PAUL G. ALLEN SCHOOL
OF COMPUTER SCIENCE & ENGINEERING

CSE341: Programming Languages

Lecture 2
Functions, Pairs, Lists

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Function definitions

Functions: the most important building block in the whole course

- Like Java methods, have arguments and result
- But no classes, **`this`**, **`return`**, etc.

Example *function binding*:

```
(* Note: correct only if y>=0 *)

fun pow (x : int, y : int) =
  if y=0
  then 1
  else x * pow(x,y-1)
```

Note: The *body* includes a (recursive) *function call*: **`pow(x,y-1)`**

Example, extended

```
fun pow (x : int, y : int) =
  if y=0
  then 1
  else x * pow(x,y-1)

fun cube (x : int) =
  pow (x,3)

val sixtyfour = cube 4

val fortytwo = pow(2,2+2) + pow(4,2) + cube(2) + 2
```

Some gotchas

Three common "gotchas"

- Bad error messages if you mess up function-argument syntax
- The use of `*` in type syntax is not multiplication
  - Example: `int * int -> int`
  - In expressions, `*` is multiplication: `x * pow(x,y-1)`
- Cannot refer to later function bindings
  - That's simply ML's rule
  - Helper functions must come before their uses
  - Need special construct for *mutual recursion* (later)

Recursion

- If you're not yet comfortable with recursion, you will be soon ☺
  - Will use for most functions taking or returning lists

- "Makes sense" because calls to same function solve "simpler" problems

- Recursion more powerful than loops
  - We won't use a single loop in ML
  - Loops often (not always) obscure simple, elegant solutions

Function bindings: 3 questions

- Syntax: `fun x0 (x1 : t1, … , xn : tn) = e`
  - (Will generalize in later lecture)

- Evaluation: ***A function is a value!*** (No evaluation yet)
  - Adds **`x0`** to environment so *later* expressions can *call* it
  - (Function-call semantics will also allow recursion)

- Type-checking:
  - Adds binding **`x0 : (t1 * … * tn) -> t`** if:
  - Can type-check body **`e`** to have type **`t`** in the static environment containing:
    - "Enclosing" static environment (earlier bindings)
    - **`x1 : t1, …, xn : tn`** (arguments with their types)
    - **`x0 : (t1 * … * tn) -> t`** (for recursion)

More on type-checking

```
fun x0 (x1 : t1, … , xn : tn) = e
```

- New kind of type: `(t1 * … * tn) -> t`
  - Result type on right
  - The overall type-checking result is to give `x0` this type in rest of program (unlike Java, not for earlier bindings)
  - Arguments can be used only in `e` (unsurprising)

- Because evaluation of a call to `x0` will return result of evaluating `e`, the return type of `x0` is the type of `e`

- The type-checker "magically" figures out `t` if such a `t` exists
  - Later lecture: Requires some cleverness due to recursion
  - More magic after hw1: Later can omit argument types too

Function Calls

A new kind of expression: 3 questions

Syntax: ***`e0 (e1,…,en)`***

- (Will generalize later)
- Parentheses optional if there is exactly one argument

Type-checking:

If:

- **`e0`** has some type **`(t1 * … * tn) -> t`**
- **`e1`** has type **`t1`**, …, **`en`** has type **`tn`**

Then:

- **`e0(e1,…,en)`** has type **`t`**

Example: **`pow(x,y-1)`** in previous example has type **`int`**

Function-calls continued

`e0(e1,…,en)`

Evaluation:

1. (Under current dynamic environment,) evaluate **`e0`** to a function **`fun`** ***`x0`*** **`(`*****`x1`*** **`:`** ***`t1`*****`,`** … **`,`** ***`xn`*** **`:`** ***`tn`*****`)`** **`=`** ***`e`***
   – Since call type-checked, result *will be* a function

2. (Under current dynamic environment,) evaluate arguments to values **`v1,`** …**`,`** **`vn`**

3. Result is evaluation of ***`e`*** in an environment extended to map **`x1`** to **`v1`**, …, **`xn`** to **`vn`**
   – ("An environment" is actually the environment where the function was defined, and includes **`x0`** for recursion)

Tuples and lists

So far: numbers, booleans, conditionals, variables, functions

- Now ways to build up data with multiple parts
- This is essential
- Java examples: classes with fields, arrays

Now:

- *Tuples*: fixed “number of pieces” that may have different types

Then:

- *Lists*: any “number of pieces” that all have the same type

Later:

- Other more general ways to create compound data

Pairs (2-tuples)

Need a way to *build* pairs and a way to *access* the pieces

Build:

- Syntax: `(e1,e2)`
- Evaluation: Evaluate **`e1`** to **`v1`** and **`e2`** to **`v2`**; result is **`(v1,v2)`**
  - A pair of values is a value
- Type-checking: If **`e1`** has type **`ta`** and **`e2`** has type **`tb`**, then the pair expression has type **`ta * tb`**
  - A new kind of type

Pairs (2-tuples)

Need a way to *build* pairs and a way to *access* the pieces

Access:

- Syntax: ***`#1 e`*** and ***`#2 e`***

- Evaluation: Evaluate **`e`** to a pair of values and return first or second piece
  - Example: If **`e`** is a variable **`x`**, then look up **`x`** in environment

- Type-checking: If **`e`** has type **`ta * tb`**, then **`#1 e`** has type **`ta`** and **`#2 e`** has type **`tb`**

Examples

Functions can take and return pairs

```
fun swap (pr : int*bool) =
   (#2 pr, #1 pr)

fun sum_two_pairs (pr1 : int*int, pr2 : int*int) =
   (#1 pr1) + (#2 pr1) + (#1 pr2) + (#2 pr2)

fun div_mod (x : int, y : int) =
   (x div y, x mod y)

fun sort_pair (pr : int*int) =
  if (#1 pr) < (#2 pr)
  then pr
  else (#2 pr, #1 pr)
```

Tuples

Actually, you can have *tuples* with more than two parts

- A new feature: a generalization of pairs

- **`(e1,e2,…,en)`**
- **`ta * tb * … * tn`**
- **`#1 e, #2 e, #3 e, …`**

Homework 1 uses triples of type **`int*int*int`** a lot

Nesting

Pairs and tuples can be nested however you want

- Not a new feature: implied by the syntax and semantics

```
val x1 = (7,(true,9)) (* int * (bool*int) *)

val x2 = #1 (#2 x1)    (* bool *)

val x3 = (#2 x1)       (* bool*int *)

val x4 = ((3,5),((4,8),(0,0)))
         (* (int*int)*((int*int)*(int*int)) *)
```

Lists

- Despite nested tuples, the type of a variable still "commits" to a particular "amount" of data

In contrast, a list:

- Can have any number of elements
- But all list elements have the same type

Need ways to *build* lists and *access* the pieces…

Building Lists

- The empty list is a value:

```
[]
```

- In general, a list of values is a value; elements separated by commas:

```
[v1,v2,…,vn]
```

- If **`e1`** evaluates to **`v`** and **`e2`** evaluates to a list **`[v1,…,vn]`**, then **`e1::e2`** evaluates to **`[v,v1,…,vn]`**

```
e1::e2 (* pronounced "cons" *)
```

Accessing Lists

Until we learn pattern-matching, we will use three standard-library functions

- **`null e`** evaluates to **`true`** if and only if **`e`** evaluates to **`[]`**
- If **`e`** evaluates to **`[v1,v2,…,vn]`** then **`hd e`** evaluates to **`v1`**
  - (raise exception if **`e`** evaluates to **`[]`**)
- If **`e`** evaluates to **`[v1,v2,…,vn]`** then **`tl e`** evaluates to **`[v2,…,vn]`**
  - (raise exception if **`e`** evaluates to **`[]`**)
  - Notice result is a list

Type-checking list operations

Lots of new types: For any type **`t`**, the type **`t list`** describes lists where all elements have type **`t`**

- Examples: **`int list`** **`bool list`** **`int list list`** **`(int * int) list`** **`(int list * int) list`**

- So **`[]`** can have type **`t list`** for *any* type **`t`**
  - SML uses type **`'a list`** to indicate this (“tick a” or “alpha”)
- For **`e1::e2`** to type-check, we need a **`t`** such that **`e1`** has type **`t`** and **`e2`** has type **`t list`**. Then the result type is **`t list`**
- **`null : 'a list -> bool`**
- **`hd   : 'a list -> 'a`**
- **`tl   : 'a list -> 'a list`**

Example list functions

```
fun sum_list (xs : int list) =
  if null xs
  then 0
  else hd(xs) + sum_list(tl(xs))

fun countdown (x : int) =
  if x=0
  then []
  else x :: countdown (x-1)

fun append (xs : int list, ys : int list) =
  if null xs
  then ys
  else hd (xs) :: append (tl(xs), ys)
```

Recursion again

Functions over lists are usually recursive

– Only way to “get to all the elements”

- What should the answer be for the empty list?
- What should the answer be for a non-empty list?
  – Typically in terms of the answer for the tail of the list!

Similarly, functions that produce lists of potentially any size will be recursive

– You create a list out of smaller lists

Lists of pairs

Processing lists of pairs requires no new features. Examples:

```
fun sum_pair_list (xs : (int*int) list) =
  if null xs
  then 0
  else #1(hd xs) + #2(hd xs) + sum_pair_list(tl xs)

fun firsts (xs : (int*int) list) =
  if null xs
  then []
  else #1(hd xs) :: firsts(tl xs)

fun seconds (xs : (int*int) list) =
  if null xs
  then []
  else #2(hd xs) :: seconds(tl xs)

fun sum_pair_list2 (xs : (int*int) list) =
 (sum_list (firsts xs)) + (sum_list (seconds xs))
```