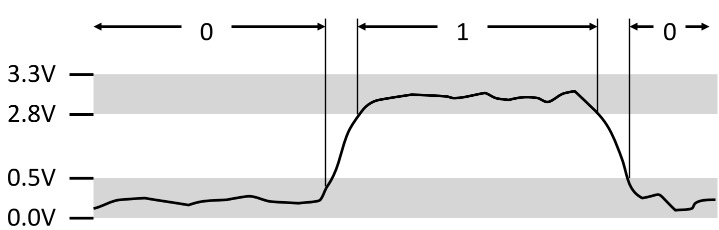
Today's topics

- **Memory and its bits, bytes, and integers**
- **Representing information as bits**
- **Bit-level manipulations**
  - Boolean algebra
  - Boolean algebra in C

Binary Representations

- **Base 2 number representation**
  - Represent 351_{10} as 0000000101011111_2 or 101011111_2
  - Represent 3.6_{10} as $11.1001100110011001[1001]..._2$
  - Represent $3.51 * 10^2$ as $1.01011111_2 * 2^8$
- **Electronic implementation**
  - Easy to store with bi-stable elements
  - Reliably transmitted on noisy and inaccurate wires

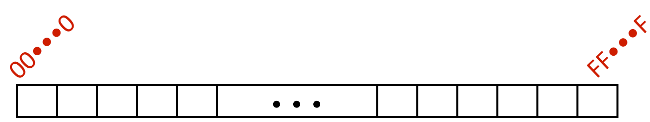


Encoding Byte Values

- **Binary** **00000000_2 -- 11111111_2**
  - Byte = 8 bits (binary digits)
- **Decimal** **0_{10} -- 255_{10}**
- **Hexadecimal** **00_{16} -- FF_{16}**
  - Byte = 2 hexadecimal (hex) or base 16 digits
  - Base-16 number representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - Write $FA1D37B_{16}$ in C
    - as `0xFA1D37B` or `0xfa1d37b`

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Byte-Oriented Memory Organization



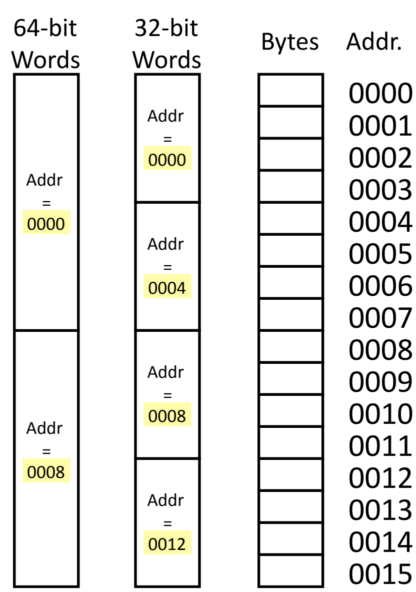
- **Programs refer to addresses**
  - Conceptually, a very large array of bytes
  - Actually implemented with hierarchy of different memory types (later…)
  - System provides an <u>address space</u> private to each “process”
    - Process = program being executed + its data + its “state”
    - Program can clobber its own data, but not that of others
    - Clobbering code or “state” often leads to crashes (or security holes)
- **Compiler + run-time system control memory allocation**
  - Where different program objects should be stored
  - All allocation within a single address space

Machine Words

- **Machine has a "word size"**
  - Nominal size of integer-valued data
    - Including addresses
  - Most current machines use 32 bits (4 bytes) words
    - Limits addresses to 4GB
    - Becoming too small for memory-intensive applications
  - High-end systems use 64 bits (8 bytes) words
    - Potential address space ≈ 1.8×10^{19} bytes
    - x86-64 machines support 48-bit addresses: 256 Terabytes
    - Can't be real physical addresses -> virtual addresses
  - Machines support multiple data formats
    - Fractions or multiples of word size
    - Always integral number of bytes

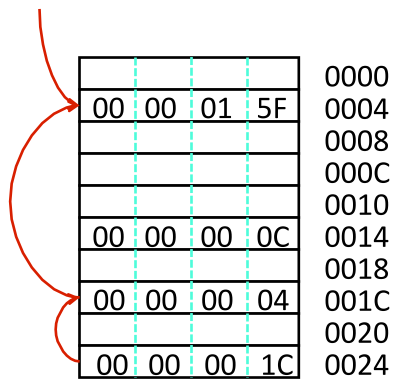
Word-Oriented Memory Organization

- **Addresses specify locations of bytes in memory**
  - Address of first byte in word
  - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Addresses and Pointers

- **Address is a location in memory**
- **Pointer is a data object that contains an address**
- **Address 0004 stores the value 351 (or $15F_{16}$)**
- **Pointer to address 0004 stored at address 001C**
- **Pointer to a pointer in 0024**
- **Address 0014 stores the value 12**
  - Is it a pointer?



Data Representations

- **Sizes of objects (in bytes)**

Java Data Type	C Data Type	Typical 32-bit	x86-64
boolean	*bool*	1	1
byte	char	1	1
char		2	2
short	short int	2	2
int	int	4	4
float	float	4	4
long	long int	4	8
double	double	8	8
	long long	8	8
	long double	8	16
(reference)	pointer *	4	8

Byte Ordering

- **How should bytes within multi-byte word be ordered in memory?**
- **Conventions**
  - Big-endian: Sun, PPC Mac, Internet
    - Least significant byte has highest address
  - Little-endian: x86
    - Least significant byte has lowest address

Byte Ordering Example

- **Big-Endian**
  - Least significant byte has highest address
- **Little-Endian**
  - Least significant byte has lowest address
- **Example**
  - Variable has 4-byte representation `0x01234567`
  - Address of variable is `0x100`

			0x100	0x101	0x102	0x103		
Big Endian			01	23	45	67		

			0x100	0x101	0x102	0x103		
Little Endian			67	45	23	01		

Reading Byte-Reversed Listings

- **Disassembly**
  - Text representation of binary machine code
  - Generated by program that reads the machine code
- **Example instruction in memory**
  - add value 0x12ab to register 'ebx' *(a special location in CPU's memory)*

Address	Instruction Code	Assembly Rendition
8048366:	81 c3 ab 12 00 00	add $0x12ab,%ebx

Deciphering numbers

- **Value:** **0x12ab**
- **Pad to 32 bits:** **0x000012ab**
- **Split into bytes:** **00 00 12 ab**
- **Reverse (little-endian):** **ab 12 00 00**

Addresses and Pointers in C

& = 'address of value'
** = 'value at address'*
or 'de-reference'

**(&x) is equivalent to x*

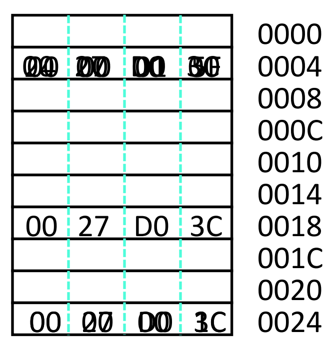
- **Pointer declarations use ***
  - int * ptr; int x, y; ptr = &x;
  - Declares a variable ptr that is a pointer to a data item that is an integer
  - Declares integer values named x and y
  - Assigns ptr to point to the address where x is stored
- **We can do arithmetic on pointers**
  - ptr = ptr + 1; *// really adds 4 (because an integer uses 4 bytes)*
  - Changes the value of the pointer so that it now points to the next data item in memory (that may be y, may not – dangerous!)
- **To use the value pointed to by a pointer we use de-reference**
  - y = *ptr + 1; is the same as y = x + 1;
  - But, if ptr = &y then y = *ptr + 1; is the same as y = y + 1;
  - *ptr is the value stored at the location to which the pointer ptr is pointing

Arrays

- **Arrays represent adjacent locations in memory storing the same type of data object**
  - E.g., int big_array[128];
    allocated 400 adjacent locations in memory starting at 0x00ff0000
- **Pointers to arrays point to a certain type of object**
  - E.g., int * array_ptr;
    array_ptr = big_array;
    array_ptr = &big_array[0];
    array_ptr = &big_array[3];
    array_ptr = &big_array[0] + 3;
    array_ptr = big_array + 3;
    *array_ptr = *array_ptr + 1;
    array_ptr = &big_array[130];
  - In general: &big_array[i] is the same as (big_array + i)
    - *which implicitly computes: &bigarray[0] + i*sizeof(bigarray[0]);*

General rules for C

- **Left-hand-side = right-hand-side**
  - LHS must evaluate to a memory LOCATION
  - RHS must evaluate to a VALUE (could be an address)
- **E.g., x at location 0x04, y at 0x18**
  - int x, y;
    x = y; // get value at y and put it in x
  - int * x; int y;
    x = &y + 3; // get address of y add 12
  - int * x; int y;
    *x = y; // value of y to location x points



Examining Data Representations

- **Code to print byte representation of data**
  - Casting pointer to `unsigned char *` creates byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, int len)
{
  int i;
  for (i = 0; i < len; i++)
    printf("0x%p\t0x%.2x\n", start+i, start[i]);
  printf("\n");
}
```

```
void show_int (int x)
{
  show_bytes( (pointer) &x, sizeof(int));
}
```

Some printf directives:

%p: Print pointer

%x: Print hexadecimal

"\n": New line

`show_bytes` Execution Example

```
int a = 12345; // represented as 0x00003039
printf("int a = 12345;\n");
show_int(a);   // show_bytes((pointer) &a, sizeof(int));
```

Result (Linux):

```
int a = 12345;
0x11ffffcb8    0x39
0x11ffffcb9    0x30
0x11ffffcba    0x00
0x11ffffcbb    0x00
```

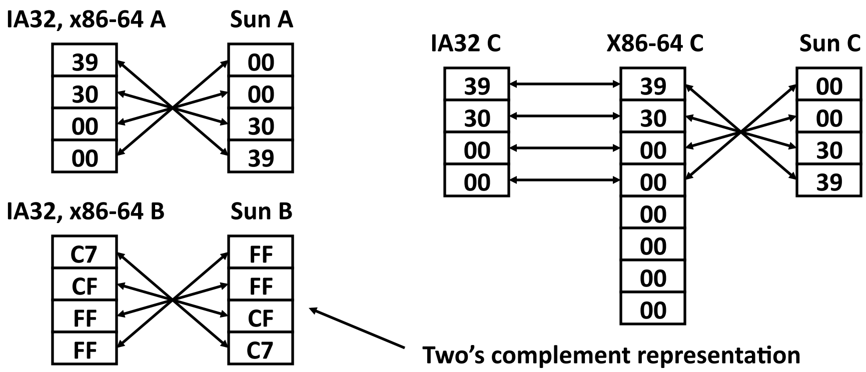
Representing Integers

- `int A = 12345;`
- `int B = -12345;`
- `long int C = 12345;`

Decimal: 12345

Binary: 0011 0000 0011 1001

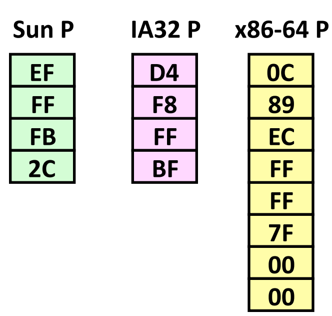
Hex: 3 0 3 9



Two's complement representation for negative integers (covered later)

Representing Pointers

- `int B = -12345;`
- `int *P = &B;`



Different compilers & machines assign different locations to objects

Representing Strings

- **Strings in C**
  - Represented by array of characters
  - Each character encoded in ASCII format
    - Standard 7-bit encoding of character set
      - Fits into 8 bits with a leading 0
    - Character “0” has code `0x30`
      - Digit *i* has code `0x30`+*i*
  - String should be null-terminated
    - Final character = 0x00
- **Compatibility**
  - Byte ordering not an issue
- **Unicode characters – up to 4 bytes/character**
  - ASCII codes still work (leading 0 bit) but can support the many characters in all languages in the world
  - Java and C have libraries for Unicode (Java commonly uses 2 bytes/char)

char S[6] = "12345";

Linux/Alpha S	Sun S
31	31
32	32
33	33
34	34
35	35
00	00

Boolean Algebra

- **Developed by George Boole in 19th Century**
  - Algebraic representation of logic
    - Encode “True” as 1 and “False” as 0
  - AND: A&B = 1 when both A is 1 and B is 1
  - OR: A|B = 1 when either A is 1 or B is 1
  - XOR: A^B = 1 when either A is 1 or B is 1, but not both
  - NOT: ~A = 1 when A is 0 and vice-versa
  - DeMorgan’s Law: ~(A | B) = ~A & ~B

&	0	1
0	0	0
1	0	1

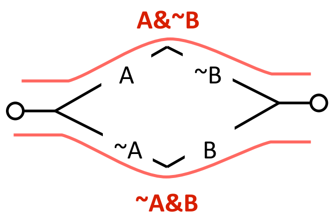
|	0	1
0	0	1
1	1	1

^	0	1
0	0	1
1	1	0

~	
0	1
1	0

Application of Boolean Algebra

- **Applied to digital systems by Claude Shannon**
  - 1937 MIT Masters Thesis
  - Reason about networks of relay switches
    - Encode closed switch as 1, open switch as 0



Connection when:
A&~B | ~A&B = A^B

General Boolean Algebras

- **Operate on bit vectors**
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

- **All of the properties of Boolean algebra apply**

```
  01010101
^ 01010101
  00000000
```

Representing & Manipulating Sets

- **Representation**
  - Width *w* bit vector represents subsets of {0, …, *w*–1}
  - $a_j = 1$ if $j \in A$

    01101001 { 0, 3, 5, 6 }
    76543210

    01010101 { 0, 2, 4, 6 }
    76543210

- **Operations**
  - & Intersection 01000001 { 0, 6 }
  - | Union 01111101 { 0, 2, 3, 4, 5, 6 }
  - ^ Symmetric difference 00111100 { 2, 3, 4, 5 }
  - ~ Complement 10101010 { 1, 3, 5, 7 }

Bit-Level Operations in C

- **Operations &, |, ^, ~ are available in C**
  - Apply to any "integral" data type
    - `long, int, short, char, unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- **Examples (char data type)**
  - `~0x41 --> 0xBE`

    $\sim 01000001_2$ --> 10111110_2
  - `~0x00 --> 0xFF`

    $\sim 00000000_2$ --> 11111111_2
  - `0x69 & 0x55 --> 0x41`

    01101001_2 & 01010101_2 --> 01000001_2
  - `0x69 | 0x55 --> 0x7D`

    01101001_2 | 01010101_2 --> 01111101_2

Contrast: Logic Operations in C

- **Contrast to logical operators**
  - `&&, ||, !`
    - View 0 as "False"
    - Anything nonzero as "True"
    - Always return 0 or 1
    - Early termination
- **Examples (char data type)**
  - `!0x41  -->  0x00`
  - `!0x00  -->  0x01`
  - `!!0x41 -->  0x01`

  - `0x69 && 0x55  -->  0x01`
  - `0x69 || 0x55  -->  0x01`
  - `p && *p++` (avoids null pointer access, null pointer = 0x00000000)