CSE 332 Winter 2024
Lecture 5: Priority Queues

Nathan Brunelle

http://www.cs.uw.edu/332

ADT: Queue

- What is it?
  - A “First In First Out” (FIFO) collection of items
- What Operations do we need?
  - Enqueue
    - Add a new item to the queue
  - Dequeue
    - Remove the “oldest” item from the queue
  - Is_empty
    - Indicate whether or not there are items still on the queue

ADT: Priority Queue

1. call mom
2. laundry
3. check email

- What is it?
  - A collection of items and their "priorities"
  - Allows quick access/removal to the "top priority" thing
- What Operations do we need?
  - insert(item, priority)
    - Add a new item to the PQ with indicated priority
    - Usually, smaller priority value means more important
  - deleteMin
    - Remove and return the "top priority" item from the queue
  - Is_empty
    - Indicate whether or not there are items still on the queue
- Note: the "priority" value can be any type/class so long as it's comparable (i.e. you can use "<" or "compareTo" with it)

Priority Queue, example

PriorityQueue PQ = new PriorityQueue();
PQ.insert(5,5)
PQ.insert(6,6)
PQ.insert(1,1)
PQ.insert(3,3)
PQ.insert(8,8)
Print(PQ.deleteMin) 1
Print(PQ.deleteMin) 3
Print(PQ.deleteMin) 5
Print(PQ.deleteMin) 6
Print(PQ.deleteMin) 8

Priority Queue, example

PriorityQueue PQ = new PriorityQueue();

PQ.insert(5,5)

PQ.insert(6,6)

PQ.insert(1,1)

Print(PQ.deleteMin) 1

PQ.insert(3,3)

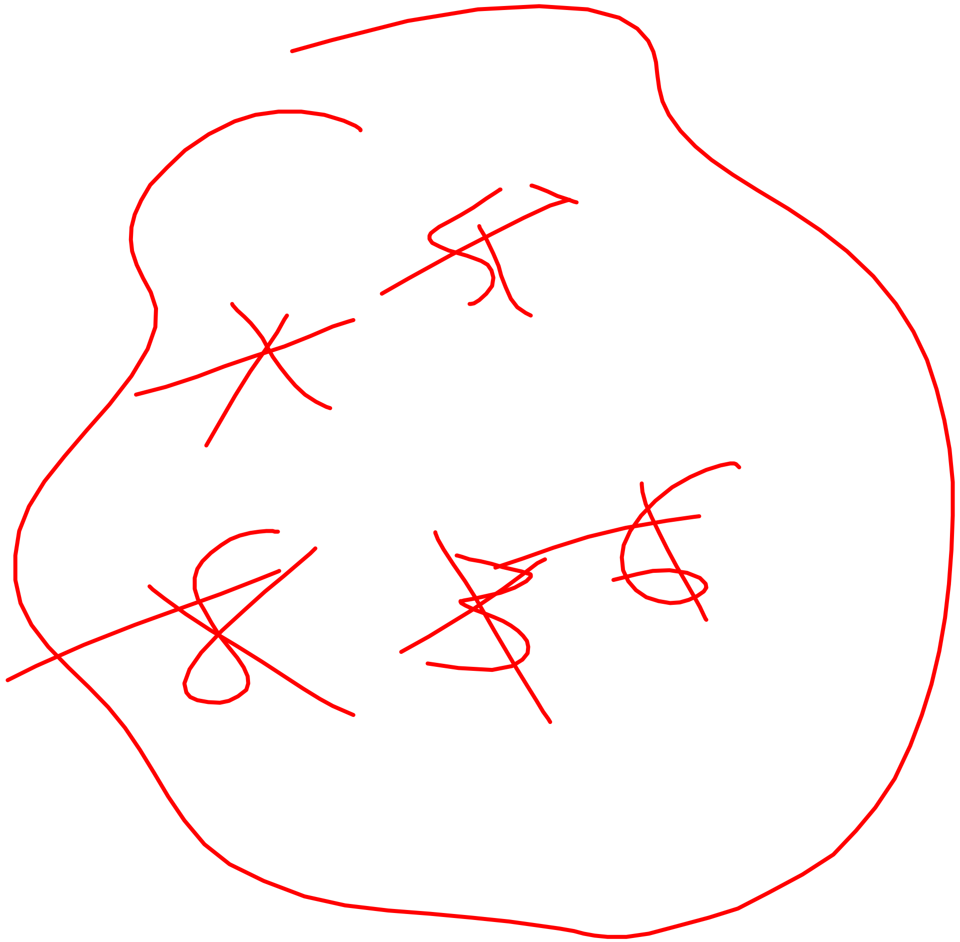
Print(PQ.deleteMin) 3

Print(PQ.deleteMin) 5

PQ.insert(8,8)

Print(PQ.deleteMin) 6

Print(PQ.deleteMin) 8



Applications?

- ER
  - Todo
  - sort
  - issue tracking
  - sm thread

- OS Scheduler

Thinking through implementations

Data Structure	Worst case time to insert	Worst case time to deleteMin
Unsorted Array	1	n
Unsorted Linked List	1	n
Sorted Array	n	n 1
Sorted Linked List	n	1
Binary Search Tree	n	n

Note: Assume we know the maximum size of the PQ in advance

Thinking through implementations

Data Structure	Worst case time to insert	Worst case time to deleteMin
Unsorted Array	$\Theta(1)$	$\Theta(n)$
Unsorted Linked List	$\Theta(1)$	$\Theta(n)$
Sorted Array	$\Theta(n)$	$\Theta(n)$
Sorted Linked List	$\Theta(n)$	$\Theta(1)$
Binary Search Tree	$\Theta(n)$	$\Theta(n)$

Note: Assume we know the maximum size of the PQ in advance

Thinking through implementations

Data Structure	Worst case time to insert	Worst case time to deleteMin
Unsorted Array	$\Theta(1)$	$\Theta(n)$
Unsorted Linked List	$\Theta(1)$	$\Theta(n)$
Sorted Array	$\Theta(n)$	$\Theta(n)$
Sorted Linked List	$\Theta(n)$	$\Theta(1)$
Binary Search Tree	$\Theta(n)$	$\Theta(n)$
Binary Heap	$\Theta(\log n)$	$\Theta(\log n)$

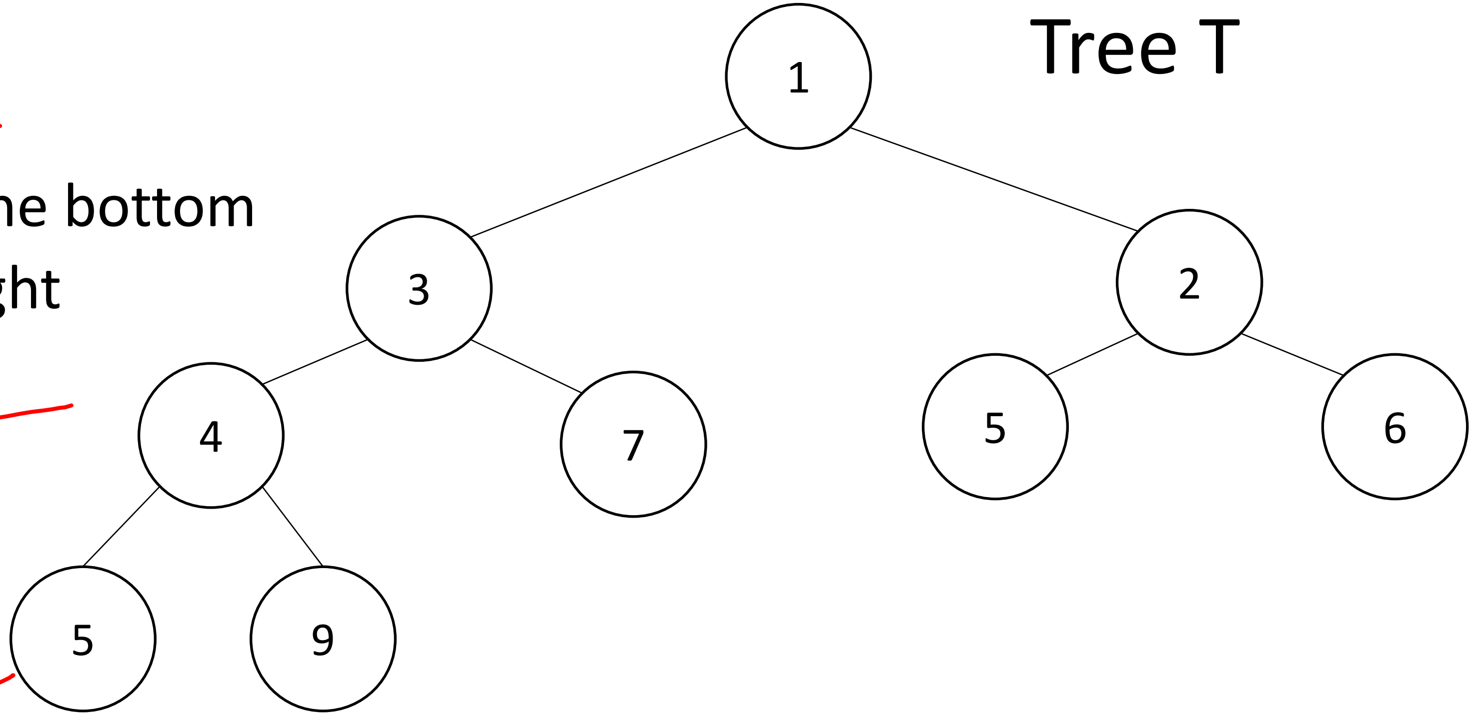
4

Kinda Sorted

Note: Assume we know the maximum size of the PQ in advance

Trees for Heaps

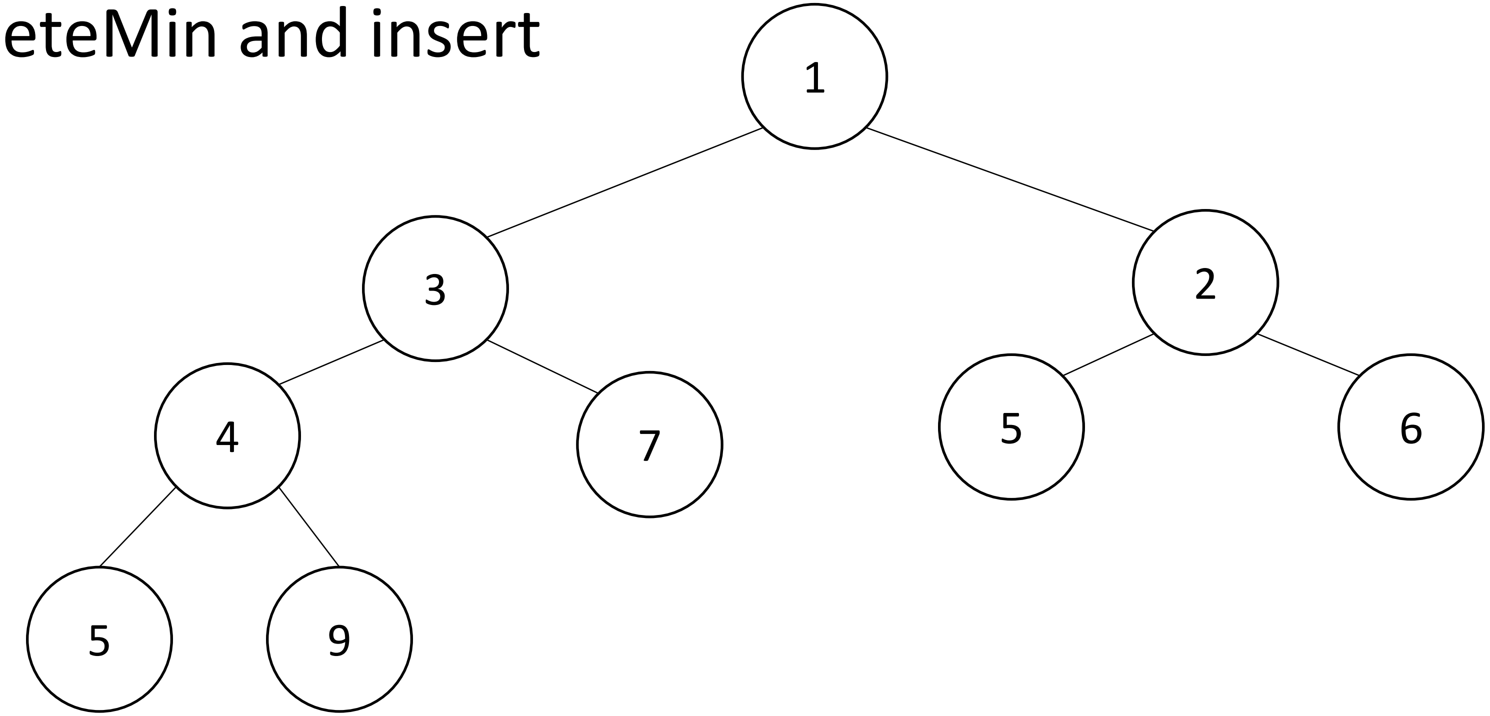
- Binary Trees:
  - The branching factor is 2
  - Every node has ≤ 2 children
- Complete Tree:
  - All “layers” are full, except the bottom
  - Bottom layer filled left-to-right



Tree T

Heap – Priority Queue Data Structure

- Idea: We need to keep some ordering, but it doesn't need to be entirely sorted
- $\Theta(\log n)$ worst case for deleteMin and insert

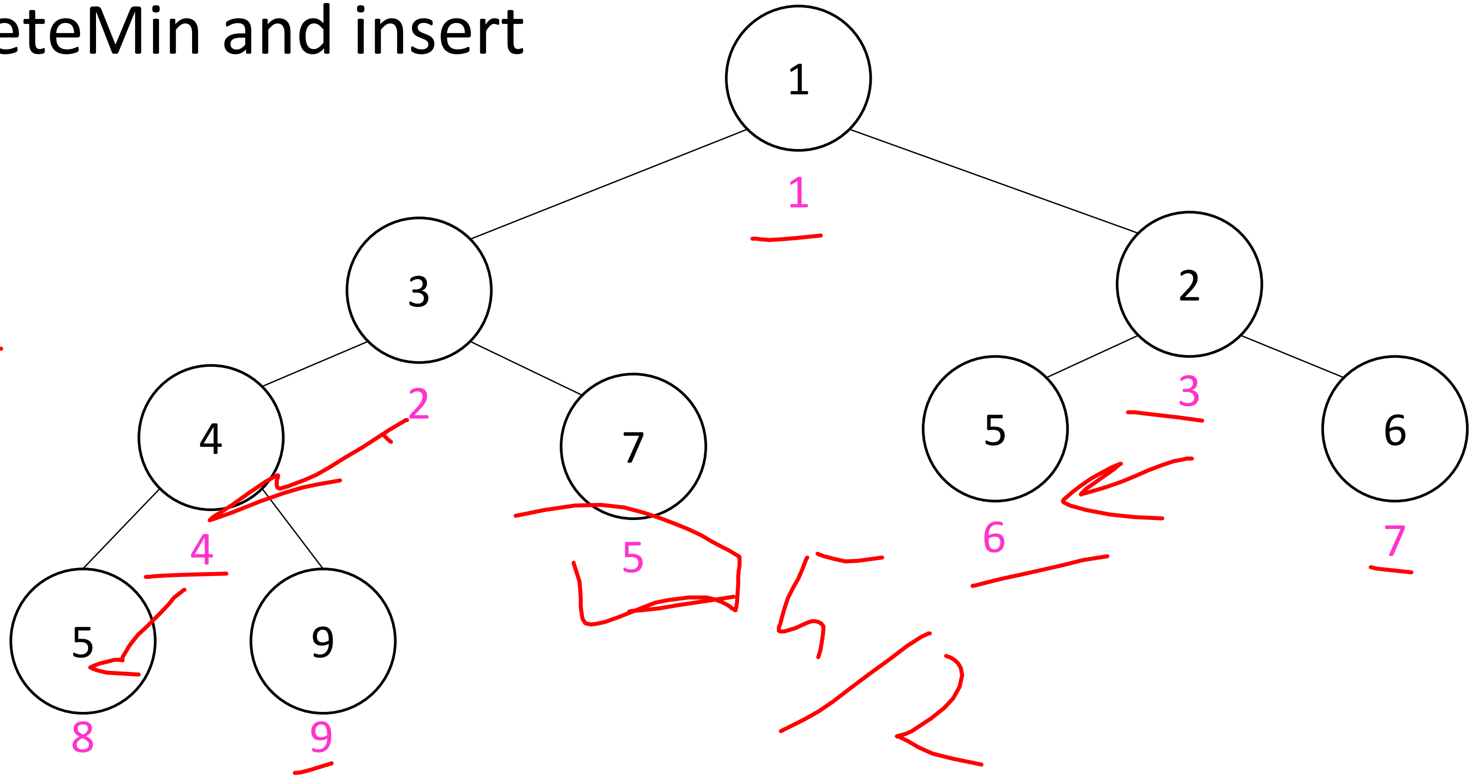


Left child(i) = 2*i

Heap – Priority Queue Data Structure

- Idea: We need to keep some ordering, but it doesn't need to be entirely sorted
- $\Theta(\log n)$ worst case for deleteMin and insert

	1	3	2	4	7	5
0	1	2	3	4	5	6

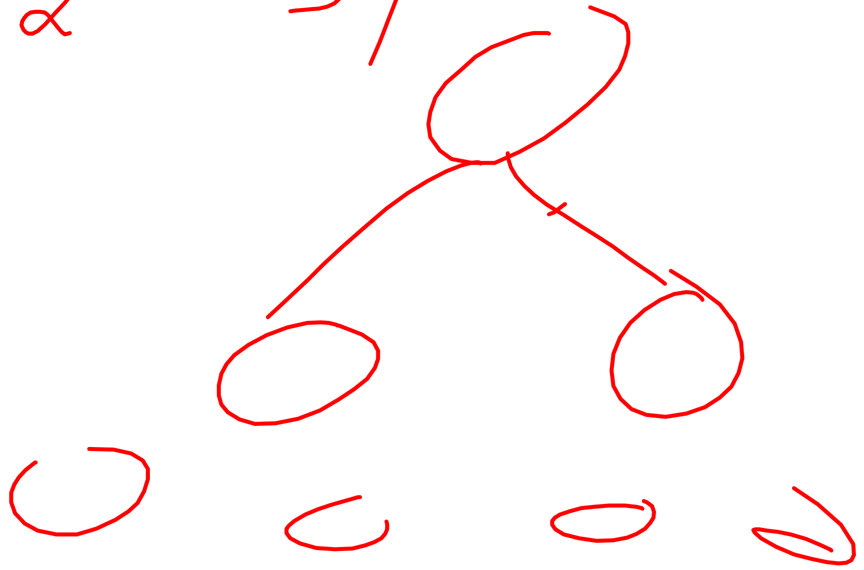


Challenge!

- What is the maximum number of total nodes in a binary tree of height h?
  - $2^{h+1} - 1$
  - $\Theta(2^h)$
- If I have n nodes in a binary tree, what is its minimum height?
  - $\boxed{\Theta(\log n)}$
- Heap Idea:
  - If n values are inserted into a complete tree, the height will be roughly $\log n$
  - Ensure each insert and deleteMin requires just one "trip" from root to leaf

h	$\#n$
1	3
2	7
3	15
h	$\underbrace{2^{h+1}-1}$

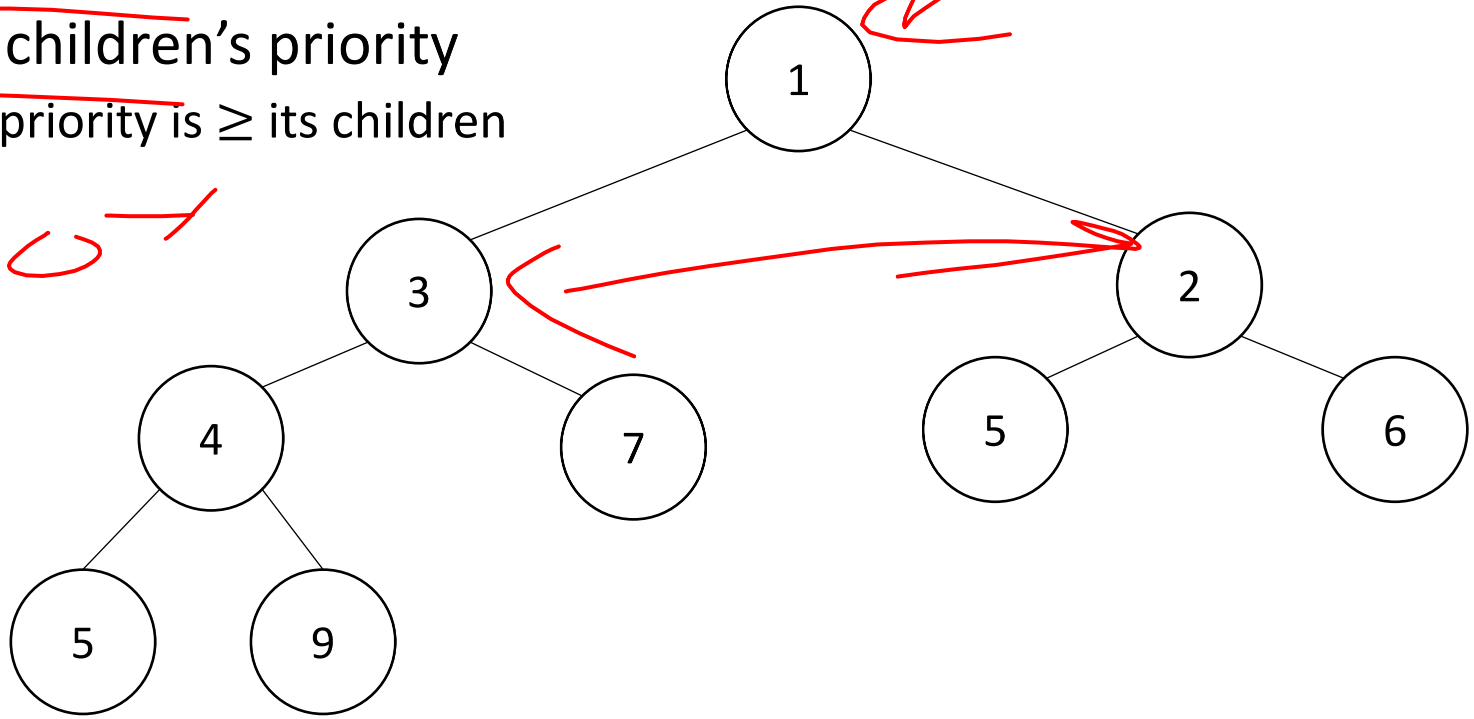
$n = 2^{h+1} - 1$



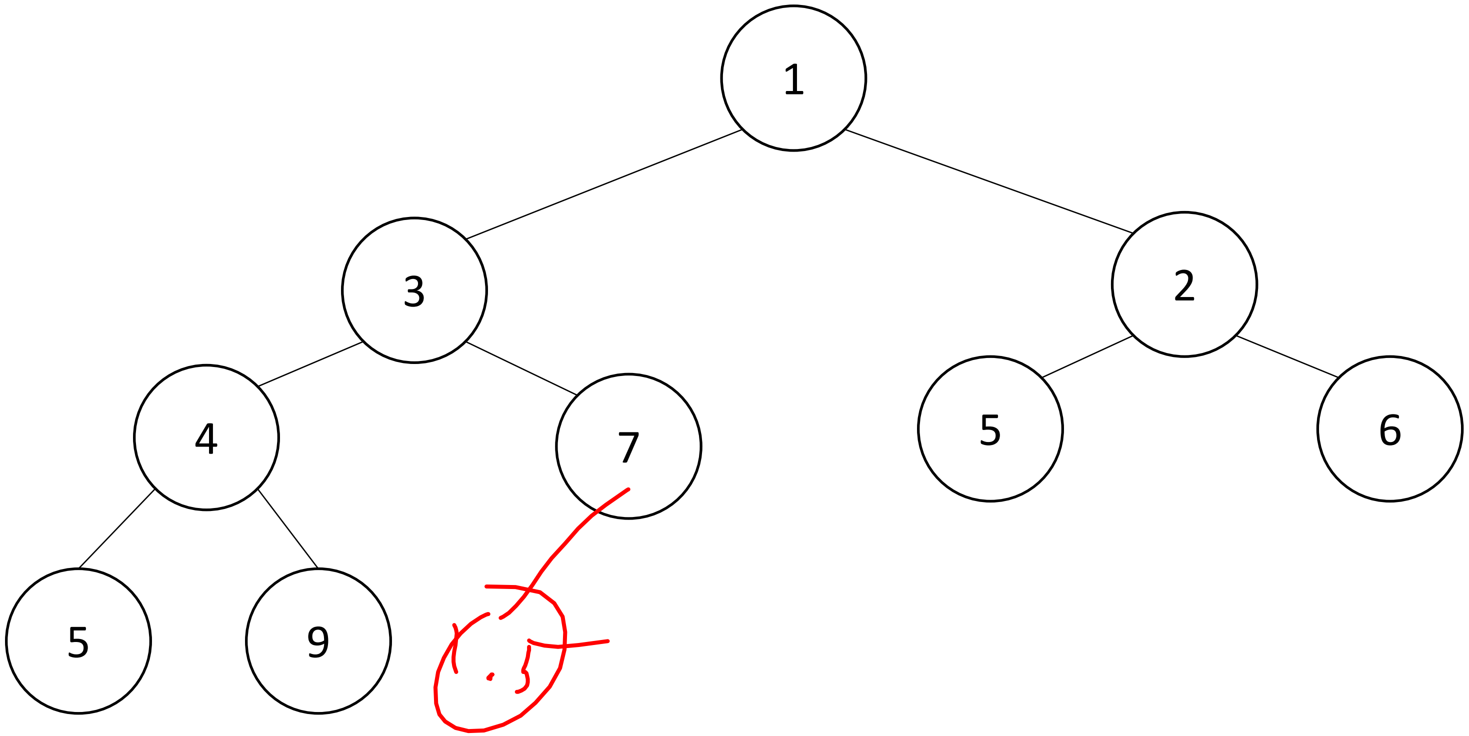
$\approx O(2^h)$

(Min) Heap Data Structure

- Keep items in a complete binary tree
- Maintain the “(Min) Heap Property” of the tree
  - Every node’s priority is $\leq$ its children’s priority
  - Max Heap Property: every node’s priority is $\geq$ its children
- Where is the min? Root
- How do I insert?
- How do I deleteMin?
- How to do it in Java?

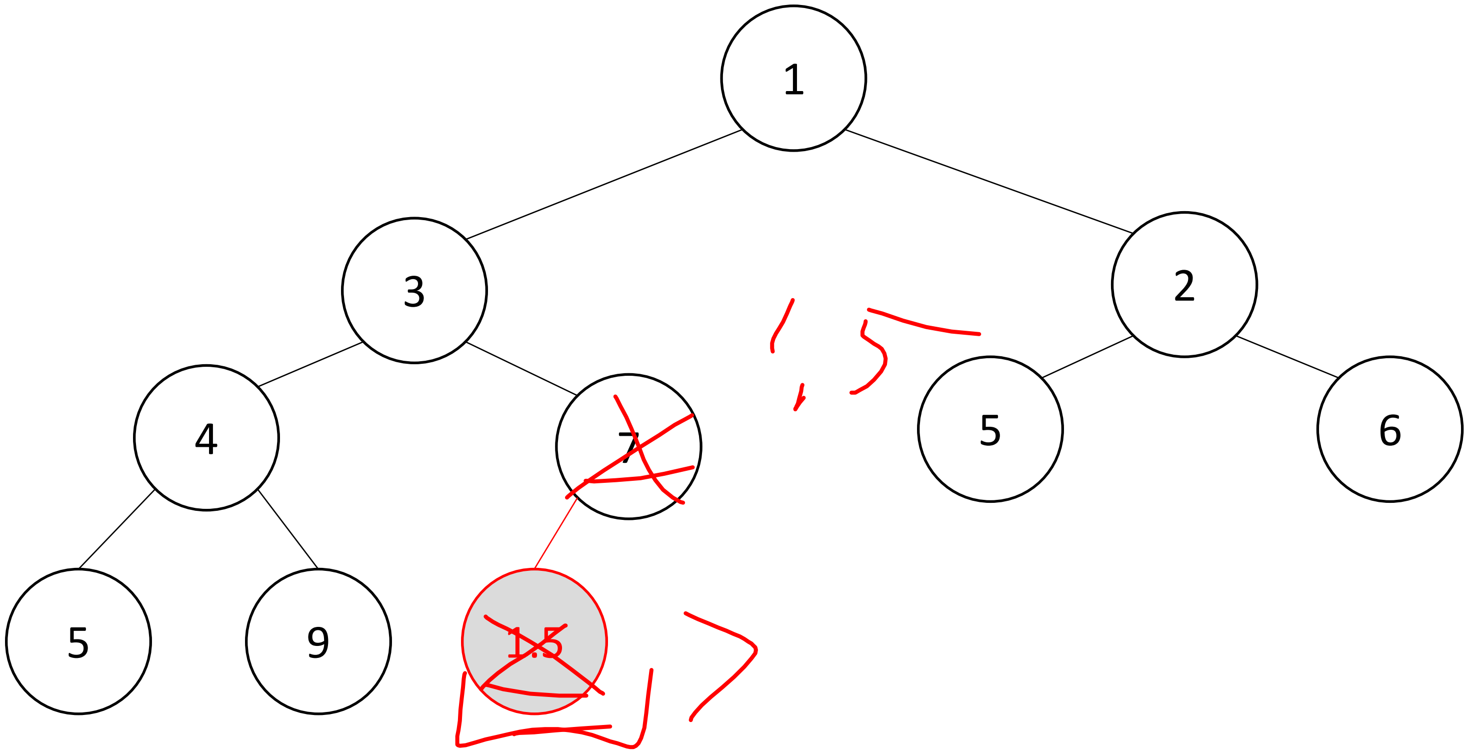


Heap Insert



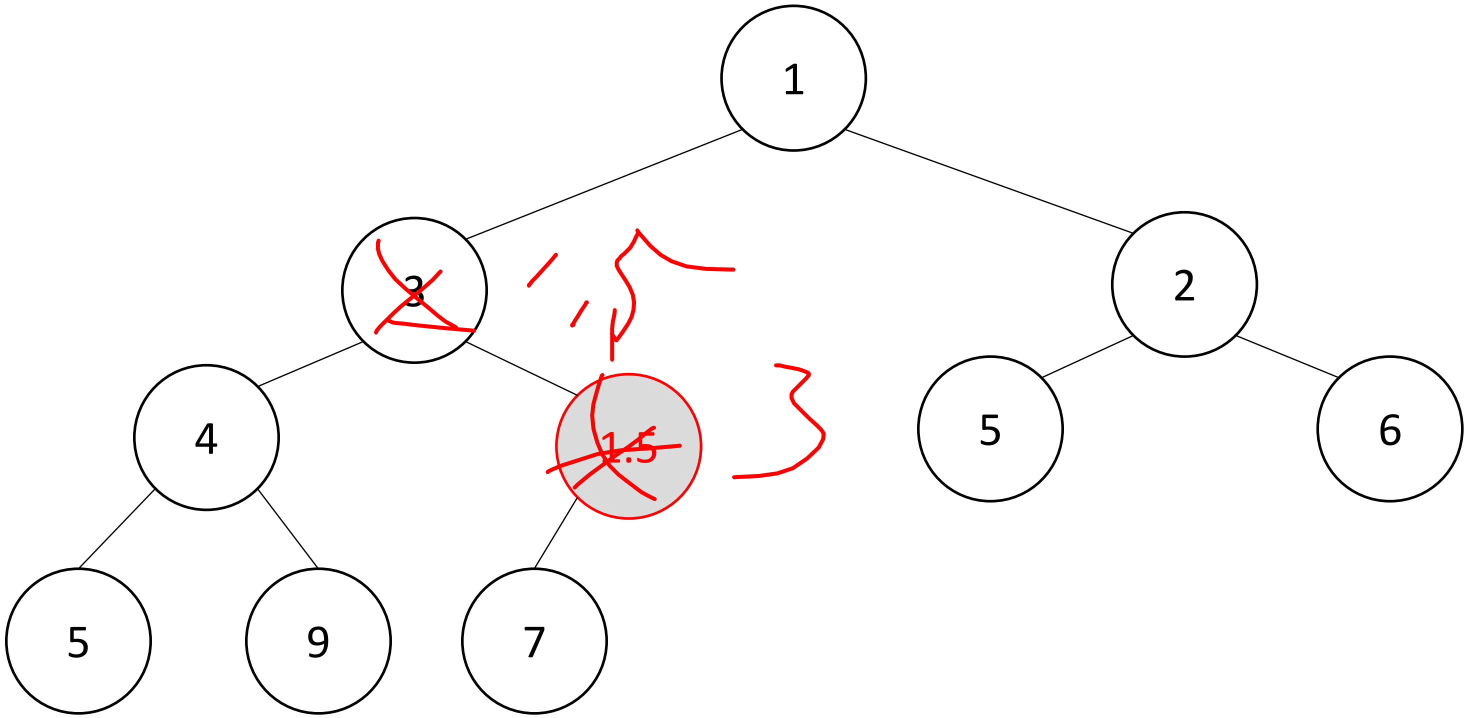

```
insert(item){
   put item in the "next open" spot (keep tree complete)
   while (item.priority < parent(item).priority){
      swap item with parent
   }
}
```

Heap Insert




```
insert(item){
    put item in the "next open" spot (keep tree complete)
    while (item.priority < parent(item).priority){
        swap item with parent
    }
}
```

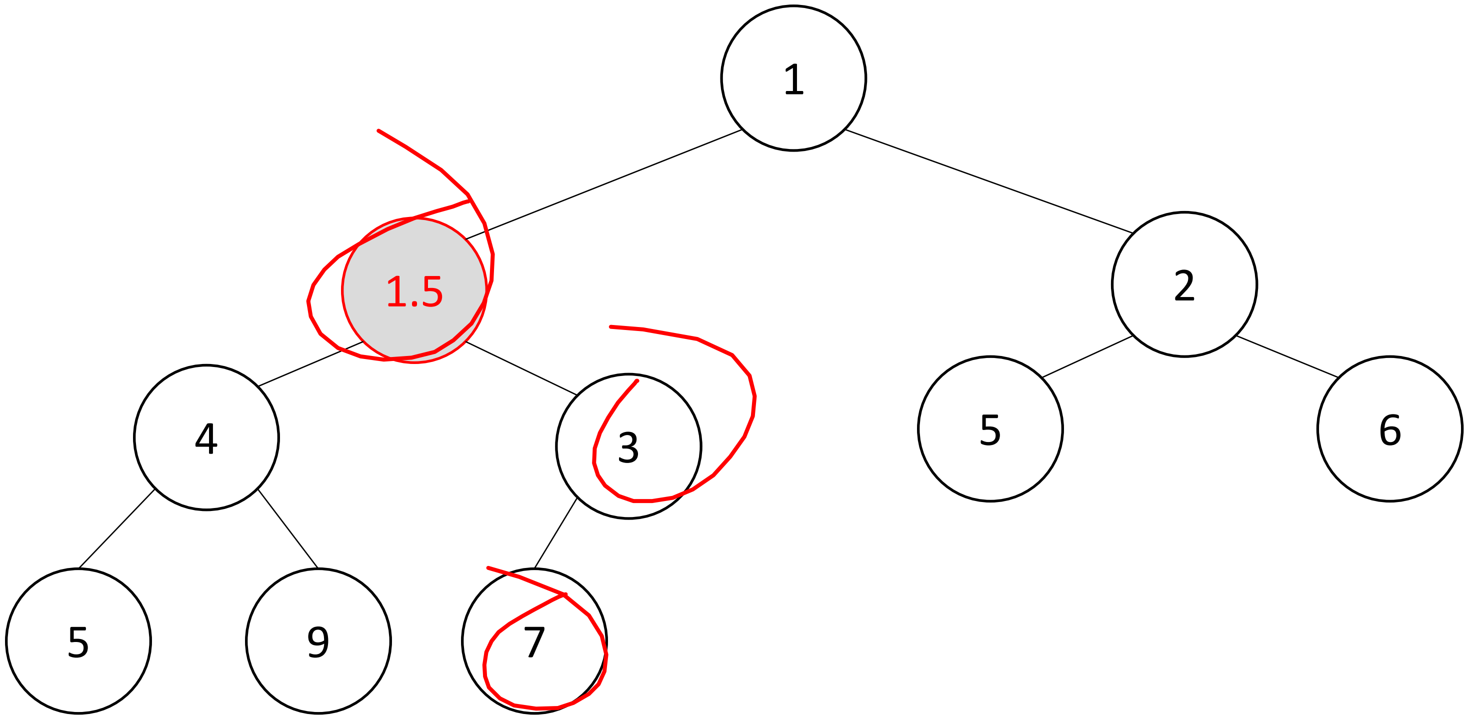
Heap Insert




```
insert(item){
    put item in the "next open" spot (keep tree complete)
    while (item.priority < parent(item).priority){
        swap item with parent
    }
}
```

Percolate Up

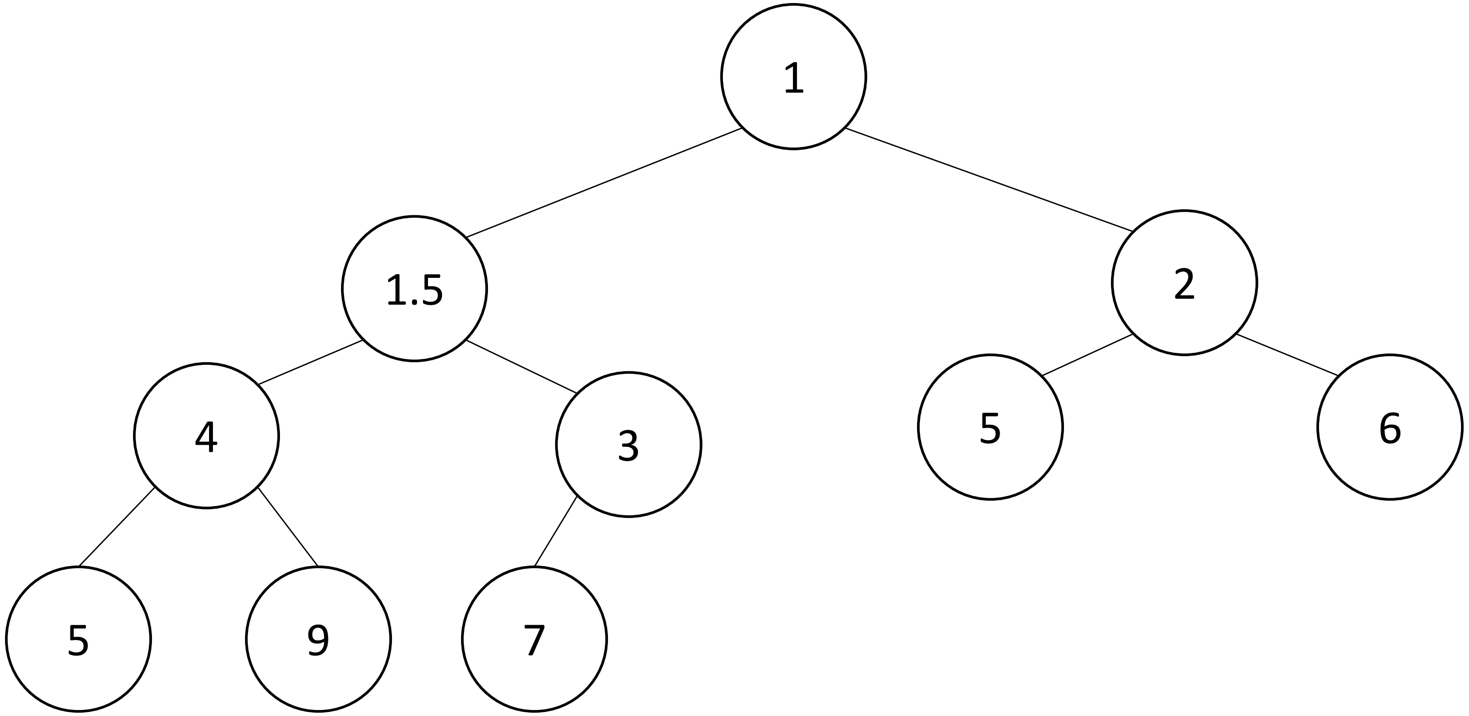
Heap Insert




```
insert(item){
    put item in the "next open" spot (keep tree complete)
    while (item.priority < parent(item).priority){
        swap item with parent
    }
}
```

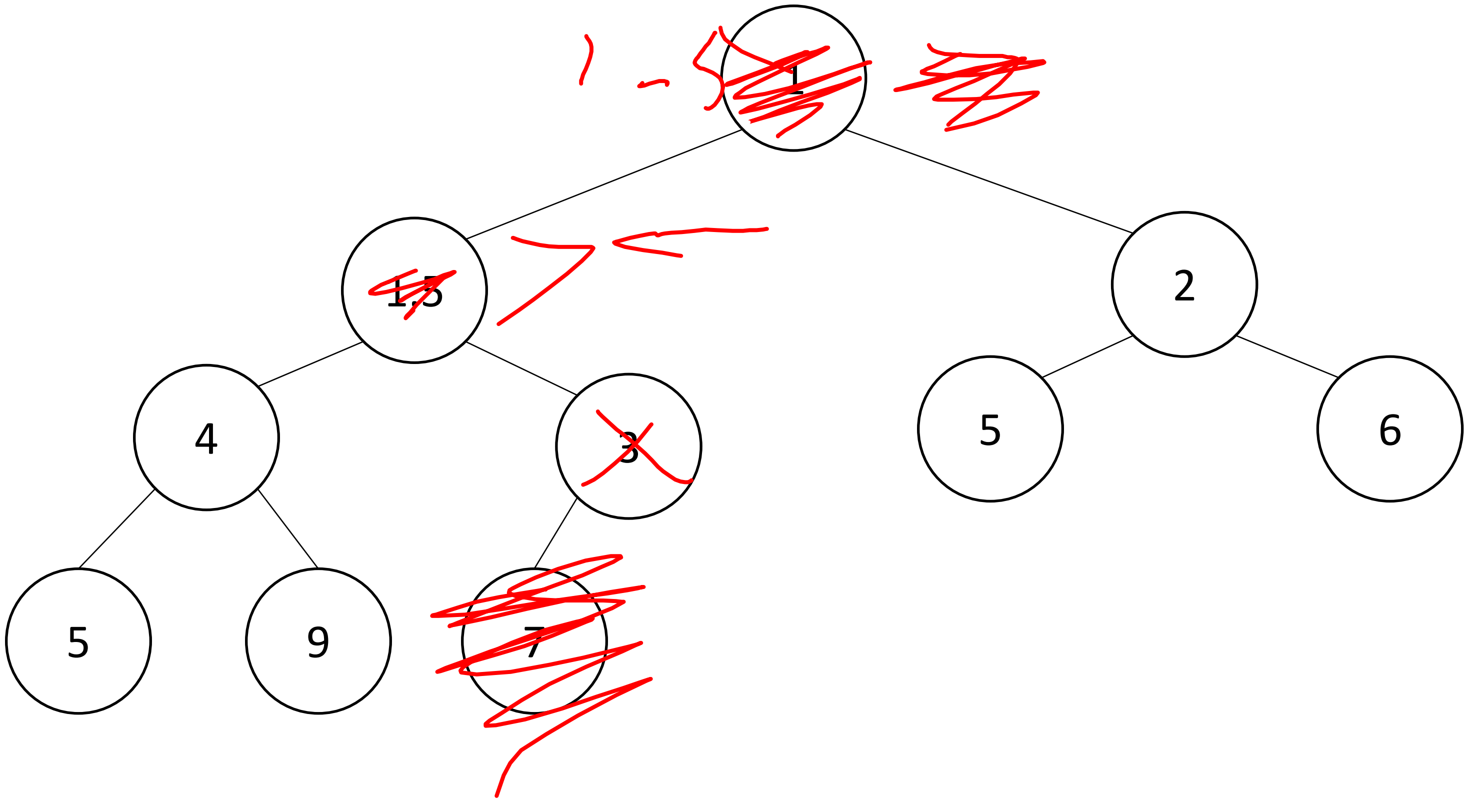
Percolate Up

Heap Insert



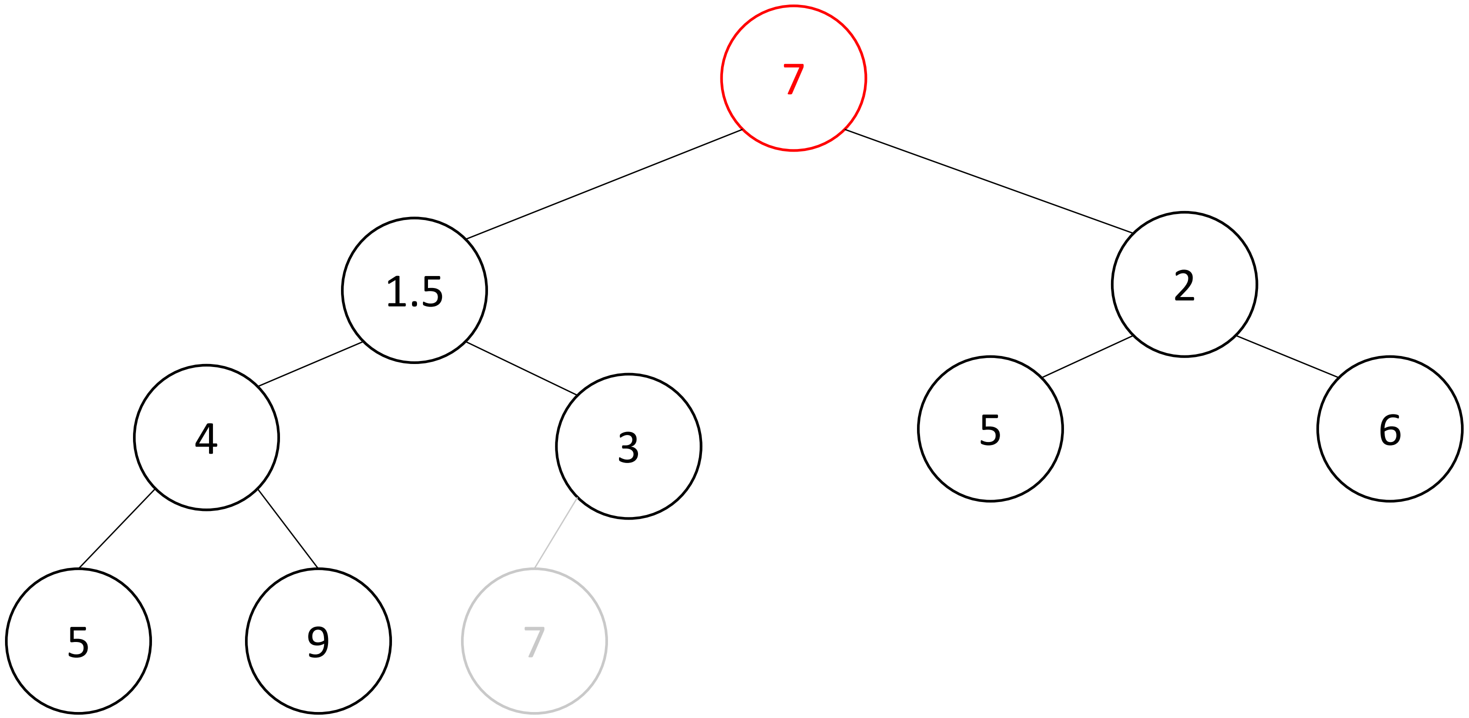

```
insert(item){
   put item in the "next open" spot (keep tree complete)
   while (item.priority < parent(item).priority){
      swap item with parent
   }
}
```

Heap deleteMin




```
deleteMin(){
    min = root
    br = bottom-right item
    move br to the root
    while(br > either of its children){
        swap br with its smallest child
    }
    return min
}
```

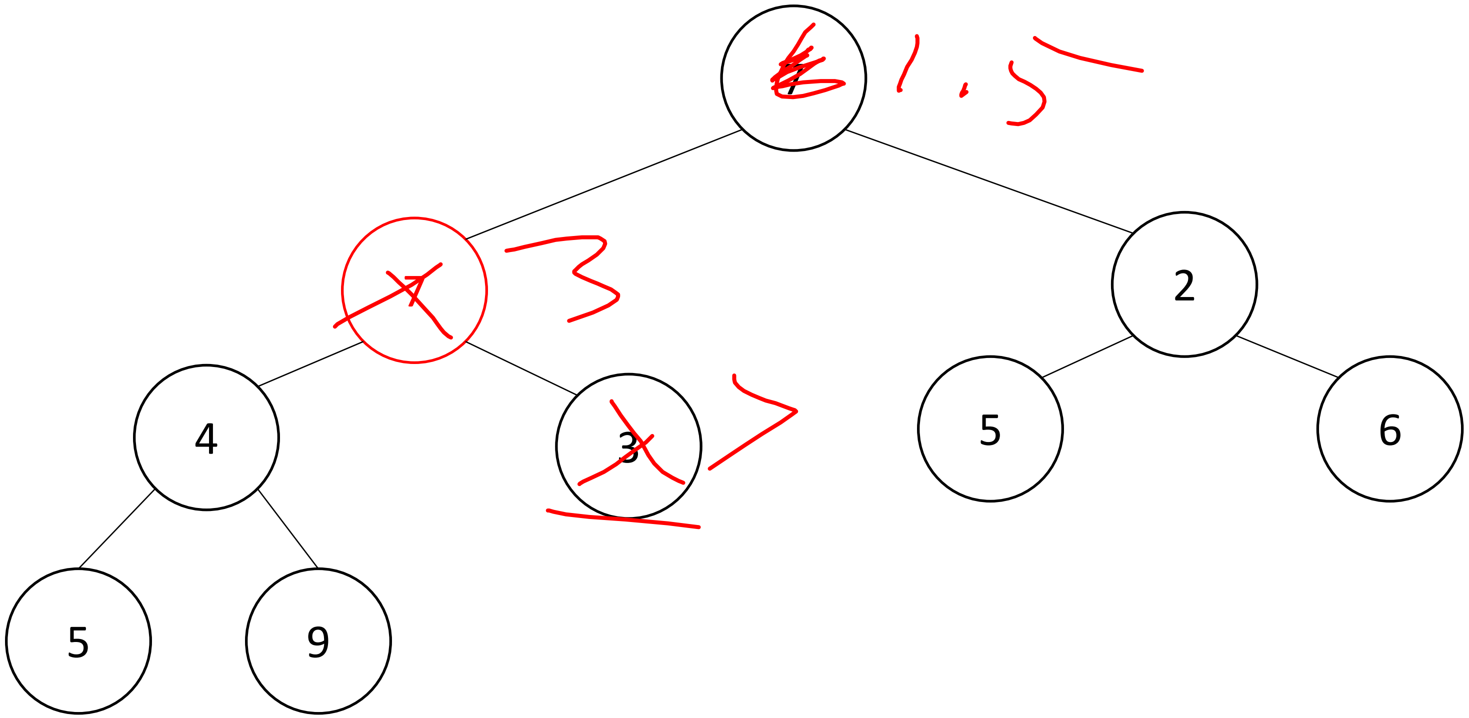
Heap deleteMin




```
deleteMin(){
    min = root
    br = bottom-right item
    move br to the root
    while(br > either of its children){
        swap br with its smallest child
    }
    return min
}
```

$\log n$

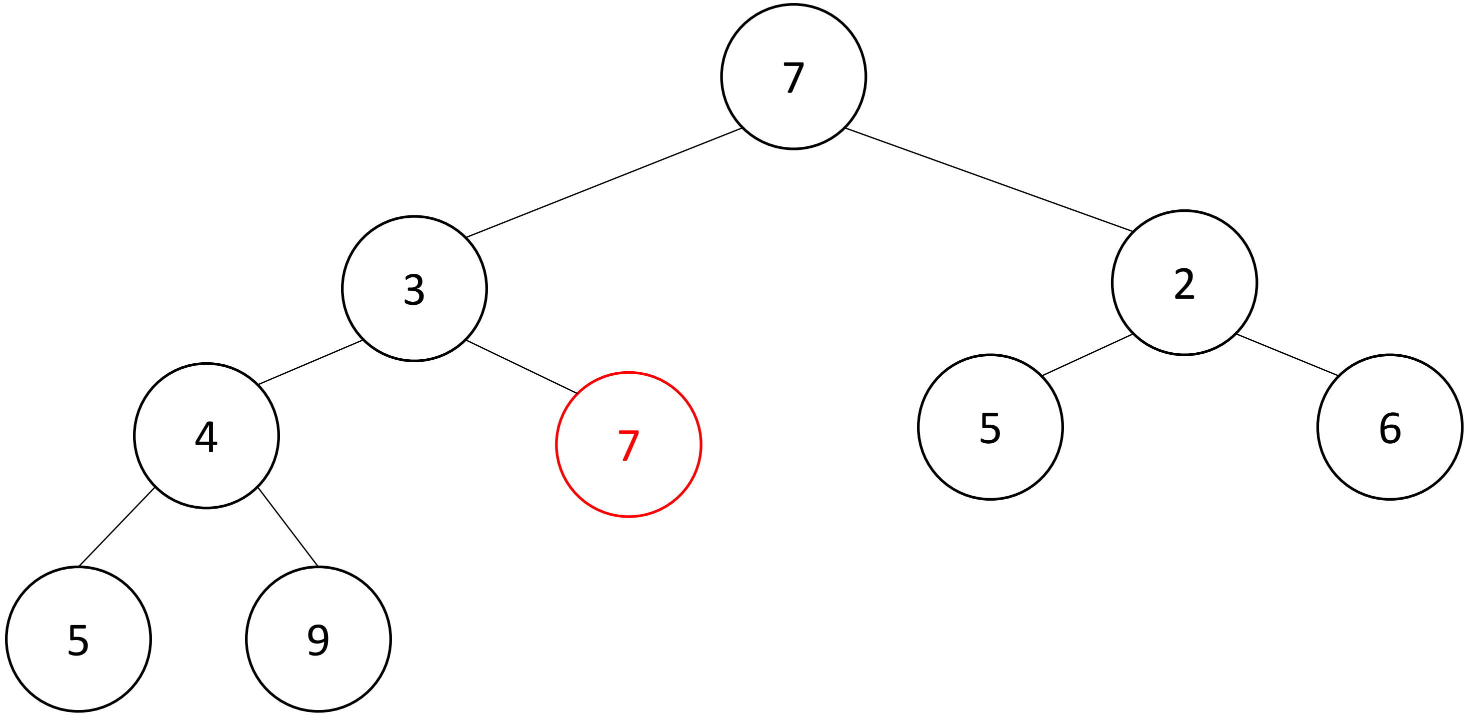
Heap deleteMin




```
deleteMin(){
    min = root
    br = bottom-right item
    move br to the root
    while(br > either of its children){
        swap br with its smallest child
    }
    return min
}
```

Percolate Down

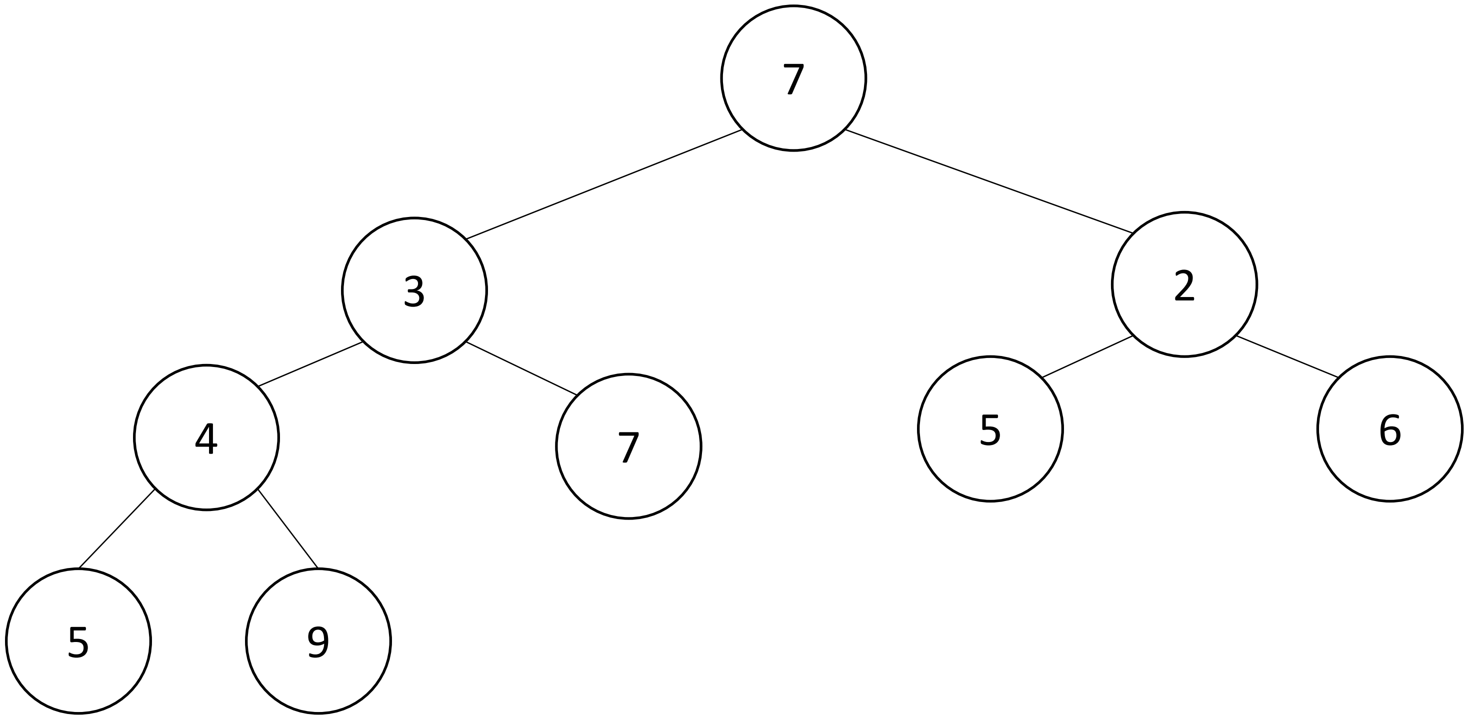
Heap deleteMin




```
deleteMin(){
   min = root
   br = bottom-right item
   move br to the root
   while(br > either of its children){
      swap br with its smallest child
   }
   return min
}
```

Percolate Down

Heap deleteMin




```
deleteMin(){
    min = root
    br = bottom-right item
    move br to the root
    while(br > either of its children){
        swap br with its smallest child
    }
    return min
}
```

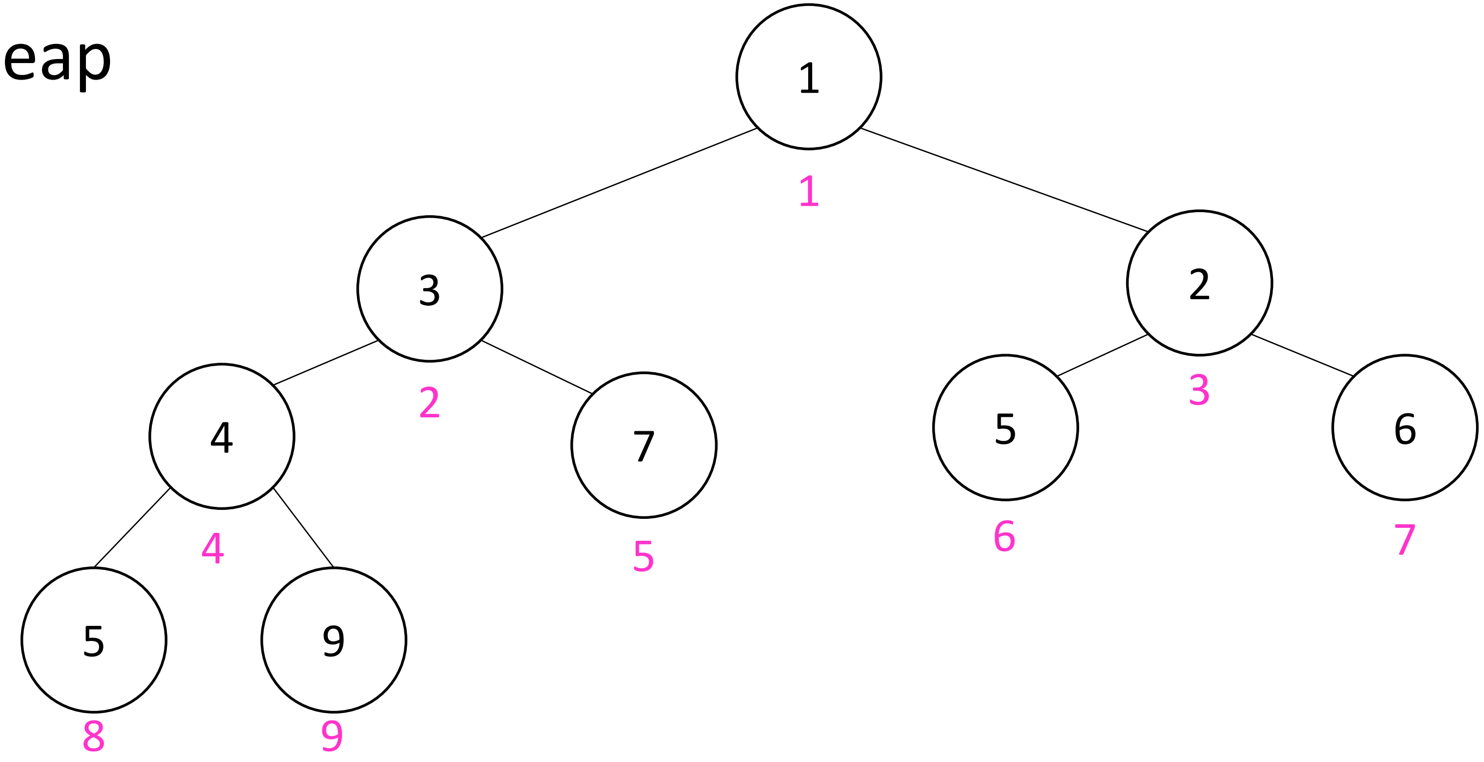
Percolate Up and Down (for a Min Heap)

- Goal: restore the “Heap Property”
- Percolate Up:
  - Take a node that may be smaller than a parent, repeatedly swap with a parent until it is larger than its parent
- Percolate Down:
  - Take a node that may be larger than one of its children, repeatedly swap with smallest child until both children are larger
- Worst case running time of each:
  - $\Theta(\log n)$

Representing a Heap

	1	3	2	4	7	5	6	5	9
0	1	2	3	4	5	6	7	8	9

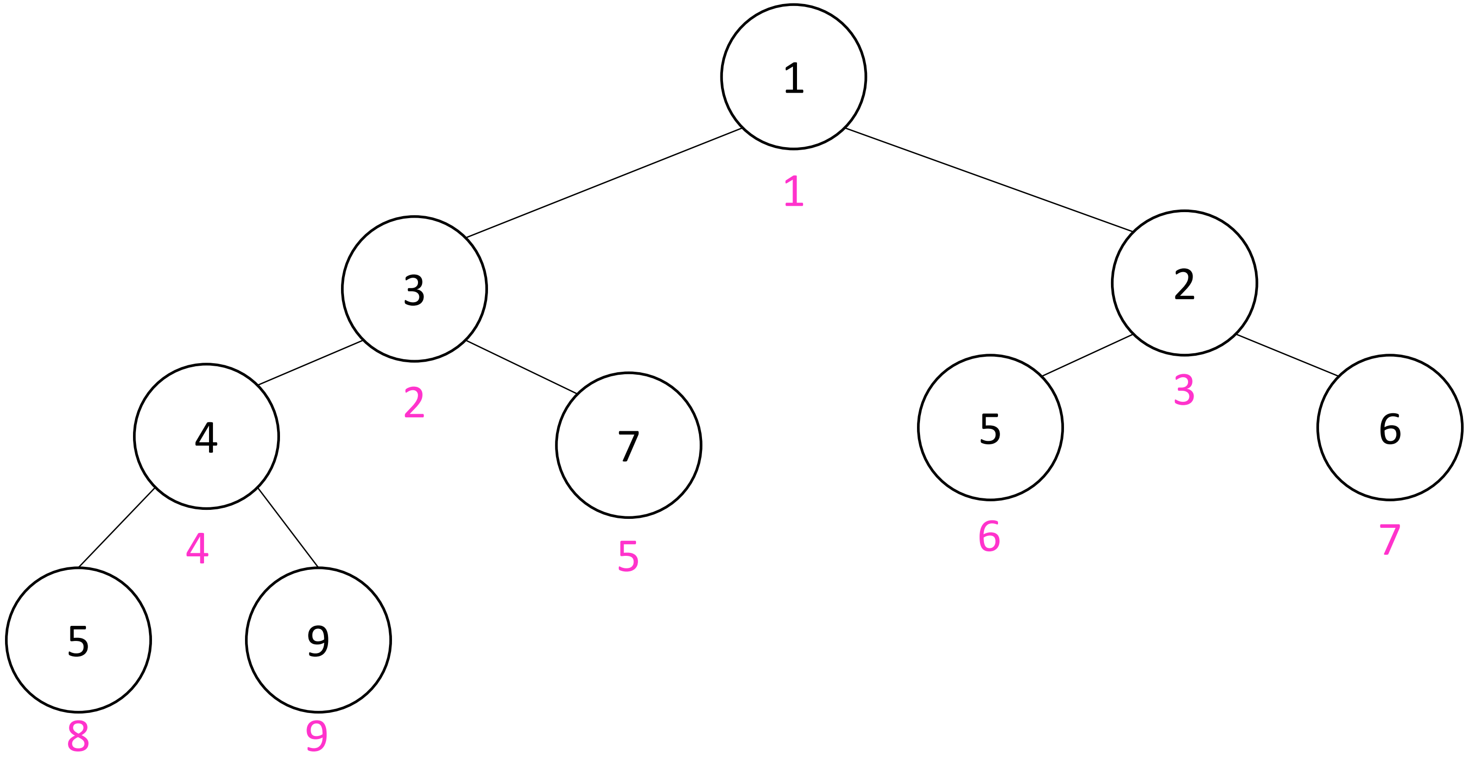
- Every complete binary tree with the same number of nodes uses the same positions and edges
- Use an array to represent the heap
- Index of root:
- Parent of node i:
- Left child of node i:
- Right child of node i:
- Location of the leaves:



Insert Psuedocode

	1	3	2	4	7	5	6	5	9	
0	1	2	3	4	5	6	7	8	9	10

```
insert(item){
    if(size == arr.length – 1){resize();}
    size++;
    arr[i] = item;
    percolateUp(i)
}
```



Percolate Up

```
percolateUp(int i){
    int parent = i/2;  \\ index of parent
    Item val = arr[i];  \\ value at current location
    while(i > 1 && arr[i].priority < arr[parent].priority){  \\ until location is root or heap property holds
        arr[i] = arr[parent];  \\ move parent value to this location
        arr[parent] = val; \\ put current value into parent's location
        i = parent;  \\ make current location the parent
        parent = i/2;  \\ update new parent
    }
}
```

DeleteMin Psuedocode

```
deleteMin(){
    theMin = arr[1];
    arr[1] = arr[size];
    size--;
    percolateDown(1);
    return theMin;
}
```

Percolate Down

```
percolateDown(int i){
    int left = i*2;  \\ index of left child
    int right = i*2+1;  \\ index of right child
    Item val = arr[i];  \\ value at location
    while(left <= size){  \\ until location is leaf
        int toSwap = right;
        if(right > size || arr[left].priority < arr[right] .priority){  \\ if there is no right child or if left child is smaller
            toSwap = left;  \\ swap with left
        } \\ now toSwap has the smaller of left/right, or left if right does not exist
        if (arr[toSwap] .priority < val.priority){  \\ if the smaller child is less than the current value
            arr[i] = arr[toSwap];
            arr[toSwap] = val; \\ swap parent with smaller child
            i = toSwap; \\ update current node to be smaller child
            left = i*2;
            right = i*2+1;
        }
        else{ return;} \\ if we don't swap, then heap property holds
    }
}
```

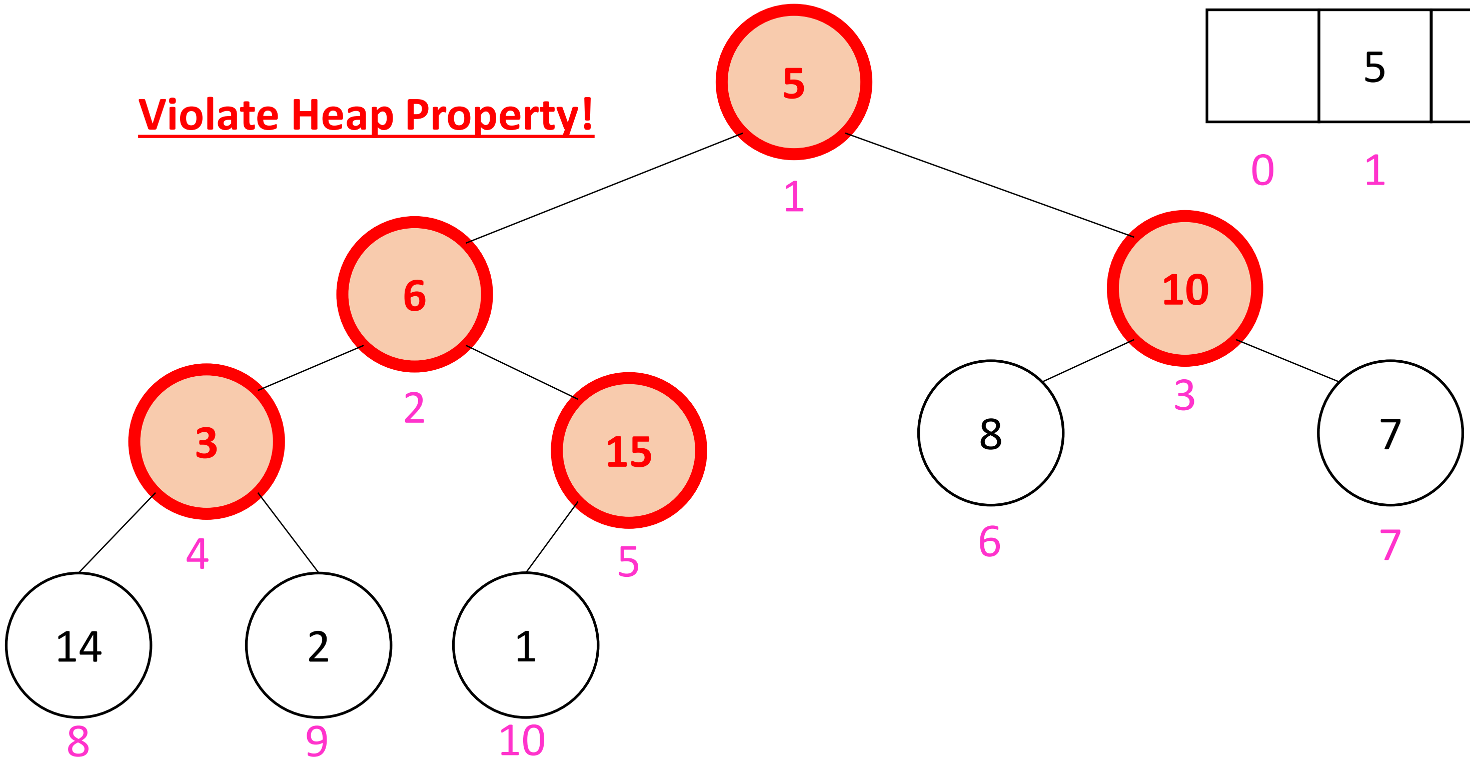
Other Operations

- Increase Key
  - Given the index of an item in the PQ, make its priority value larger
    - Min Heap: Then percolate down
    - Max Heap: Then percolate up
- Decrease Key
  - Given the index of an item in the PQ, make its priority value smaller
    - Min Heap: Then percolate up
    - Max Heap: Then percolate down
- Remove
  - Given the item at the given index from the PQ

Aside: Expected Running time of Insert

Building a Heap From “Scratch”

- Suppose we had n items and wanted to “heapify” them



	5	6	10	3	15	8	7	14	2	1
0	1	2	3	4	5	6	7	8	9	10

Two ways for “fix” the heap:

1) Percolate Up
2) Percolate Down

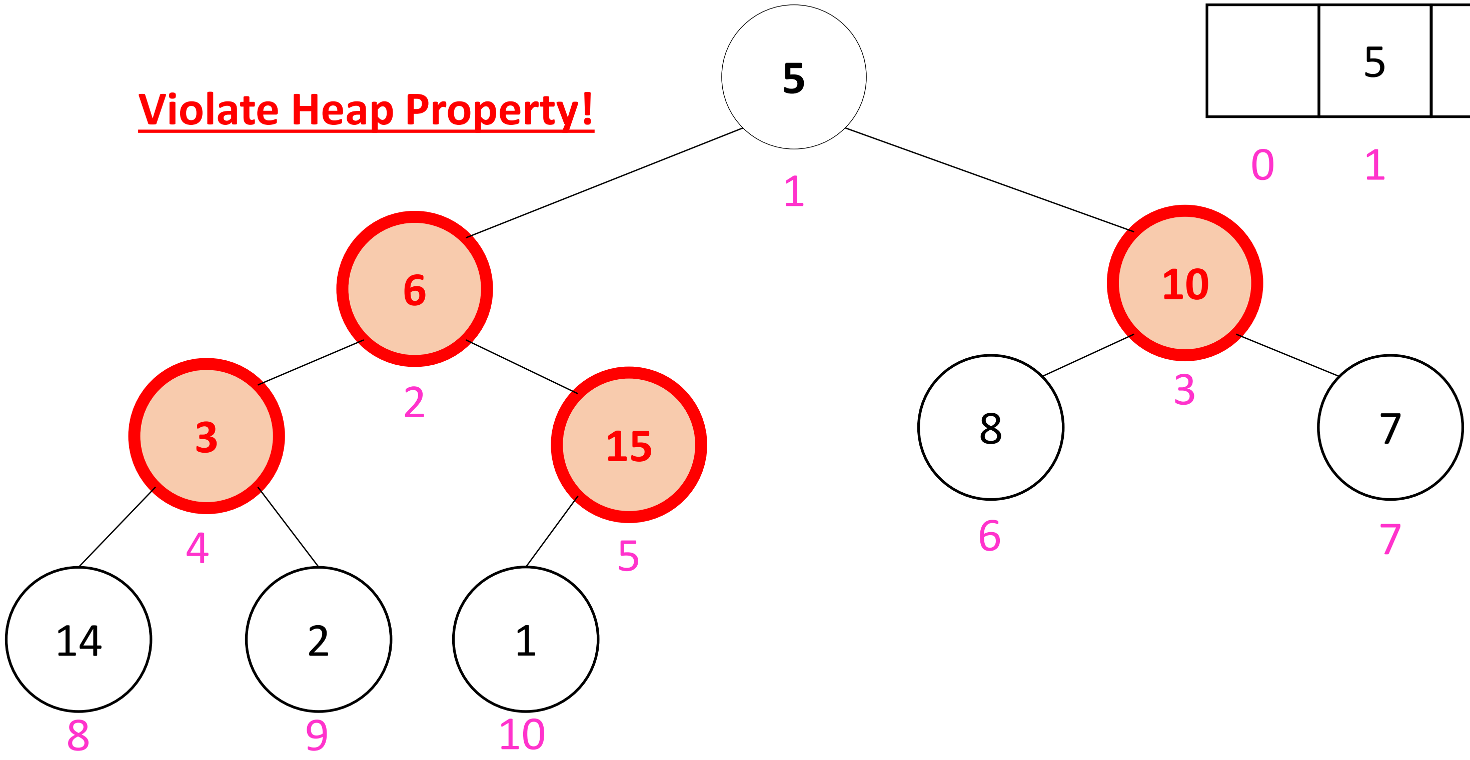
Floyd's buildHeap method

- Working towards the root, one row at a time, percolate down

```
buildHeap(){
    for(int i = size; i>0; i--){
        percolateDown(i);
    }
}
```

Floyd's buildHeap method

- Suppose we had n items and wanted to "heapify" them

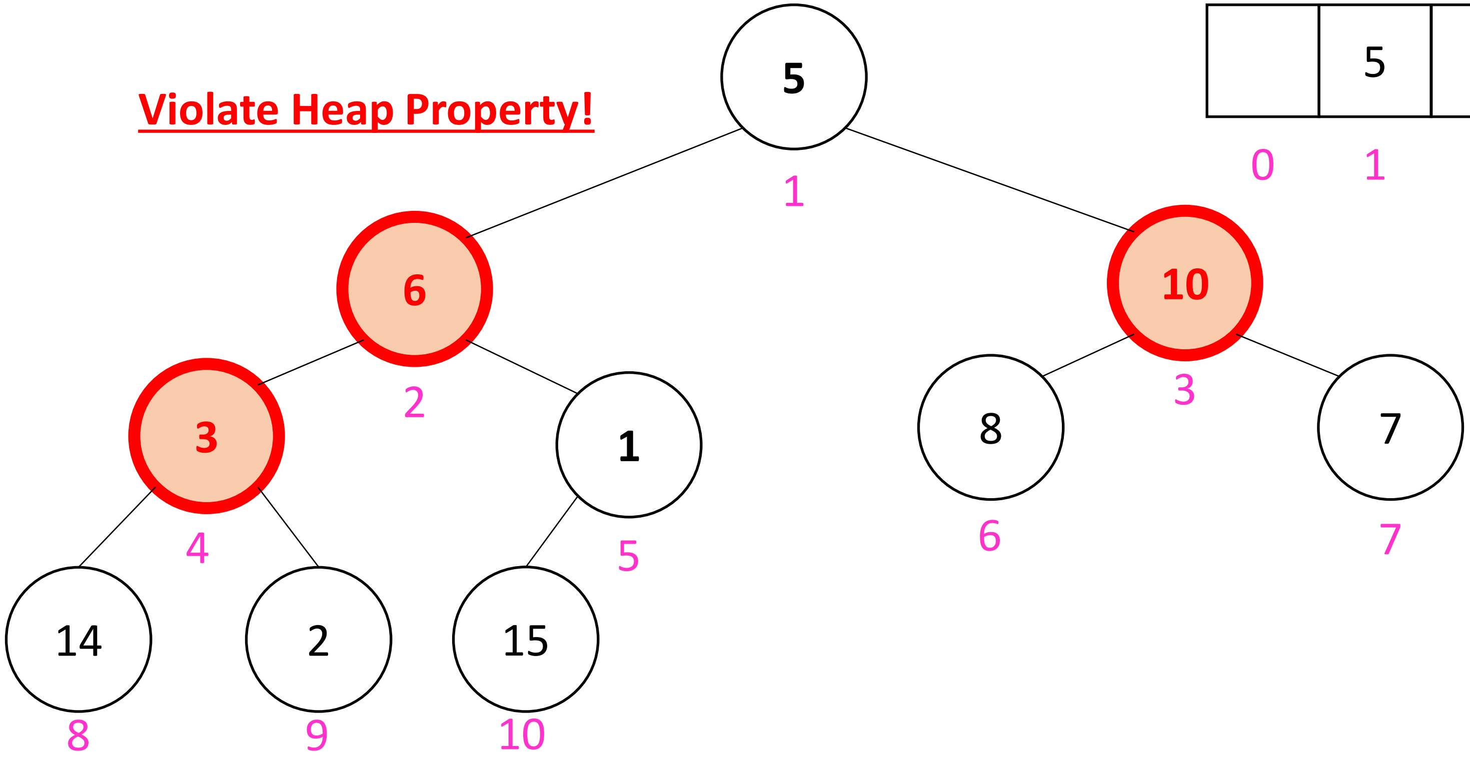


0	1	2	3	4	5	6	7	8	9	10
	5	6	10	3	15	8	7	14	2	1

```
buildHeap(){
   for(int i = size; i>0; i--){
      percolateDown(i);
   }
}
```

Floyd's buildHeap method

- Suppose we had n items and wanted to "heapify" them

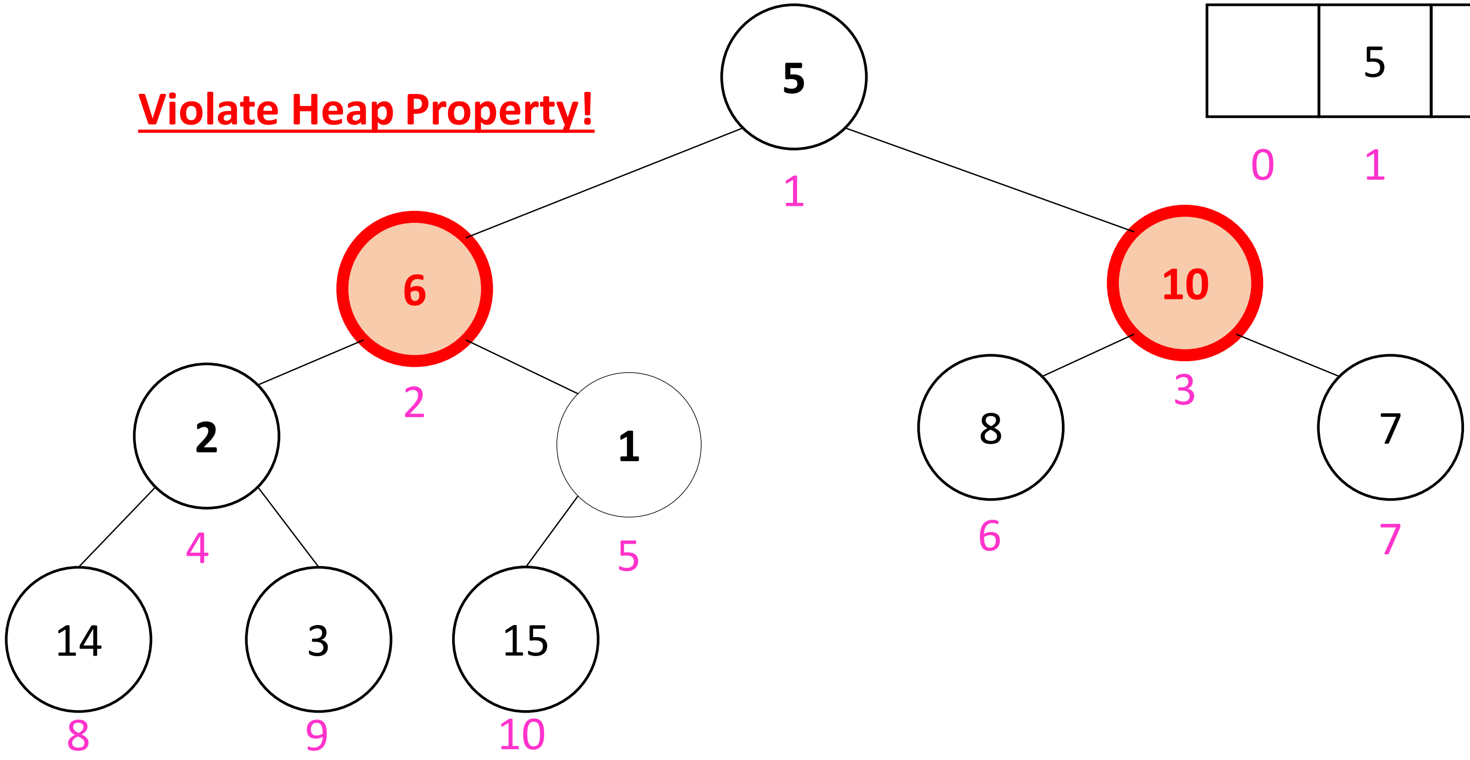


	5	6	10	3	15	8	7	14	2	1
0	1	2	3	4	5	6	7	8	9	10

```
buildHeap(){
   for(int i = size; i>0; i--){
      percolateDown(i);
   }
}
```

Floyd's buildHeap method

- Suppose we had n items and wanted to "heapify" them

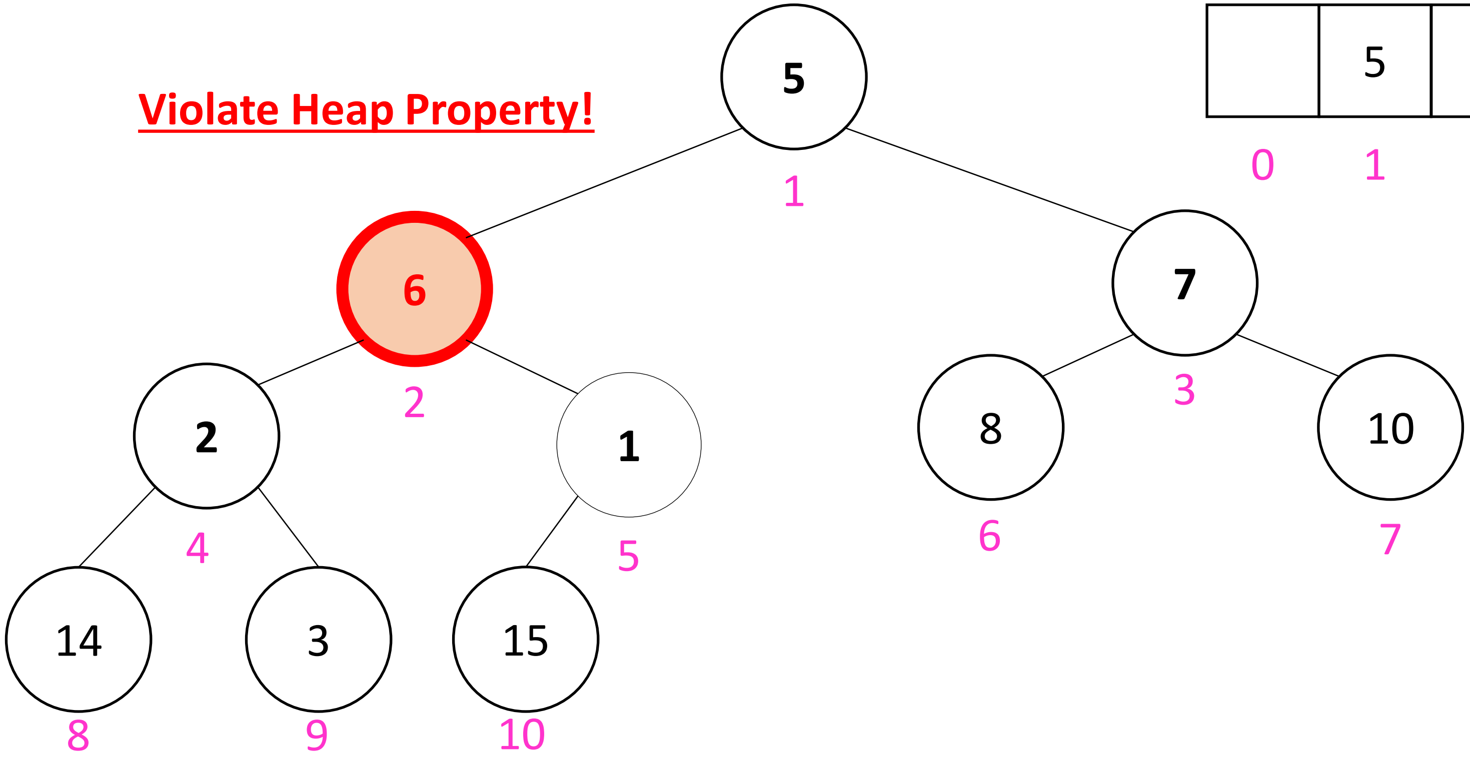


	5	6	10	3	15	8	7	14	2	1
0	1	2	3	4	5	6	7	8	9	10

```
buildHeap(){
    for(int i = size; i>0; i--){
        percolateDown(i);
    }
}
```

Floyd’s buildHeap method

- Suppose we had n items and wanted to “heapify” them

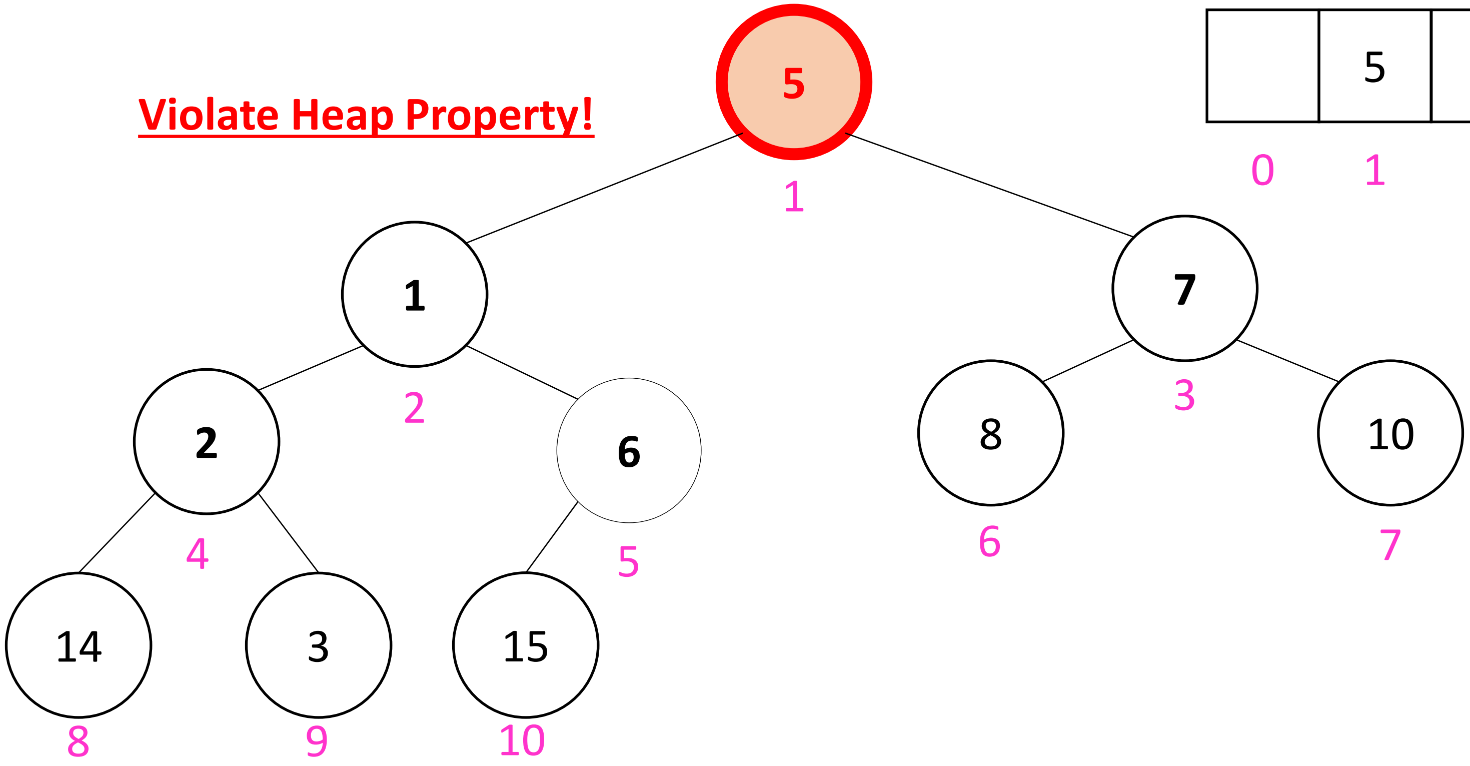


	5	6	10	3	15	8	7	14	2	1
0	1	2	3	4	5	6	7	8	9	10

```
buildHeap(){
   for(int i = size; i>0; i--){
      percolateDown(i);
   }
}
```

Floyd's buildHeap method

- Suppose we had n items and wanted to "heapify" them



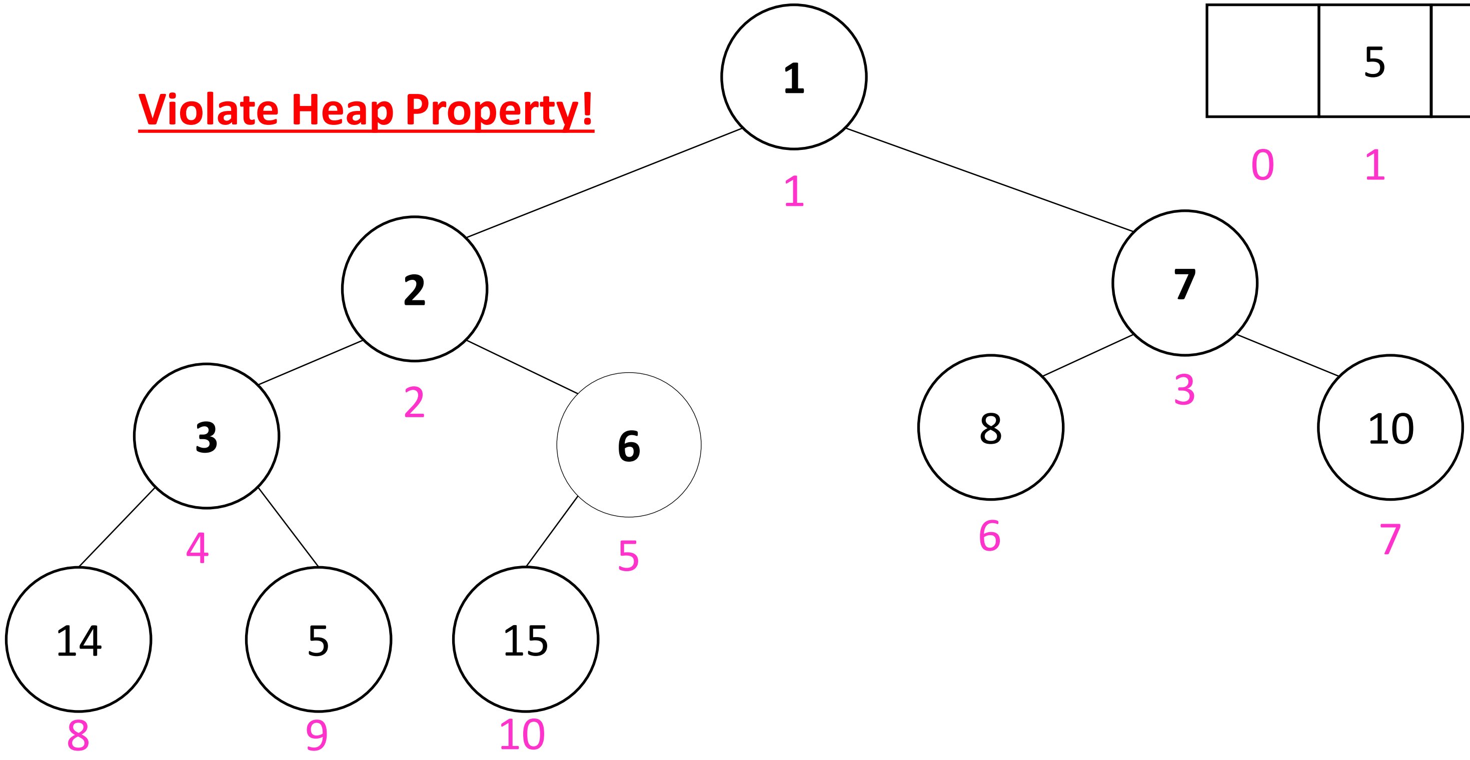
	5	6	10	3	15	8	7	14	2	1
0	1	2	3	4	5	6	7	8	9	10

```
buildHeap(){
   for(int i = size; i>0; i--){
      percolateDown(i);
   }
}
```

Floyd's buildHeap method

- Suppose we had n items and wanted to "heapify" them

Violate Heap Property!



	5	6	10	3	15	8	7	14	2	1
0	1	2	3	4	5	6	7	8	9	10

```
buildHeap(){
   for(int i = size; i>0; i--){
      percolateDown(i);
   }
}
```

How long did this take?

```
buildHeap(){
    for(int i = size; i>0; i--){
        percolateDown(i);
    }
}
```

- Worst case running time of buildHeap:
- No node can percolate down more than the height of its subtree
  - When i is a leaf:
  - When i is second-from-last level:
  - When i is third-from-last level:
- Overall Running time: