CSE 331
Software Design & Implementation

Autumn 2022

Section 3 – ADTs, AFs, and HW3

Administrivia

- HW2 due yesterday (10/12) at 11PM!
- HW3 due next Wednesday (10/19) at 11PM!
- Any questions?

Abstract Data Types (ADTs)

- Abstraction representing some set of data
  - Meant to express the meaning/concept behind some Java class

- Different from implementation/Java fields!
  - Same ADT can have many different implementations

Abstract data types by example

Review ADT concepts through two examples:

- A **`Line`** ADT
- A **`Rectangle`** ADT

On the course website, see “Resources” → “Class and Method Specifications” for a handy guide with full details.

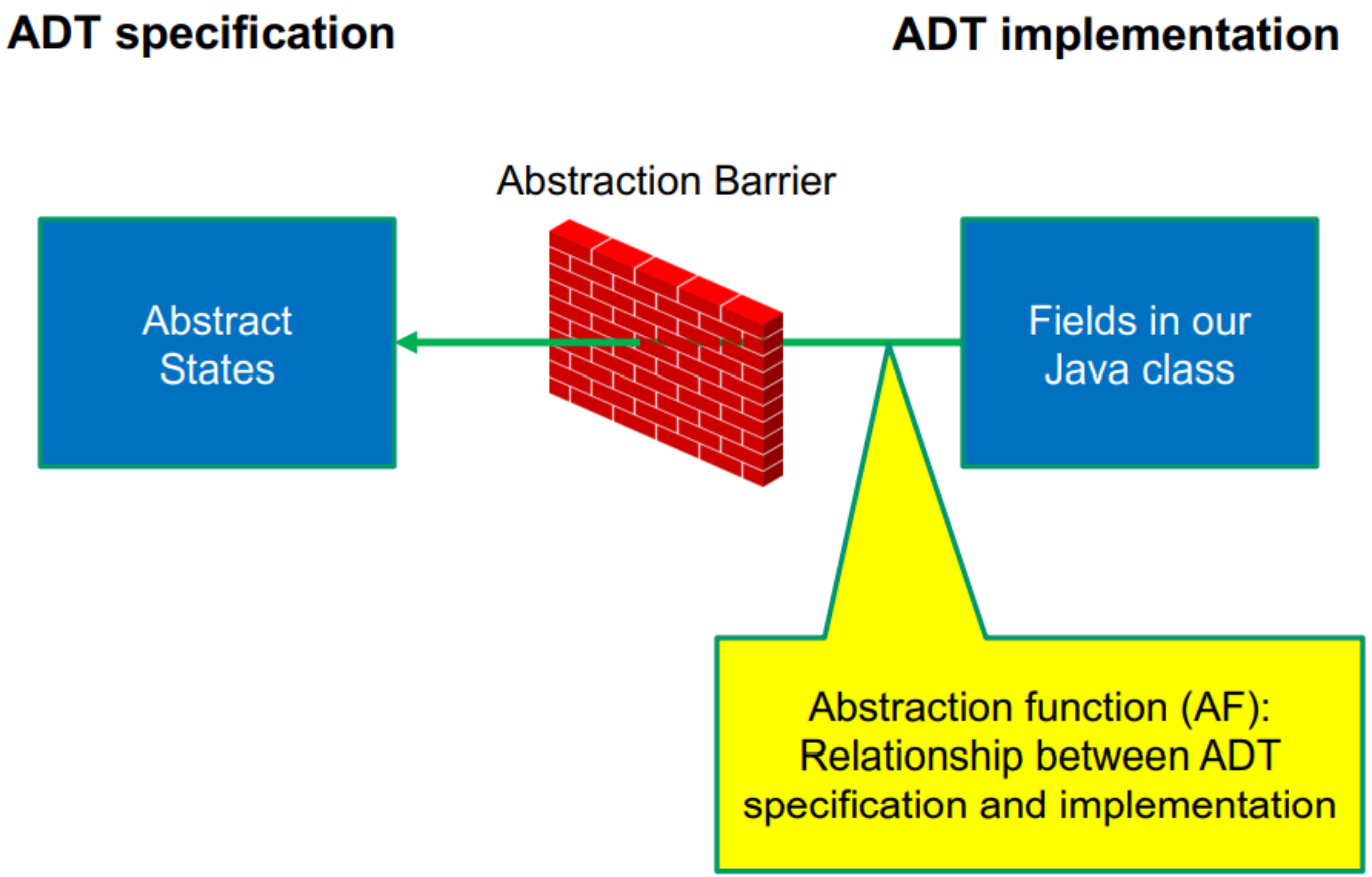
Abstraction Functions (AFs)

- Let's say we have an ADT
  - And we choose some way to implement it

- How does the concrete implementation relate to our ADT?

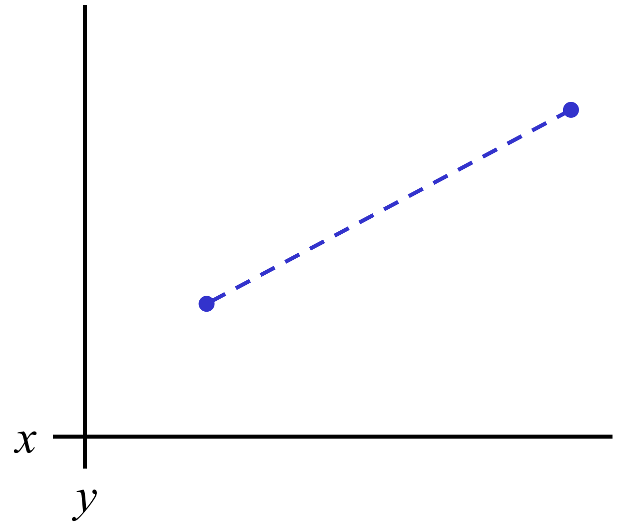
- This is an **abstraction function**
  - Maps object implementation (our Java fields) to the abstract state
  - Ex: "How does a Triangle object from Triangle.java represent a Triangle ADT?"
  - Note: specific to implementation

Diagram


ADT specification
ADT implementation
Abstraction Barrier
Abstract
States
Fields in our
Java class
Abstraction function (AF):
Relationship between ADT
specification and implementation

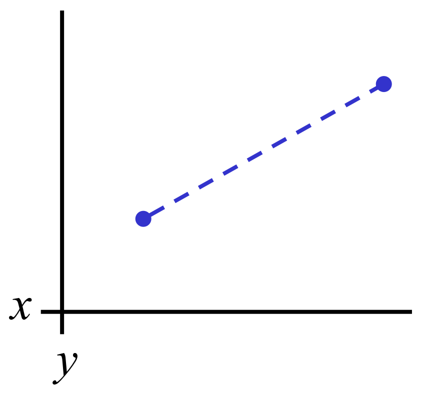
Line ADT

Concept: A line segment in the Cartesian co-ordinate plane



Line ADT: Class specification

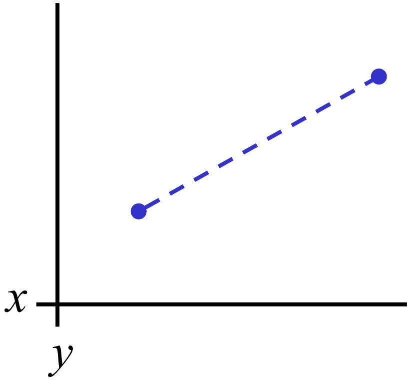
```
/**
 * A Line is a mutable 2D line segment with endpoints
 * p1 and p2.
 */
public class Line {
  ... // rep invariant, fields, methods, etc.
}
```



Line ADT: Representation #1

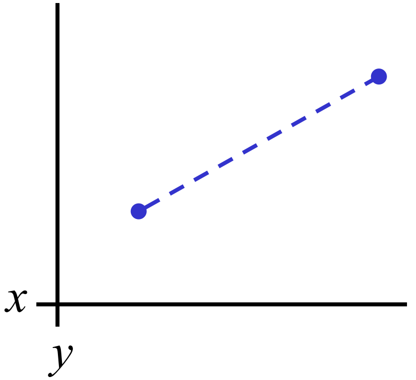
```
/**
 * A Line is a mutable 2D line segment with endpoints
 * p1 and p2.
 */
public class Line {
  // Abstract state is ______
  private Point p1, p2;
}
```

What is our abstraction function?



Line ADT: Representation #1

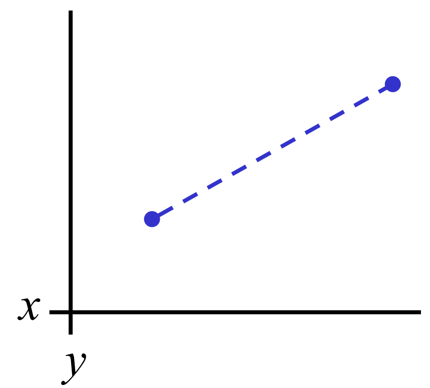
```
/**
 * A Line is a mutable 2D line segment with endpoints
 * p1 and p2.
 */
public class Line {
  // Abstract state is line with endpoints p1 and p2
  private Point p1, p2;
}
```



Line ADT: Representation #2

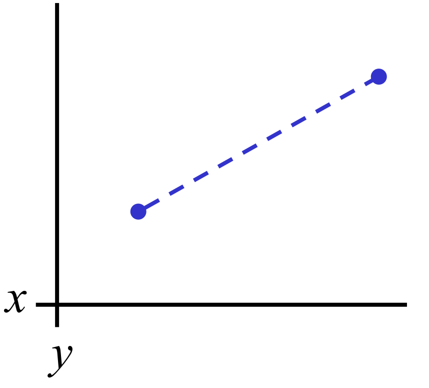
```
/**
 * A Line is a mutable 2D line segment with endpoints
 * p1 and p2.
 */
public class Line {
  // Abstract state is _____
  private int x1, x2;
  private int y1, y2;
}
```

What is our abstraction function?



Line ADT: Representation #2

```
/**
 * A Line is a mutable 2D line segment with endpoints
 * p1 and p2.
 */
public class Line {
  // Abstract state is line with endpoints (x1, y1) and
  //                                       (x2, y2)
  private int x1, x2;
  private int y1, y2;
}
```

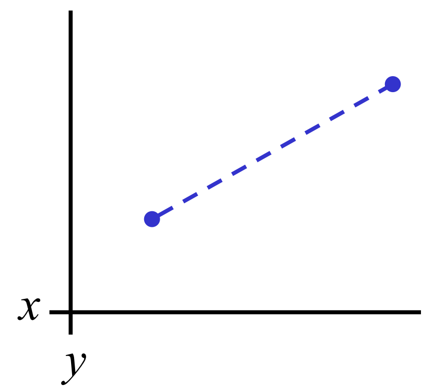


Does this representation have any advantages over #1?

Line ADT: Representation #3

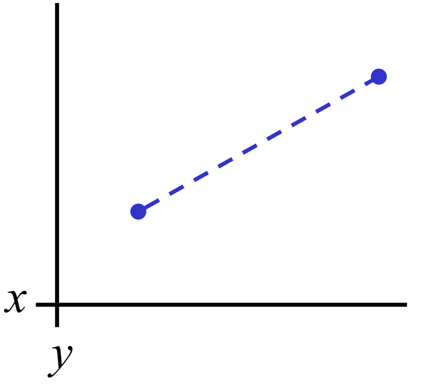
```
/**
 * A Line is a mutable 2D line segment with endpoints
 * p1 and p2.
 */
public class Line {
  // Abstract state is _____
  private int x1, y1;
  private double angle;
  private double len;
}
```

What is our abstraction function?



Line ADT: Representation #3

```
/**
 * A Line is a mutable 2D line segment with endpoints
 * p1 and p2.
 */
public class Line {
  // Abstract state is line with endpoints (x1, y1) and
  //  (x1 + len * cos(angle), y1 + len * sin(angle))
  private int x1, y1;
  private double angle;
  private double len;
}
```



Does this representation have any advantages over #1?

Try it yourself!

Write your own specification of a Rectangle ADT on the handout.

Then give two different possible representations for your Rectangle ADT and write abstraction functions for them.

HW3

In HW3, you will be writing methods in the **Natural** class

Let's look at the specification:

```
/**
 * Represents an immutable, non-negative integer value
 * along with a base in which to print its digits, which we
 * can think of as a pair (base, value).
 * For example, (2, 5) represents the integer 5 (in decimal),
 * but it will show its digits as 101 (in binary) when
 * printed.
 *
 * We require that the base is at least 2 and at most 36 for
 * simplicity.
 */
public class Natural { … }
```

Different Base Examples

Let's take the value **10.** We can use the constructor:

`public Natural(int base, int value){…}`

`new Natural(10, 10)` => "10"

`new Natural(2, 10)` => "1010"

`new Natural(3, 10)` => "101"

`new Natural(4, 10)` => "22"

`Natural` Fields

Now, let's look at the fields, RI, and AF:

```
// Shorthand: b = this.base, D = this.digits, and
//            n = this.digits.length
//
//   RI: 2 <= b <= 36 and D != null and n >= 1 and
//       if n > 1, then D[n-1] != 0 (no leading zeros) and
//       for i = 0 .. n-1, we have 0 <= D[i] < b
//
//   AF(this) = (b, D[0] + D[1] b + D[2] b^2 + ... +
//                  D[n-1] b^{n-1})

private final int base;
private final int[] digits;
```

Least significant digits come **first** in the array

`new Natural(2, 10)` => [0, 1, 0, 1] => "1010"

leftShift()

Now let's take a look at the left shift method:

```
/**
 * Produces a number whose digits, in this base, are the result of taking the
 * digits of this number and shifting them to the left m positions, writing
 * zeros in the now empty positions.
 * @return (this.base, this.value * this.base^m)
 */
public Natural leftShift(int m) { … }
```

How do we multiply something by 10 in base-10? Add a zero

How do we multiply something by 2 in binary? Add a zero

How do we multiply something by 100 (10^2) in decimal? Add two zeroes

What's the pattern? How can we do this in our code?

leftShift()

Now let's take a look at the left shift code:

```
public Natural leftShift(int m) {
  int[] digits = new int[this.digits.length + m];
  System.arraycopy(this.digits, 0, digits, m, this.digits.length);
  return new Natural(this.base, digits);
}
```

`new Natural(10, 36) => [6,3] => leftShift(2) => ?`

leftShift()

Now let's take a look at the left shift code:

```
public Natural leftShift(int m) {
  int[] digits = new int[this.digits.length + m];
  System.arraycopy(this.digits, 0, digits, m, this.digits.length);
  return new Natural(this.base, digits);
}
```

`new Natural(10, 36) => [6,3] => leftShift(2) => [0,0,6,3] = (10,3600)`

leftShift()

Now let's take a look at the left shift code:

```
public Natural leftShift(int m) {
  int[] digits = new int[this.digits.length + m];
  System.arraycopy(this.digits, 0, digits, m, this.digits.length);
  return new Natural(this.base, digits);
}
```

`new Natural(10, 36) => [6,3] => leftShift(2)`
`=> [0,0,6,3] = (10,3600)`

`new Natural(2, 10) => [0,1,0,1] => leftShift(3)`
`=> ?`

leftShift()

Now let's take a look at the left shift code:

```
public Natural leftShift(int m) {
  int[] digits = new int[this.digits.length + m];
  System.arraycopy(this.digits, 0, digits, m, this.digits.length);
  return new Natural(this.base, digits);
}
```

new Natural(10, 36) => [6,3] => leftShift(2) => [0,0,6,3] = (10,3600)

new Natural(2, 10) => [0,1,0,1] => leftShift(3) => [0,0,0,0,1,0,1] = (2,80)

Does this make sense?

getValue()

```
public int getValue() {

  int i = this.digits.length - 1;
  int j = 0;
  int val = this.digits[i];

  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //       i + j = n - 1, where D = this.digits,
  //       n = this.digits.length, and b = this.base
  while (j != this.digits.length - 1) {
    j = j + 1;
    i = i - 1;
    val = val * this.base + this.digits[i];
  }

  // Post: val = D[0] + D[1] b + D[2] b^2 + ... + D[n-1] b^{n-1}
  return val;
}
```

What is this method doing?

Proving `getValue()`

Let's first prove that the invariant is established before the loop:

```
public int getValue() {
  {{ RI, which includes n >= 1 }}
  int i = this.digits.length - 1;
  {{ ? }}
  int j = 0;

  int val = this.digits[i];



  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //      i + j = n - 1, where D = this.digits,
  //      n = this.digits.length, and b = this.base
  …
}
```

Proving `getValue()`

Let's first prove that the invariant is established before the loop:

```
public int getValue() {
  {{ RI, which includes n >= 1 }}
  int i = this.digits.length - 1;
  {{ n >= 1 and i = n - 1 }}
  int j = 0;
  {{ ? }}
  int val = this.digits[i];



  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //      i + j = n - 1, where D = this.digits,
  //      n = this.digits.length, and b = this.base
  …
}
```

Proving `getValue()`

Let's first prove that the invariant is established before the loop:

```
public int getValue() {
  {{ RI, which includes n >= 1 }}
  int i = this.digits.length - 1;
  {{ n >= 1 and i = n - 1 }}
  int j = 0;
  {{ n >= 1 and i = n - 1 and j = 0}}
  int val = this.digits[i];
  {{ ? }}


  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //      i + j = n - 1, where D = this.digits,
  //      n = this.digits.length, and b = this.base
  …
}
```

Proving `getValue()`

Let's first prove that the invariant is established before the loop:

```
public int getValue() {
  {{ RI, which includes n >= 1 }}
  int i = this.digits.length - 1;
  {{ n >= 1 and i = n - 1 }}
  int j = 0;
  {{ n >= 1 and i = n - 1 and j = 0}}
  int val = this.digits[i];
  {{ n >= 1 and i = n - 1 and j = 0 and val = D[i]}}
```

Does this imply the invariant?

```
  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //      i + j = n - 1, where D = this.digits,
  //      n = this.digits.length, and b = this.base
  …
}
```

Proving `getValue()`

Let's prove the part after the loop:

```
public int getValue() {
  …
  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //      i + j = n - 1, where D = this.digits,
  //      n = this.digits.length, and b = this.base
  while (j != this.digits.length - 1) {
    …
  }
  {{ ? }}




  // Post: val = D[0] + D[1] b + D[2] b^2 + ... + D[n-1] b^{n-1}
  return val;
}
```

Proving getValue()

Let's prove the part after the loop:

```
public int getValue() {
  …
  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //      i + j = n - 1, where D = this.digits,
  //      n = this.digits.length, and b = this.base
  while (j != this.digits.length - 1) {
    …
  }
  {{ val = D[i] b^0 + D[i+1] b^1 + … + D[n-1] b^j and i+j = n-1
     and j = n-1 }}
  ⇔ {{ ? }}


  // Post: val = D[0] + D[1] b + D[2] b^2 + ... + D[n-1] b^{n-1}
  return val;
}
```

Proving `getValue()`

Let's prove the part after the loop:

```
public int getValue() {
  …
  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //      i + j = n - 1, where D = this.digits,
  //      n = this.digits.length, and b = this.base
  while (j != this.digits.length - 1) {
    …
  }
  {{ val = D[i] b^0 + D[i+1] b^1 + … + D[n-1] b^j and i+j = n-1
     and j = n-1 }}
  ⇔{{ val = D[0] b^0 + D[1] b^1 + … + D[n-1] b^{n-1} and i=0
       and j = n-1 }}

  // Post: val = D[0] + D[1] b + D[2] b^2 + ... + D[n-1] b^{n-1}
  return val;
}
```

Proving **getValue()**

Now let's prove the loop body:

```
public int getValue() {
  …
  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //      i + j = n - 1, where D = this.digits,
  //      n = this.digits.length, and b = this.base
  while (j != this.digits.length - 1) {
    {{ ? }}

    j = j + 1;


    i = i - 1;


    val = val * this.base + this.digits[i];



  }
  …
}
```

Proving `getValue()`

Now let's prove the loop body:

```
public int getValue() {
  …
  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //      i + j = n - 1, where D = this.digits,
  //      n = this.digits.length, and b = this.base
  while (j != this.digits.length - 1) {
    {{ val = D[i]b^0 + D[i+1]b^1 + ... + D[n-1]b^j and i+j = n-1
                                                      and j != n-1 }}
    j = j + 1;
    {{ ? }}

    i = i - 1;


    val = val * this.base + this.digits[i];



  }
  …
}
```

Proving getValue()

Now let's prove the loop body:

```
public int getValue() {
  …
  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //      i + j = n - 1, where D = this.digits,
  //      n = this.digits.length, and b = this.base
  while (j != this.digits.length - 1) {
    {{ val = D[i]b^0 + D[i+1]b^1 + ... + D[n-1]b^j and i+j = n-1
                                                   and j != n-1 }}
    j = j + 1;
    {{ val = D[i]b^0 + D[i+1]b^1 + ... + D[n-1]b^{j-1} and i+j-1 = n-1
                                                     and j != n }}
    i = i - 1;
    {{ ? }}

    val = val * this.base + this.digits[i];



  }
  …
}
```

Proving **getValue()**

Now let's prove the loop body:

```
public int getValue() {
  …
  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //      i + j = n - 1, where D = this.digits,
  //      n = this.digits.length, and b = this.base
  while (j != this.digits.length - 1) {
    {{ val = D[i]b^0 + D[i+1]b^1 + ... + D[n-1]b^j and i+j = n-1
                                                    and j != n-1 }}
    j = j + 1;
    {{ val = D[i]b^0 + D[i+1]b^1 + ... + D[n-1]b^{j-1} and i+j-1 = n-1
                                                      and j != n }}
    i = i - 1;
    {{ val = D[i+1]b^0 + D[i+2]b^1 + ... + D[n-1]b^{j-1} and i+j = n-1
                                                      and j != n }}
    val = val * this.base + this.digits[i];
    {{ ? }}


  }
  …
}
```

Proving `getValue()`

Now let's prove the loop body:

```
public int getValue() {
  …
  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //      i + j = n - 1, where D = this.digits,
  //      n = this.digits.length, and b = this.base
  while (j != this.digits.length - 1) {
    {{ val = D[i]b^0 + D[i+1]b^1 + ... + D[n-1]b^j and i+j = n-1
                                                       and j != n-1 }}
    j = j + 1;
    {{ val = D[i]b^0 + D[i+1]b^1 + ... + D[n-1]b^{j-1} and i+j-1 = n-1
                                                         and j != n }}
    i = i - 1;
    {{ val = D[i+1]b^0 + D[i+2]b^1 + ... + D[n-1]b^{j-1} and i+j = n-1
                                                         and j != n }}
    val = val * this.base + this.digits[i];
    {{ (val - D[i])/b = D[i+1]b^0 + D[i+2]b^1 + ... + D[n-1]b^{j-1}
                                         and i+j = n-1 and j != n }}
    ⇔ {{ ? }}
  }
  …
}
```

Proving `getValue()`

Now let's prove the loop body:

```
public int getValue() {
  …
  // Inv: val = D[i] b^0 + D[i+1] b^1 + ... + D[n-1] b^j and
  //       i + j = n - 1, where D = this.digits,
  //       n = this.digits.length, and b = this.base
  while (j != this.digits.length - 1) {
    {{ val = D[i]b^0 + D[i+1]b^1 + ... + D[n-1]b^j and i+j = n-1
                                                        and j != n-1 }}
    j = j + 1;
    {{ val = D[i]b^0 + D[i+1]b^1 + ... + D[n-1]b^{j-1} and i+j-1 = n-1
                                                          and j != n }}
    i = i - 1;
    {{ val = D[i+1]b^0 + D[i+2]b^1 + ... + D[n-1]b^{j-1} and i+j = n-1
                                                          and j != n }}
    val = val * this.base + this.digits[i];
    {{ (val - D[i])/b = D[i+1]b^0 + D[i+2]b^1 + ... + D[n-1]b^{j-1}
                                              and i+j = n-1 and j != n }}
    ⇔ {{ val = D[i] + D[i+1]b^1 + … + D[n-1]b^j and i+j = n-1 and j != n }}
  }
  …
}
```

It's correct!

Starter Code

Let's end with skimming through the starter code and run the tests…