 |  5  0  4  -1  0  0  5
              5  0-10 -25
              0  0 14  24
                   14  24 0
                   14  24 0  0
                   14  0 -28 -70
                   0  24  28  70
                      24  28  70 5
                       24  0  -48 -120
```

Polynomial Division

```
                    5  0 14 24
1  0 -2 -5 |  5  0  4  -1  0  0  5
              5  0-10 -25
              0  0 14  24
                   14  24 0
                   14  24 0  0
                   14  0 -28 -70
                   0  24  28  70
                      24  28  70 5
                      24  0  -48 -120
                      0   28 118 125
```

Polynomial Division

$(5x^6 + 4x^4 - x^3 + 5) / (x^3 - 2x - 5)$

$5x^3 + 14x + 24$

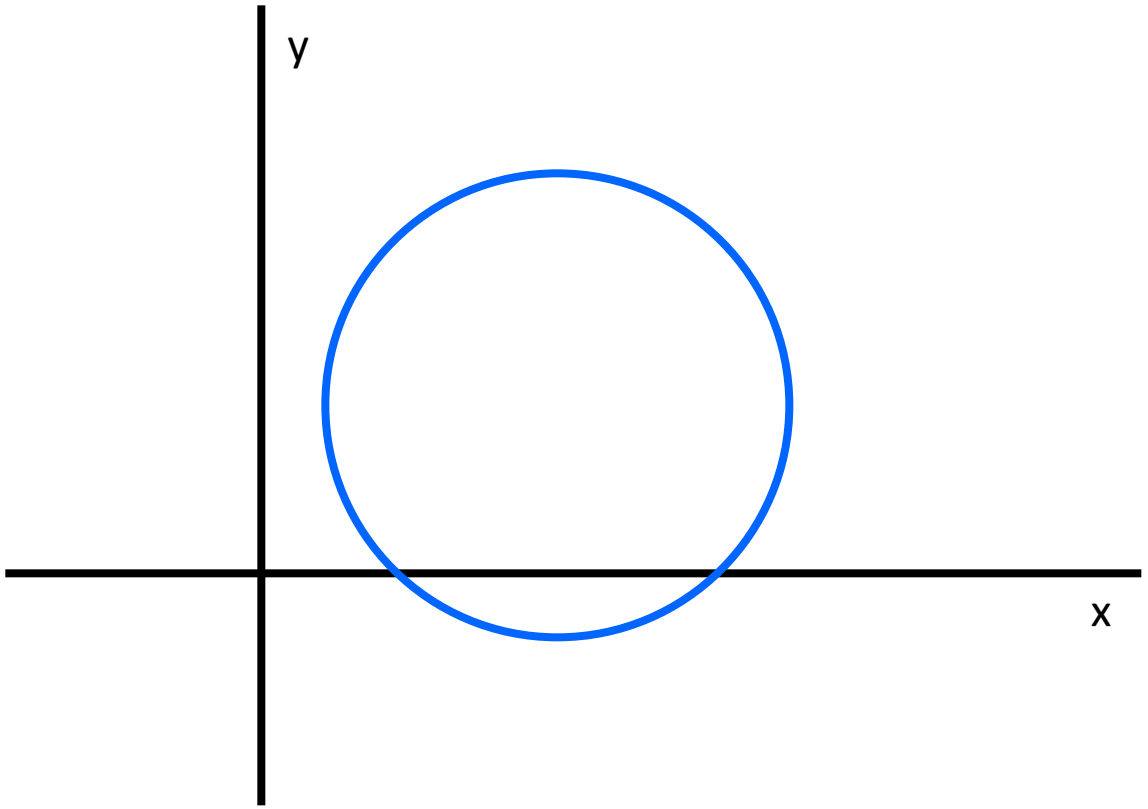
Polynomial Division

$$(5x^6 + 4x^4 - x^3 + 5) \;/\; (x^3 - 2x - 5)$$

$$5x^3 + 14x + 24 + \frac{28x^2 + 118x + 125}{x^3 - 2x - 5}$$

ADT Example: Circle

- Suppose we want to make a `Circle` class that represents circles on the Cartesian plane



ADT Example: Circle

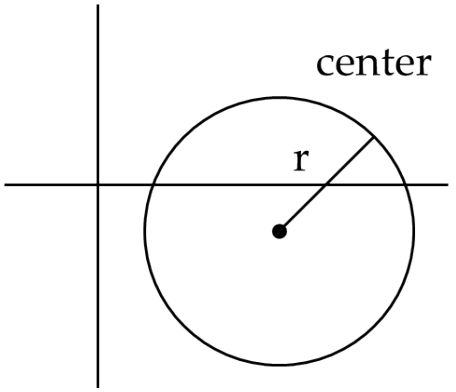
```
/**
 * This class represents the mathematical concept of a circle.
 *
*/
public class Circle {

        // What goes here?

}
```

Circle: Class Specification

- What represents the abstract state of a Circle?
- What are some properties of a circle we can determine?
- How can we implement this?
- What are some ways to “break” a circle?

Circle Implementation 1

```
public class Circle1 {
     private Point center;
     private double rad;

     // Rep invariant:
     //

     //   ...
}
```



Representation Invariants

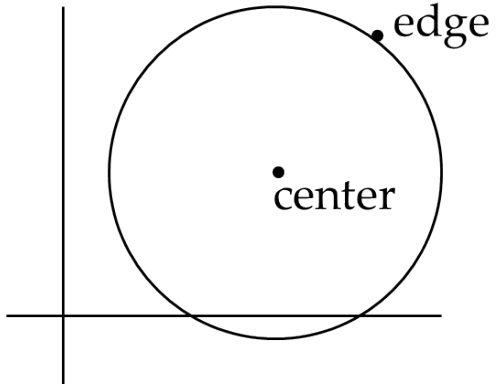
- Constrains an object's internal state
- Maps concrete representation of object to a boolean
- If representation invariant is false/violated, the object is "broken" – doesn't map to any abstract value

Circle Implementation 1

```
public class Circle1 {
    private Point center;
    private double rad;

    // Rep invariant:
    // center != null && rad > 0

    //   ...
}
```

Circle Implementation 2

```
public class Circle2 {
    private Point center;
    private Point edge;

    // Rep invariant:
    //

    //    ...
}
```

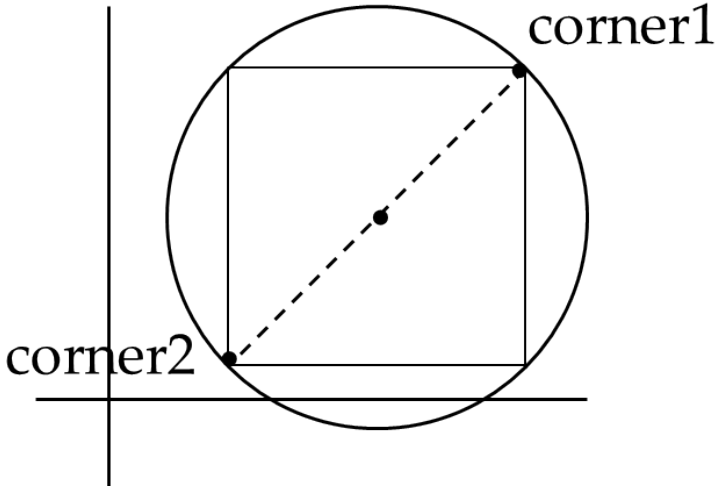


Circle Implementation 2

```
public class Circle2 {
    private Point center;
    private Point edge;

    // Rep invariant:
    // center != null &&
    // edge != null &&
    // !center.equals(edge)
    //    ...
}
```

Circle Implementation 3

```
public class Circle3 {
    private Point corner1, corner2;

    // Rep invariant:
    //


    //    ...
}
```



Circle Implementation 3

```
public class Circle3 {
    private Point corner1, corner2;

    // Rep invariant:
    // corner1 != null &&
    // corner2 != null &&
    // !corner1.equals(corner2)
    //    ...
}
```

Checking Rep Invariants

- Representation invariant should hold before and after every public method
- Write and use `checkRep()`
  - Call before and after public methods
  - Make use of Java's assert syntax!
  - OK that it adds extra code
    - Asserts won't be included on release builds
    - Important for finding bugs

checkRep() Example with Asserts

```
public class Circle1 {
        private Point center;
        private double rad;

        private void checkRep() {
                assert center != null : “This does not have a
                                        center”;
                assert radius > 0 : “This circle has a negative
                                    radius”;
        }
}
```

Using Asserts

- To enable asserts: Go to Run->Run Configurations…->Arguments tab-> input
  - **–ea** in VM arguments section
    - Do this for every test file

Using Asserts

- Demo

Abstraction Function

- Abstraction function: a **mapping** from **internal state** to **abstract value**
- Abstract fields may not map directly to representation fields
  - Circle has **radius** but not necessarily

```
private int radius;
```

- Internal representation can be anything as long as it somehow encodes the abstract value
- Representation Invariant excludes values for which the abstraction function has no meaning

Circle Implementation 1

```
public class Circle1 {
	private Point center;
	private double rad;

	// Abstraction function:
	// AF(this) = a circle c such that
	//	c.center =
	//	c.radius =

	// Rep invariant:
	// center != null && rad > 0

	//  ...
}
```

Circle Implementation 1

```
public class Circle1 {
	private Point center;
	private double rad;

	// Abstraction function:
	// AF(this) = a circle c such that
	//	c.center = this.center
	//	c.radius = this.rad

	// Rep invariant:
	// center != null && rad > 0

	//  ...
}
```

Circle Implementation 2

```
public class Circle2 {
        private Point center;
        private Point edge;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center =
        //      c.radius =

        // Rep invariant:
        // center != null && edge ! null &&
           !center.equals(edge)

        //    ...
}
```

Circle Implementation 2

```
public class Circle2 {
        private Point center;
        private Point edge;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center = this.center
        //      c.radius = sqrt((center.x-edge.x)^2 +
                           (center.y-edge.y)^2)

        // Rep invariant:
        // center != null && edge ! null &&
           !center.equals(edge)

        //    ...
}
```

Circle Implementation 3

```
public class Circle3 {
        private Point corner1, corner2;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center =
        //      c.radius =

        // Rep invariant:
        // corner1 != null && corner2 != null &&
           !corner1.equals(corner2)

        //    ...
}
```

Circle Implementation 3

```
public class Circle3 {
        private Point corner1, corner2;

        // Abstraction function:
        // AF(this) = a circle c such that
        //      c.center = <(corner1.x + corner2.x) / 2,
                            (corner.y + corner2.y) / 2>

        //      c.radius = (1/2)*sqrt((corner1.x-
                            corner2.x)^2 + (corner1.y-
                            corner2.y)^2)

        // Rep invariant:
        // corner1 != null && corner2 != null &&
           !corner1.equals(corner2)

        //    ...
}
```

ADT Example: NonNullStringList

```
public class NonNullStringList  {
       // Abstraction function:
       // ??

       // Rep invariant:
       // ??

       public void add(String s) { ... }
       public boolean remove(String s) { ... }
       public String get(int i) { ... }
}
```

NonNullStringList Implementation 1

```
public class NonNullStringList  {
       // Abstraction function:
       // Index i in arr contains the (i+1)th element in
          the list

       // Rep invariant:
       // All elements in arr from index 0 to count-1 are
          not null

       private String[] arr;
       private int count;

       public void add(String s) { ... }
       public boolean remove(String s) { ... }
       public String get(int i) { ... }
}
```

NonNullStringList Implementation 2

```
public class NonNullStringList  {
       // Abstraction function:
       // Value in the nth node after head contains the
          nth item in the list

       // Rep invariant:
       // Head has size nodes after it, each of whose
          value is non-null, no cycle in ListNodes

       private int size;
       private ListNode head;

       public void add(String s) { ... }
       public boolean remove(String s) { ... }
       public String get(int i) { ... }
}
```