PAUL G. ALLEN SCHOOL OF COMPUTER SCIENCE & ENGINEERING

CSE341: Programming Languages

Lecture 24
Subtyping

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Last major topic: Subtyping

Build up key ideas from first principles
- In pseudocode because:
  - No time for another language
  - Simpler to first show subtyping without objects

Then:

- How does subtyping relate to types for OOP?
  - Brief sketch only
- What are the relative strengths of subtyping and generics?
- How can subtyping and generics combine synergistically?

A tiny language

- Can cover most core subtyping ideas by just considering *records with mutable fields*
- Will make up our own syntax
  - ML has records, but no subtyping or field-mutation
  - Racket and Ruby have no type system
  - Java uses class/interface names and rarely fits on a slide

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Records (half like ML, half like Java)

Record creation (field names and contents):

`{f1=e1, f2=e2, …, fn=en}` Evaluate `ei`, make a record

Record field access:

`e.f` Evaluate `e` to record `v` with an `f` field, get contents of `f` field

Record field update

`e1.f = e2` Evaluate `e1` to a record `v1` and `e2` to a value `v2`;
Change `v1`'s `f` field (which must exist) to `v2`;
Return `v2`

A Basic Type System

Record types: What fields a record has and type for each field

`{f1:t1, f2:t2, …, fn:tn}`

Type-checking expressions:

- If `e1` has type `t1`, …, `en` has type `tn`, then `{f1=e1, …, fn=en}` has type `{f1:t1, …, fn:tn}`
- If `e` has a record type containing `f : t`, then `e.f` has type `t`
- If `e1` has a record type containing `f : t` and `e2` has type `t`, then `e1.f = e2` has type `t`

This is sound

These evaluation rules and typing rules prevent ever trying to access a field of a record that does not exist

Example program that type-checks (in a made-up language):

```
fun distToOrigin (p:{x:real,y:real}) =
  Math.sqrt(p.x*p.x + p.y*p.y)

val pythag : {x:real,y:real} = {x=3.0, y=4.0}
val five : real = distToOrigin(pythag)
```

Motivating subtyping

But according to our typing rules, this program does not type-check

- It does nothing wrong and seems worth supporting

```
fun distToOrigin (p:{x:real,y:real}) =
  Math.sqrt(p.x*p.x + p.y*p.y)

val c : {x:real,y:real,color:string} =
   {x=3.0, y=4.0, color="green"}

val five : real = distToOrigin(c)
```


A good idea: allow extra fields

Natural idea: If an expression has type

`{f1:t1, f2:t2, …, fn:tn}`

Then it can *also* have a type with some fields removed

This is what we need to type-check these function calls:

```
fun distToOrigin (p:{x:real,y:real}) = …
fun makePurple (p:{color:string}) =
    p.color = "purple"

val c :{x:real,y:real,color:string} =
   {x=3.0, y=4.0, color="green"}

val _ = distToOrigin(c)
val _ = makePurple(c)
```


Keeping subtyping separate

A programming language already has a lot of typing rules and we do not want to change them

- Example: The type of an actual function argument must ***equal*** the type of the function parameter

We can do this by adding "just two things to our language"

- *Subtyping*: Write `t1 <: t2` for `t1` is a subtype of `t2`
- One new typing rule that uses subtyping:
  If `e` has type `t1` and `t1 <: t2`,
  then `e` (also) has type `t2`

Now all we need to do is define `t1 <: t2`

Subtyping is not a matter of opinion

- Misconception: If we are making a new language, we can have whatever typing and subtyping rules we want
- Not if you want to prevent what you claim to prevent [soundness]
  - Here: No accessing record fields that do not exist
- Our typing rules were *sound* before we added subtyping
  - We should keep it that way
- Principle of *substitutability*: If `t1 <: t2`, then any value of type `t1` must be usable in every way a `t2` is
  - Here: Any value of subtype needs all fields any value of supertype has

Four good rules

For our record types, these rules all meet the substitutability test:

1. "Width" subtyping: A supertype can have a subset of fields with the same types
2. "Permutation" subtyping: A supertype can have the same set of fields with the same types in a different order
3. Transitivity: If `t1 <: t2` and `t2 <: t3`, then `t1 <: t3`
4. Reflexivity: Every type is a subtype of itself

(4) may seem unnecessary, but it composes well with other rules in a full language and "does no harm"

More record subtyping?

[Warning: I am misleading you ☺️]

Subtyping rules so far let us drop fields but not change their types

Example: A circle has a center field holding another record

```
fun circleY (c:{center:{x:real,y:real}, r:real}) =
   c.center.y

val sphere:{center:{x:real,y:real,z:real}, r:real} =
  {center={x=3.0,y=4.0,z=0.0}, r=1.0}

val _ = circleY(sphere)
```

For this to type-check, we need:

```
{center:{x:real,y:real,z:real}, r:real}
                  <:
    {center:{x:real,y:real}, r:real}
```

Do not have this subtyping – could we?

```
{center:{x:real,y:real,z:real}, r:real}
                 <:
   {center:{x:real,y:real}, r:real}
```

- No way to get this yet: we can drop `center`, drop `r`, or permute order, but cannot "reach into a field type" to do subtyping
- So why not add another subtyping rule… "Depth" subtyping:
  If `ta <: tb`, then `{f1:t1, …, f:ta, …, fn:tn} <:`
  `{f1:t1, …, f:tb, …, fn:tn}`
- Depth subtyping (along with width on the field's type) lets our example type-check

Stop!

- It is nice and all that our new subtyping rule lets our example type-check
- But it is not worth it if it breaks soundness
  - Also allows programs that can access missing record fields
- Unfortunately, it breaks soundness ☹

Mutation strikes again

If `ta <: tb`,
then `{f1:t1, …, f:ta, …, fn:tn} <:`
`{f1:t1, …, f:tb, …, fn:tn}`

```
fun setToOrigin (c:{center:{x:real,y:real}, r:real})=
   c.center = {x=0.0, y=0.0}
val sphere:{center:{x:real,y:real,z:real}, r:real} =
  {center={x=3.0, y=4.0, z=0.0}, r=1.0}
val _ = setToOrigin(sphere)
val _ = sphere.center.z (* kaboom! (no z field) *)
```


Moral of the story

- In a language with records/objects with getters and setters, depth subtyping is unsound
  - Subtyping cannot change the type of fields
- If fields are immutable, then depth subtyping is sound!
  - Yet another benefit of outlawing mutation!
  - Choose two of three: setters, depth subtyping, soundness
- Remember: subtyping is not a matter of opinion

Picking on Java (and C#)

Arrays should work just like records in terms of depth subtyping
- But in Java, if `t1 <: t2`, then `t1[] <: t2[]`
- So this code type-checks, surprisingly

```
class Point { … }
class ColorPoint extends Point { … }
…
void m1(Point[] pt_arr) {
  pt_arr[0] = new Point(3,4);
}
String m2(int x) {
  ColorPoint[] cpt_arr = new ColorPoint[x];
  for(int i=0; i < x; i++)
     cpt_arr[i] = new ColorPoint(0,0,"green");
  m1(cpt_arr); // !
  return cpt_arr[0].color; // !
}
```


Why did they do this?

- More flexible type system allows more programs but prevents fewer errors
  - Seemed especially important before Java/C# had generics
- Good news: despite this "inappropriate" depth subtyping
  - `e.color` will never fail due to there being no `color` field
  - Array *reads* `e1[e2]` always return a (subtype of) `t` if `e1` is a `t[]`
- Bad news: to get the good news
  - `e1[e2]=e3` can fail even if `e1` has type `t[]` and `e3` has type `t`
  - Array *stores* check the *run-time class* of `e1`'s elements and do not allow storing a supertype
  - No type-system help to avoid such bugs / performance cost

So what happens

```
void m1(Point[] pt_arr) {
  pt_arr[0] = new Point(3,4); // can throw
}
String m2(int x) {
  ColorPoint[] cpt_arr = new ColorPoint[x];
  …
  m1(cpt_arr); // "inappropriate" depth subtyping
  ColorPoint c = cpt_arr[0]; // fine, cpt_arr
    // will always hold (subtypes of) ColorPoints
  return c.color; // fine, a ColorPoint has a color
}
```

- Causes code in `m1` to throw an `ArrayStoreException`
  - Even though logical error is in `m2`
  - At least run-time checks occur only on array stores, not on field accesses like `c.color`

null

- Array stores probably the most *surprising* choice for flexibility over static checking
- But `null` is the most *common* one in practice
  - `null` is not an object; it has *no* fields or methods
  - But Java and C# let it have *any* object type (backwards, huh?!)
  - So, in fact, we do *not* have the static guarantee that evaluating `e` in `e.f` or `e.m(…)` produces an object that has an `f` or `m`
  - The "or `null`" caveat leads to run-time checks and errors, as you have surely noticed
- Sometimes `null` is convenient (like ML's option types)
  - But also having "cannot be `null`" types would be nice

Now functions

- Already know a caller can use subtyping for arguments passed
  - Or on the result
- More interesting: When is one function type a subtype of another?
  - Important for higher-order functions: If a function expects an argument of type `t1 -> t2`, can you pass a `t3 -> t4` instead?
  - Coming next: Important for understanding methods
    - (An object type is a lot like a record type where "method positions" are immutable and have function types)

Example

```
fun distMoved (f : {x:real,y:real}->{x:real,y:real},
               p : {x:real,y:real}) =
   let val p2 : {x:real,y:real} = f p
       val dx : real = p2.x - p.x
       val dy : real = p2.y - p.y
   in Math.sqrt(dx*dx + dy*dy) end

fun flip p = {x = ~p.x, y=~p.y}
val d = distMoved(flip, {x=3.0, y=4.0})
```

No subtyping here yet:
- `flip` has exactly the type `distMoved` expects for `f`
- Can pass `distMoved` a record with extra fields for `p`, but that's old news

Return-type subtyping

```
fun distMoved (f : {x:real,y:real}->{x:real,y:real},
               p : {x:real,y:real}) =
   let val p2 : {x:real,y:real} = f p
       val dx : real = p2.x - p.x
       val dy : real = p2.y - p.y
   in Math.sqrt(dx*dx + dy*dy) end

fun flipGreen p = {x = ~p.x, y=~p.y, color="green"}
val d = distMoved(flipGreen, {x=3.0, y=4.0})
```

- Return type of `flipGreen` is `{x:real,y:real,color:string}`, but `distMoved` expects a return type of `{x:real,y:real}`
- Nothing goes wrong: If `ta <: tb`, then `t -> ta <: t -> tb`
  - A function can return "*more* than it needs to"
  - Jargon: "Return types are *covariant*"

This is wrong

```
fun distMoved (f : {x:real,y:real}->{x:real,y:real},
               p : {x:real,y:real}) =
   let val p2 : {x:real,y:real} = f p
       val dx : real = p2.x - p.x
       val dy : real = p2.y - p.y
   in Math.sqrt(dx*dx + dy*dy) end

fun flipIfGreen p = if p.color = "green" (*kaboom!*)
                    then {x = ~p.x, y=~p.y}
                    else {x = p.x, y=p.y}
val d = distMoved(flipIfGreen, {x=3.0, y=4.0})
```

- Argument type of `flipIfGreen` is `{x:real,y:real,color:string}`, but it is called with a `{x:real,y:real}`
- Unsound! `ta <: tb` does **NOT** allow `ta -> t <: tb -> t`

The other way works!

```
fun distMoved (f : {x:real,y:real}->{x:real,y:real},
               p : {x:real,y:real}) =
   let val p2 : {x:real,y:real} = f p
       val dx : real = p2.x - p.x
       val dy : real = p2.y - p.y
   in Math.sqrt(dx*dx + dy*dy) end

fun flipX_Y0 p = {x = ~p.x, y=0.0}
val d = distMoved(flipX_Y0, {x=3.0, y=4.0})
```

- Argument type of `flipX_Y0` is `{x:real}`, but it is called with a `{x:real,y:real}`, which is fine
- If `tb <: ta`, then `ta -> t <: tb -> t`
  - A function can assume "*less* than it needs to" about arguments
  - Jargon: "Argument types are *contravariant*"

Can do both

```
fun distMoved (f : {x:real,y:real}->{x:real,y:real},
               p : {x:real,y:real}) =
   let val p2 : {x:real,y:real} = f p
       val dx : real = p2.x - p.x
       val dy : real = p2.y - p.y
   in Math.sqrt(dx*dx + dy*dy) end

fun flipXMakeGreen p = {x = ~p.x, y=0.0, color="green"}
val d = distMoved(flipXMakeGreen, {x=3.0, y=4.0})
```

- `flipXMakeGreen` has type
  `{x:real} -> {x:real,y:real,color:string}`
- Fine to pass a function of such a type as function of type
  `{x:real,y:real} -> {x:real,y:real}`
- If `t3 <: t1` and `t2 <: t4`, then `t1 -> t2 <: t3 -> t4`

Conclusion

- If `t3 <: t1` and `t2 <: t4`, then `t1 -> t2 <: t3 -> t4`
  - Function subtyping contravariant in argument(s) and covariant in results
- Also essential for understanding subtyping and methods in OOP
- Most unintuitive concept in the course
  - Smart people often forget and convince themselves covariant arguments are okay
  - These people are always mistaken
  - At times, you or your boss or your friend may do this
  - Remember: A guy with a PhD in PL ***jumped up and down*** insisting that function/method subtyping is always contravariant in its argument -- covariant is unsound