CSE332: Data Abstractions

Lecture 19: Analysis of Fork-Join Parallel Programs

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Outline

Done:

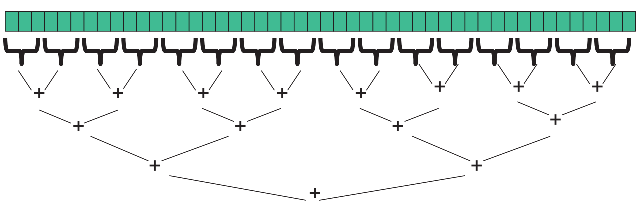
- How to use **`fork`** and **`join`** to write a parallel algorithm
- Why using divide-and-conquer with lots of small tasks is best
  - Combines results in parallel
- Some Java and ForkJoin Framework specifics
  - More pragmatics (e.g., installation) in separate notes

Now:

- More examples of simple parallel programs
- Arrays & balanced trees support parallelism better than linked lists
- Asymptotic analysis for fork-join parallelism
- Amdahl's Law

What else looks like this?

- Saw summing an array went from $O(n)$ sequential to $O(\log n)$ parallel (*assuming **a lot** of processors and very large n!*)
  - Exponential speed-up in theory ($n / \log n$ grows exponentially)



- Anything that can use results from two halves and merge them in $O(1)$ time has the same property…

Examples

- Maximum or minimum element
- Is there an element satisfying some property (e.g., is there a 17)?
- Left-most element satisfying some property (e.g., first 17)
  - What should the recursive tasks return?
  - How should we merge the results?
- Corners of a rectangle containing all points (a "bounding box")
- Counts, for example, number of strings that start with a vowel
  - This is just summing with a different base case
  - Many problems are!

Reductions

- Computations of this form are called reductions (or reduces?)
- Produce single answer from collection via an associative operator
  - Examples: max, count, leftmost, rightmost, sum, product, …
  - Non-examples: median, subtraction, exponentiation
- (Recursive) results don't have to be single numbers or strings. They can be arrays or objects with multiple fields.
  - Example: Histogram of test results is a variant of sum
- But some things are inherently sequential
  - How we process **`arr[i]`** may depend entirely on the result of processing **`arr[i-1]`**

Even easier: Maps (Data Parallelism)

- A map operates on each element of a collection independently to create a new collection of the same size
  - No combining results
  - For arrays, this is so trivial some hardware has direct support
- Canonical example: Vector addition

```
int[] vector_add(int[] arr1, int[] arr2){
  assert (arr1.length == arr2.length);
  result = new int[arr1.length];
  FORALL(i=0; i < arr1.length; i++) {
    result[i] = arr1[i] + arr2[i];
  }
  return result;
}
```

Maps in ForkJoin Framework

```
class VecAdd extends RecursiveAction {
  int lo; int hi; int[] res; int[] arr1; int[] arr2;
  VecAdd(int l,int h,int[] r,int[] a1,int[] a2){ … }
  protected void compute(){
    if(hi – lo < SEQUENTIAL_CUTOFF) {
      for(int i=lo; i < hi; i++)
        res[i] = arr1[i] + arr2[i];
    } else {
      int mid = (hi+lo)/2;
      VecAdd left = new VecAdd(lo,mid,res,arr1,arr2);
      VecAdd right= new VecAdd(mid,hi,res,arr1,arr2);
      left.fork();
      right.compute();
      left.join();
    }
  }
}
static final ForkJoinPool fjPool = new ForkJoinPool();
int[] add(int[] arr1, int[] arr2){
  assert (arr1.length == arr2.length);
  int[] ans = new int[arr1.length];
  fjPool.invoke(new VecAdd(0,arr.length,ans,arr1,arr2);
  return ans;
}
```


Maps and reductions

Maps and reductions: the "workhorses" of parallel programming

- By far the two most important and common patterns
  - Two more-advanced patterns in next lecture
- Learn to recognize when an algorithm can be written in terms of maps and reductions
- Use maps and reductions to describe (parallel) algorithms
- Programming them becomes "trivial" with a little practice
  - Exactly like sequential for-loops seem second-nature

Digression: MapReduce on clusters

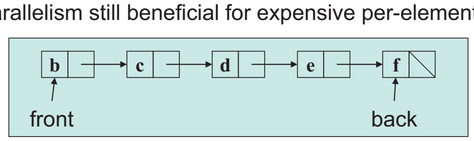
- You may have heard of Google's "map/reduce"
  - Or the open-source version Hadoop
- Idea: Perform maps/reduces on data using many machines
  - The system takes care of distributing the data and managing fault tolerance
  - You just write code to map one element and reduce elements to a combined result
- Separates how to do recursive divide-and-conquer from what computation to perform
  - Old idea in higher-order functional programming transferred to large-scale distributed computing
  - Complementary approach to declarative queries for databases

Trees

- Maps and reductions work just fine on balanced trees
  - Divide-and-conquer each child rather than array subranges
  - Correct for unbalanced trees, but won't get much speed-up
- Example: minimum element in an unsorted but balanced binary tree in $O(\log n)$ time given enough processors
- How to do the sequential cut-off?
  - Store number-of-descendants at each node (easy to maintain)
  - Or could approximate it with, e.g., AVL-tree height

Linked lists

- Can you parallelize maps or reduces over linked lists?
  - Example: Increment all elements of a linked list
  - Example: Sum all elements of a linked list
  - Parallelism still beneficial for expensive per-element operations



- Once again, data structures matter!
- For parallelism, balanced trees generally better than lists so that we can get to all the data exponentially faster $O(\log n)$ vs. $O(n)$
  - Trees have the same flexibility as lists compared to arrays

Analyzing algorithms

- Like all algorithms, parallel algorithms should be:
  - Correct
  - Efficient
- For our algorithms so far, correctness is "obvious" so we'll focus on efficiency
  - Want asymptotic bounds
  - Want to analyze the algorithm without regard to a specific number of processors
  - The key "magic" of the ForkJoin Framework is getting expected run-time performance asymptotically optimal for the available number of processors
    - So we can analyze algorithms assuming this guarantee

Work and Span

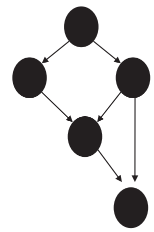
Let T_P be the running time if there are **P** processors available

Two key measures of run-time:

- Work: How long it would take 1 processor = T_1
  - Just "sequentialize" the recursive forking
- Span: How long it would take infinity processors = T_∞
  - The longest dependence-chain
  - Example: $O($`log` $n)$ for summing an array
    - Notice having > $n/2$ processors is no additional help
  - Also called "critical path length" or "computational depth"

The DAG

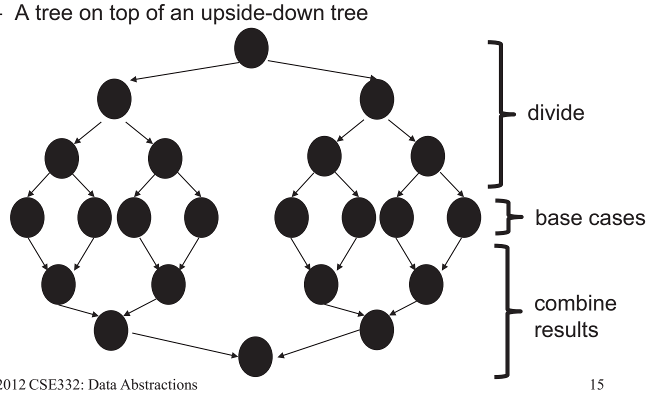
- A program execution using **`fork`** and **`join`** can be seen as a DAG
  - Nodes: Pieces of work
  - Edges: Source must finish before destination starts



- A **`fork`** "ends a node" and makes two outgoing edges
  - New thread
  - Continuation of current thread
- A **`join`** "ends a node" and makes a node with two incoming edges
  - Node just ended
  - Last node of thread joined on

Our simple examples

- **`fork`** and **`join`** are very flexible, but divide-and-conquer maps and reductions use them in a very basic way:
  - A tree on top of an upside-down tree



More interesting DAGs?

- The DAGs are not always this simple
- Example:
  - Suppose combining two results might be expensive enough that we want to parallelize each one
  - Then each node in the inverted tree on the previous slide would itself expand into another set of nodes for that parallel computation

Connecting to performance

- Recall: T_P = running time if there are **P** processors available
- Work = T_1 = sum of run-time of all nodes in the DAG
  - That lonely processor does everything
  - Any topological sort is a legal execution
  - $O(n)$ for simple maps and reductions
- Span = T_∞ = sum of run-time of all nodes on the most-expensive path in the DAG
  - Note: costs are on the nodes not the edges
  - Our infinite army can do everything that is ready to be done, but still has to wait for earlier results
  - $O($`log` $n)$ for simple maps and reductions

Definitions

A couple more terms:

- Speed-up on **P** processors: T_1 / T_P
- If speed-up is **P** as we vary **P**, we call it perfect linear speed-up
  - Perfect linear speed-up means doubling **P** halves running time
  - Usually our goal; hard to get in practice
- Parallelism is the maximum possible speed-up: T_1 / T_∞
  - At some point, adding processors won't help
  - What that point is depends on the span

Parallel algorithms is about decreasing span without increasing work too much

Optimal T_P: Thanks ForkJoin library!

- So we know T_1 and T_∞ but we want T_P (e.g., **P**=4)
- Ignoring memory-hierarchy issues (caching), T_P can't beat
  - T_1 / **P** why not?
  - T_∞ why not?
- So an *asymptotically* optimal execution would be:

$$T_P = O((T_1 / P) + T_\infty)$$

  - First term dominates for small **P**, second for large **P**
- The ForkJoin Framework gives an *expected-time guarantee* of asymptotically optimal!
  - Expected time because it flips coins when *scheduling*
  - How? For an advanced course (few need to know)
  - Guarantee requires a few assumptions about your code…

Division of responsibility

- Our job as ForkJoin Framework users:
  - Pick a good algorithm, write a program
  - When run, program creates a DAG of things to do
  - *Make all the nodes a small-ish and approximately equal amount of work*
- The framework-writer's job:
  - Assign work to available processors to avoid idling
    - Let framework-user ignore all scheduling issues
  - Keep constant factors low
  - Give the expected-time optimal guarantee assuming framework-user did his/her job

$$T_P = O((T_1 / P) + T_\infty)$$

Examples

$$T_P = O((T_1 / P) + T_\infty)$$

- In the algorithms seen so far (e.g., sum an array):
  - $T_1 = O(n)$
  - $T_\infty = O(\log n)$
  - So expect (ignoring overheads): $T_P = O(n/P + \log n)$
- Suppose instead:
  - $T_1 = O(n^2)$
  - $T_\infty = O(n)$
  - So expect (ignoring overheads): $T_P = O(n^2/P + n)$

Amdahl's Law (mostly bad news)

- So far: analyze parallel programs in terms of work and span
- In practice, typically have parts of programs that parallelize well…
  - Such as maps/reductions over arrays and trees

  …and parts that don't parallelize at all

  - Such as reading a linked list, getting input, doing computations where each needs the previous step, etc.

"Nine women can't make a baby in one month"

Amdahl's Law (mostly bad news)

Let the ***work*** (time to run on 1 processor) be 1 unit time

Let **S** be the portion of the execution that can't be parallelized

Then: $T_1 = S + (1-S) = 1$

Suppose we get perfect linear speedup *on the parallel portion*

Then: $T_P = S + (1-S)/P$

So the overall speedup with **P** processors is (Amdahl's Law):

$$T_1 / T_P = 1 / (S + (1-S)/P)$$

And the parallelism (infinite processors) is:

$$T_1 / T_\infty = 1 / S$$

Why such bad news

$T_1 / T_P = 1 / (S + (1-S)/P)$ $T_1 / T_\infty = 1 / S$

- Suppose 33% of a program's execution is sequential
  - Then a billion processors won't give a speedup over 3
- Suppose you miss the good old days (1980-2005) where 12ish years was long enough to get 100x speedup
  - Now suppose in 12 years, clock speed is the same but you get 256 processors instead of 1
  - For 256 processors to get at least 100x speedup, we need
    $100 \le 1 / (S + (1-S)/256)$
    Which means $S \le .0061$ (i.e., 99.4% perfectly parallelizable)

Plots you have to see

1. Assume 256 processors
   – x-axis: sequential portion **S**, ranging from .01 to .25
   – y-axis: speedup $\mathbf{T_1 / T_P}$ (will go down as **S** increases)

2. Assume **S** = .01 or .1 or .25 (three separate lines)
   – x-axis: number of processors **P**, ranging from 2 to 32
   – y-axis: speedup $\mathbf{T_1 / T_P}$ (will go up as **P** increases)

Do this as a homework problem!
   – Chance to use a spreadsheet or other graphing program
   – Compare against your intuition
   – A picture is worth 1000 words, especially if you made it

All is not lost

Amdahl's Law is a bummer!
   – Unparallelized parts become a bottleneck very quickly
   – But it doesn't mean additional processors are worthless

- We can find new parallel algorithms
   – Some things that seem sequential are actually parallelizable

- We can change the problem or do new things
   – Example: Video games use tons of parallel processors
      - They are not rendering 10-year-old graphics faster
      - They are rendering more beautiful(?) monsters

Moore and Amdahl


- Moore's "Law" is an observation about the progress of the semiconductor industry
   – Transistor density doubles roughly every 18 months
- Amdahl's Law is a mathematical theorem
   – Diminishing returns of adding more processors
- Both are incredibly important in designing computer systems