PAUL G. ALLEN SCHOOL
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CSE341: Programming Languages

Lecture 2
Functions, Pairs, Lists

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Function definitions

Functions: the most important building block in the whole course

- Like Java methods, have arguments and result
- But no classes, **`this`**, **`return`**, etc.

Example *function binding*:

```
(* Note: correct only if y>=0 *)

fun pow (x : int, y : int) =
  if y=0
  then 1
  else x * pow(x,y-1)
```

Note: The *body* includes a (recursive) *function call*: **`pow(x,y-1)`**

Example, extended

```
fun pow (x : int, y : int) =
  if y=0
  then 1
  else x * pow(x,y-1)

fun cube (x : int) =
  pow (x,3)

val sixtyfour = cube 4

val fortytwo = pow(2,2+2) + pow(4,2) + cube(2) + 2
```


Some gotchas

Three common "gotchas"

- Bad error messages if you mess up function-argument syntax
- The use of **`*`** in type syntax is not multiplication
  - Example: **`int * int -> int`**
  - In expressions, **`*`** is multiplication: **`x * pow(x,y-1)`**
- Cannot refer to later function bindings
  - That's simply ML's rule
  - Helper functions must come before their uses
  - Need special construct for *mutual recursion* (later)

Recursion

- If you're not yet comfortable with recursion, you will be soon ☺
  - Will use for most functions taking or returning lists
- "Makes sense" because calls to same function solve "simpler" problems
- Recursion more powerful than loops
  - We won't use a single loop in ML
  - Loops often (not always) obscure simple, elegant solutions

Function bindings: 3 questions

- Syntax: `fun x0 (x1 : t1, … , xn : tn) = e`
  - (Will generalize in later lecture)
- Evaluation: ***A function is a value!*** (No evaluation yet)
  - Adds **`x0`** to environment so *later* expressions can *call* it
  - (Function-call semantics will also allow recursion)
- Type-checking:
  - Adds binding `x0 : (t1 * … * tn) -> t` if:
  - Can type-check body `e` to have type `t` in the static environment containing:
    - "Enclosing" static environment (earlier bindings)
    - `x1 : t1, …, xn : tn` (arguments with their types)
    - `x0 : (t1 * … * tn) -> t` (for recursion)

More on type-checking

```
fun x0 (x1 : t1, … , xn : tn) = e
```

- New kind of type: `(t1 * … * tn) -> t`
  - Result type on right
  - The overall type-checking result is to give `x0` this type in rest of program (unlike Java, not for earlier bindings)
  - Arguments can be used only in `e` (unsurprising)
- Because evaluation of a call to `x0` will return result of evaluating `e`, the return type of `x0` is the type of `e`
- The type-checker "magically" figures out `t` if such a `t` exists
  - Later lecture: Requires some cleverness due to recursion
  - More magic after hw1: Later can omit argument types too

Function Calls

A new kind of expression: 3 questions

Syntax: `e0 (e1,…,en)`
- (Will generalize later)
- Parentheses optional if there is exactly one argument

Type-checking:

If:
- `e0` has some type `(t1 * … * tn) -> t`
- `e1` has type `t1`, …, `en` has type `tn`

Then:
- `e0(e1,…,en)` has type `t`

Example: `pow(x,y-1)` in previous example has type `int`

Function-calls continued

`e0(e1,…,en)`

Evaluation:

1. (Under current dynamic environment,) evaluate **e0** to a function `fun x0 (x1 : t1, … , xn : tn) = e`
   – Since call type-checked, result *will be* a function
2. (Under current dynamic environment,) evaluate arguments to values **v1, …, vn**
3. Result is evaluation of **e** in an environment extended to map **x1** to **v1**, …, **xn** to **vn**
   – ("An environment" is actually the environment where the function was defined, and includes **x0** for recursion)

Tuples and lists

So far: numbers, booleans, conditionals, variables, functions
- Now ways to build up data with multiple parts
- This is essential
- Java examples: classes with fields, arrays

Now:
- *Tuples*: fixed "number of pieces" that may have different types

Then:
- *Lists*: any "number of pieces" that all have the same type

Later:
- Other more general ways to create compound data

Pairs (2-tuples)

Need a way to *build* pairs and a way to *access* the pieces

Build:

- Syntax: `(e1,e2)`
- Evaluation: Evaluate **e1** to **v1** and **e2** to **v2**; result is **(v1,v2)**
  – A pair of values is a value
- Type-checking: If **e1** has type **ta** and **e2** has type **tb**, then the pair expression has type **ta * tb**
  – A new kind of type

Pairs (2-tuples)

Need a way to *build* pairs and a way to *access* the pieces

Access:

- Syntax: `#1 e` and `#2 e`
- Evaluation: Evaluate **e** to a pair of values and return first or second piece
  – Example: If **e** is a variable **x**, then look up **x** in environment
- Type-checking: If **e** has type **ta * tb**, then **#1 e** has type **ta** and **#2 e** has type **tb**

Examples

Functions can take and return pairs

```
fun swap (pr : int*bool) =
  (#2 pr, #1 pr)

fun sum_two_pairs (pr1 : int*int, pr2 : int*int) =
  (#1 pr1) + (#2 pr1) + (#1 pr2) + (#2 pr2)

fun div_mod (x : int, y : int) =
  (x div y, x mod y)

fun sort_pair (pr : int*int) =
  if (#1 pr) < (#2 pr)
  then pr
  else (#2 pr, #1 pr)
```


Tuples

Actually, you can have *tuples* with more than two parts

– A new feature: a generalization of pairs

- **(e1,e2,…,en)**
- **ta * tb * … * tn**
- **#1 e, #2 e, #3 e, …**

Homework 1 uses triples of type **int*int*int** a lot

Nesting

Pairs and tuples can be nested however you want

– Not a new feature: implied by the syntax and semantics

```
val x1 = (7,(true,9))  (* int * (bool*int) *)

val x2 = #1 (#2 x1)    (* bool *)

val x3 = (#2 x1)       (* bool*int *)

val x4 = ((3,5),((4,8),(0,0)))
         (* (int*int)*((int*int)*(int*int)) *)
```


Lists

- Despite nested tuples, the type of a variable still “commits” to a particular “amount” of data

In contrast, a list:

– Can have any number of elements

– But all list elements have the same type

Need ways to *build* lists and *access* the pieces…

Building Lists

- The empty list is a value:

  `[]`

- In general, a list of values is a value; elements separated by commas:

  `[v1,v2,…,vn]`

- If `e1` evaluates to `v` and `e2` evaluates to a list `[v1,…,vn]`, then `e1::e2` evaluates to `[v,v1,…,vn]`

```
e1::e2 (* pronounced "cons" *)
```


Accessing Lists

Until we learn pattern-matching, we will use three standard-library functions

- `null e` evaluates to `true` if and only if `e` evaluates to `[]`
- If `e` evaluates to `[v1,v2,…,vn]` then `hd e` evaluates to `v1`
  - (raise exception if `e` evaluates to `[]`)
- If `e` evaluates to `[v1,v2,…,vn]` then `tl e` evaluates to `[v2,…,vn]`
  - (raise exception if `e` evaluates to `[]`)
  - Notice result is a list

Type-checking list operations

Lots of new types: For any type `t`, the type `t list` describes lists where all elements have type `t`

- Examples: `int list` `bool list` `int list list` `(int * int) list` `(int list * int) list`

- So `[]` can have type `t list` for *any* type `t`
  - SML uses type `'a list` to indicate this ("tick a" or "alpha")
- For `e1::e2` to type-check, we need a `t` such that `e1` has type `t` and `e2` has type `t list`. Then the result type is `t list`
- `null : 'a list -> bool`
- `hd   : 'a list -> 'a`
- `tl   : 'a list -> 'a list`

Example list functions

```
fun sum_list (xs : int list) =
  if null xs
  then 0
  else hd(xs) + sum_list(tl(xs))

fun countdown (x : int) =
  if x=0
  then []
  else x :: countdown (x-1)

fun append (xs : int list, ys : int list) =
  if null xs
  then ys
  else hd (xs) :: append (tl(xs), ys)
```

Recursion again

Functions over lists are usually recursive

- Only way to "get to all the elements"

- What should the answer be for the empty list?
- What should the answer be for a non-empty list?
  - Typically in terms of the answer for the tail of the list!

Similarly, functions that produce lists of potentially any size will be recursive

- You create a list out of smaller lists

Lists of pairs

Processing lists of pairs requires no new features. Examples:

```
fun sum_pair_list (xs : (int*int) list) =
  if null xs
  then 0
  else #1(hd xs) + #2(hd xs) + sum_pair_list(tl xs)

fun firsts (xs : (int*int) list) =
  if null xs
  then []
  else #1(hd xs) :: firsts(tl xs)

fun seconds (xs : (int*int) list) =
  if null xs
  then []
  else #2(hd xs) :: seconds(tl xs)

fun sum_pair_list2 (xs : (int*int) list) =
 (sum_list (firsts xs)) + (sum_list (seconds xs))
```