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# CSE 341

## Section 4

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With thanks to Alexander Lent, Nick Mooney, Spencer  
Pearson

# Today's Agenda

- Standard-Library Docs
- More Currying and Higher Order Functions
- Mutual Recursion

# Standard Basis Documentation

## Online Documentation

<http://www.standardml.org/Basis/index.html>

<http://www.smlnj.org/doc/smlnj-lib/Manual/toc.html>

## Helpful Subset

Top-Level <http://www.standardml.org/Basis/top-level-chapter.html>

List <http://www.standardml.org/Basis/list.html>

ListPair <http://www.standardml.org/Basis/list-pair.html>

Real <http://www.standardml.org/Basis/real.html>

String <http://www.standardml.org/Basis/string.html>

# Higher-Order Functions Review

- A function that returns a function or takes a function as an argument.

## Canonical Examples

- `map : ('a -> 'b) * 'a list -> 'b list`
  - Applies a function to every element of a list and return a list of the resulting values.
  - Example: `map (fn x => x*3, [1,2,3]) === [3,6,9]`
- `filter : ('a -> bool) * 'a list -> 'a list`
  - Returns the list of elements from the original list that, when a predicate function is applied, result in true.
  - Example: `filter (fn x => x>2, [~5,3,2,5]) === [3,5]`

**Note:** `List.map` and `List.filter` are similarly defined in SML but use currying.

# Higher-Order Functions Review

- `foldl`:  $(f: 'a * 'b \rightarrow 'b) (acc: 'b) (l: 'a \text{ list}) \rightarrow 'b$ 
  - $f(l_n, f(\dots, (f(l_2, f(l_1, acc))))$
  - Apply function to the current element and the accumulator as soon as possible
- `foldr`:  $(f: 'a * 'b \rightarrow 'b) (acc: 'b) (l: 'a \text{ list}) \rightarrow 'b$ 
  - $f(l_1, f(l_2, f(\dots, f(l_n, acc))))$
  - Wait until the rest of the list has been evaluated and then apply function to the current element and result from rest of the list
- We've written `foldl` in lecture, write `foldr`

# Broader Idea

## **Functions are Awesome!**

- SML functions can be passed around like any other value.
- They can be passed as function arguments, returned, and even stored in data structures or variables.
- Functions like `map` are very pervasive in functional languages.
  - A function like `map` can even be written for other data structures such as trees.

# Currying and High Order Functions

- Some functions from standard library:
  - List.map
  - List.filter
  - List.foldl
  - List.foldr
- Write our own higher order functions
  - Alternating 0 and 1

# Mutual Recursion

- What if we need function `f` to call `g`, and function `g` to call `f`?
- This is a common idiom

```
fun earlier x =  
    ...  
    later x  
    ...  
fun later x =  
    ...  
    earlier x  
    ...
```

Unfortunately this  
does not work 😞



# Mutual Recursion Workaround

- We can use higher order functions to get this working
- It works, but there has got to be a better way!

```
fun earlier f x =  
  ...  
  f x  
  ...  
fun later x =  
  ...  
  earlier later x  
  ...
```

# Mutual Recursion with **and**

- SML has a keyword for that
- Works with mutually recursive **datatype** bindings too

```
fun earlier x =  
    ...  
    later x  
    ...  
and later x =  
    ...  
    earlier x  
    ...
```