Today's Topics

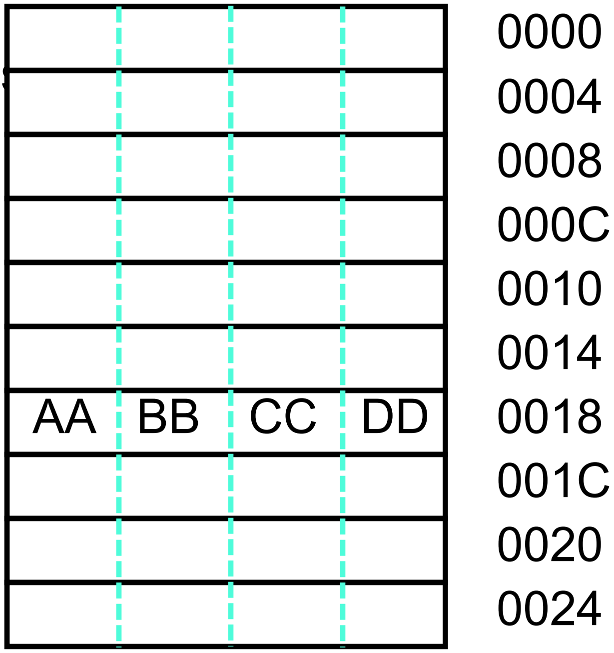
- Strings
- Boolean algebra
- Representation of integers: unsigned and signed
- Casting
- Arithmetic and shifting
- Sign extension

Quick review…

x at location 0x04, y at 0x18

```
int * x; int y;
    x = &y + 3; // get address of y add 12

int * x; int y;
    *x = y; // value of y to location x point
```



Representing strings?

Representing strings

A C-style string is represented by an array of bytes.

- Elements are one-byte ASCII codes for each character.
- A 0 value marks the end of the array.

32	space	48	0	64	@	80	P	96	`	112	p
33	!	49	1	65	A	81	Q	97	a	113	q
34	”	50	2	66	B	82	R	98	b	114	r
35	#	51	3	67	C	83	S	99	c	115	s
36	$	52	4	68	D	84	T	100	d	116	t
37	%	53	5	69	E	85	U	101	e	117	u
38	&	54	6	70	F	86	V	102	f	118	v
39	’	55	7	71	G	87	W	103	g	119	w
40	(	56	8	72	H	88	X	104	h	120	x
41	)	57	9	73	I	89	Y	105	I	121	y
42	*	58	:	74	J	90	Z	106	j	122	z
43	+	59	;	75	K	91	[	107	k	123	{
44	,	60	<	76	L	92	\	108	l	124	|
45	-	61	=	77	M	93	]	109	m	125	}
46	.	62	>	78	N	94	^	110	n	126	~
47	/	63	?	79	O	95	_	111	o	127	del

Null-terminated Strings

For example, “Harry Potter” can be stored as a 13-byte array.

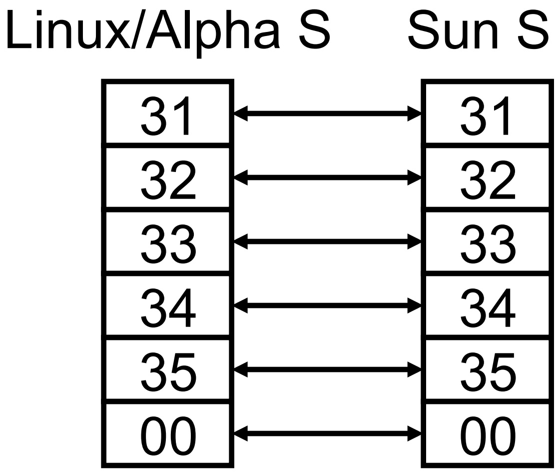
72	97	114	114	121	32	80	111	116	116	101	114	0
H	a	r	r	y		P	o	t	t	e	r	\0

Why do we put a a 0, or null, at the end of the string?

Computing string length?

Compatibility

`char S[6] = "12345";`



- **Byte ordering not an issue**
- **Unicode characters – up to 4 bytes/character**
  - ASCII codes still work (leading 0 bit) but can support the many characters in all languages in the world
  - Java and C have libraries for Unicode (Java commonly uses 2 bytes/char)

Boolean Algebra

Developed by George Boole in 19th Century

Algebraic representation of logic

Encode “True” as 1 and “False” as 0

AND: A&B = 1 when both A is 1 and B is 1

OR: A|B = 1 when either A is 1 or B is 1

XOR: A^B = 1 when either A is 1 or B is 1, but not both

NOT: ~A = 1 when A is 0 and vice-versa

DeMorgan’s Law: ~(A | B) = ~A & ~B

&	0	1
0	0	0
1	0	1

|	0	1
0	0	1
1	1	1

^	0	1
0	0	1
1	1	0

~	
0	1
1	0

General Boolean Algebras

Operate on bit vectors

Operations applied bitwise

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
```

All of the properties of Boolean algebra apply

```
  01010101
^ 01010101
```

How does this relate to set operations?

Representing & Manipulating Sets

Representation

Width *w* bit vector represents subsets of {0, …, *w*–1}

$a_j = 1$ if $j \in A$

`01101001` { 0, 3, 5, 6 }

`76543210`

`01010101` { 0, 2, 4, 6 }

`76543210`

Operations

& Intersection `01000001` { 0, 6 }

| Union `01111101` { 0, 2, 3, 4, 5, 6 }

^ Symmetric difference `00111100` { 2, 3, 4, 5 }

~ Complement `10101010` { 1, 3, 5, 7 }

Bit-Level Operations in C

Operations &, |, ^, ~ are available in C

Apply to any "integral" data type

`long, int, short, char, unsigned`

View arguments as bit vectors

Arguments applied bit-wise

Examples (char data type)

`~0x41 --> 0xBE`

$\sim 01000001_2 \quad \text{-->} \quad 10111110_2$

`~0x00 --> 0xFF`

$\sim 00000000_2 \quad \text{-->} \quad 11111111_2$

`0x69 & 0x55 --> 0x41`

$01101001_2 \ \& \ 01010101_2 \ \text{-->} \ 01000001_2$

`0x69 | 0x55 --> 0x7D`

$01101001_2 \mid 01010101_2 \ \text{-->} \ 01111101_2$

Contrast: Logic Operations in C

Contrast to logical operators

`&&, ||, !`

- View 0 as "False"
- Anything nonzero as "True"
- Always return 0 or 1
- Early termination

Examples (char data type)

```
!0x41   -->  0x00
!0x00   -->  0x01
!!0x41  -->  0x01
```

```
0x69 && 0x55  -->  0x01
0x69 || 0x55  -->  0x01
```

`p && *p++` (avoids null pointer access, null pointer = 0x00000000)

Encoding Integers

- **The hardware (and C) supports two flavors of integers:**
    - unsigned – only the non-negatives
    - signed – both negatives and non-negatives

- **There are only 2^W distinct bit patterns of W bits, so...**
    - Can't represent all the integers
    - Unsigned values are 0 ... 2^W-1
    - Signed values are -2^{W-1} ... 2^{W-1}-1

Unsigned Integers

- Unsigned values are just what you expect
  - $b_7b_6b_5b_4b_3b_2b_1b_0 = b_7 2^7 + b_6 2^6 + b_5 2^5 + \ldots + b_1 2^1 + b_0 2^0$
    - Interesting aside: $1+2+4+8+...+2^{N-1} = 2^N - 1$

$$\boxed{\begin{array}{r} 00111111 \\ +\underline{00000001} \\ 01000000 \end{array}}$$

$$\boxed{\begin{array}{r} 63 \\ +\underline{\ \ 1} \\ 64 \end{array}}$$

- You add/subtract them using the normal "carry/borrow" rules, just in binary

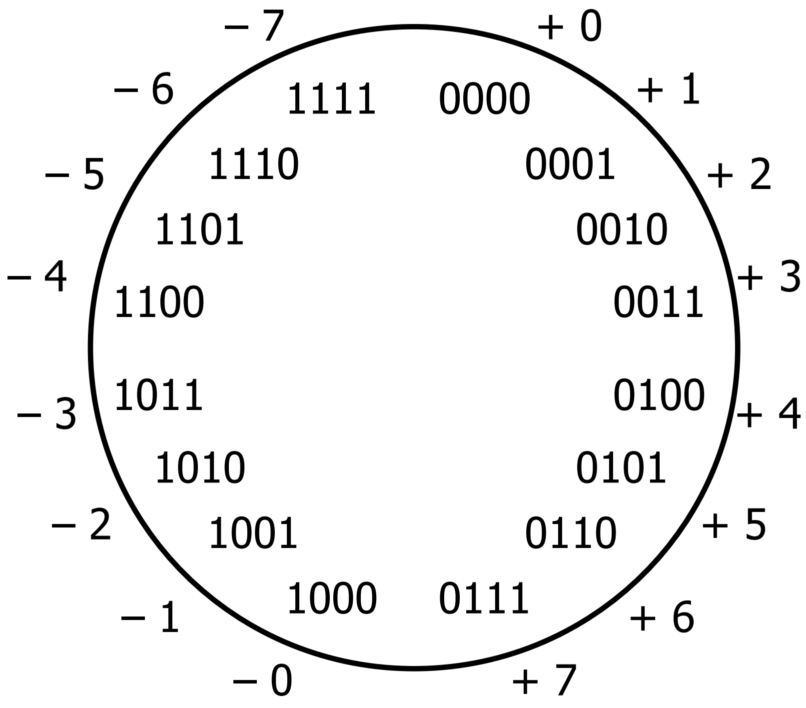
- An important use of unsigned integers in C is pointers
  - There are no negative memory addresses

Signed Integers

- **Let's do the natural thing for the positives**
  - They correspond to the unsigned integers of the same value
    - Example (8 bits): 0x00 = 0, 0x01 = 1, …, 0x7F = 127
- **But, we need to let about half of them be negative**
  - Use the high order bit to indicate 'negative'
  - Call it “the sign bit”
  - Examples (8 bits):
    - 0x00 = 00000000_2 is non-negative, because the sign bit is 0
    - 0x7F = 01111111_2 is non-negative
    - 0x80 = 10000000_2 is negative

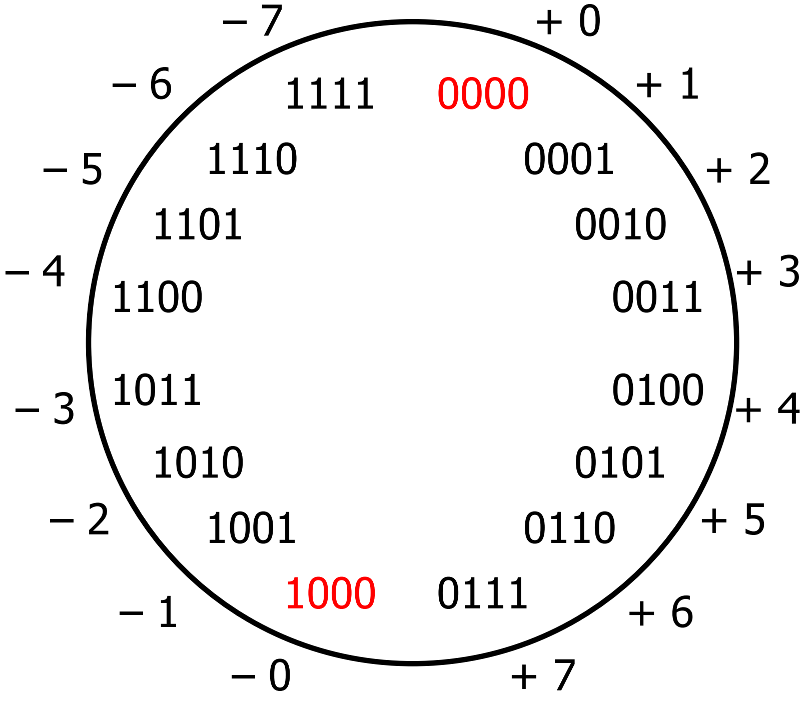
Sign-and-Magnitude Negatives

- **How should we represent -1 in binary?**
  - <u>Possibility 1:</u> 10000001_2
    Use the MSB for “+ or -”, and the other bits to give magnitude



Sign-and-Magnitude Negatives

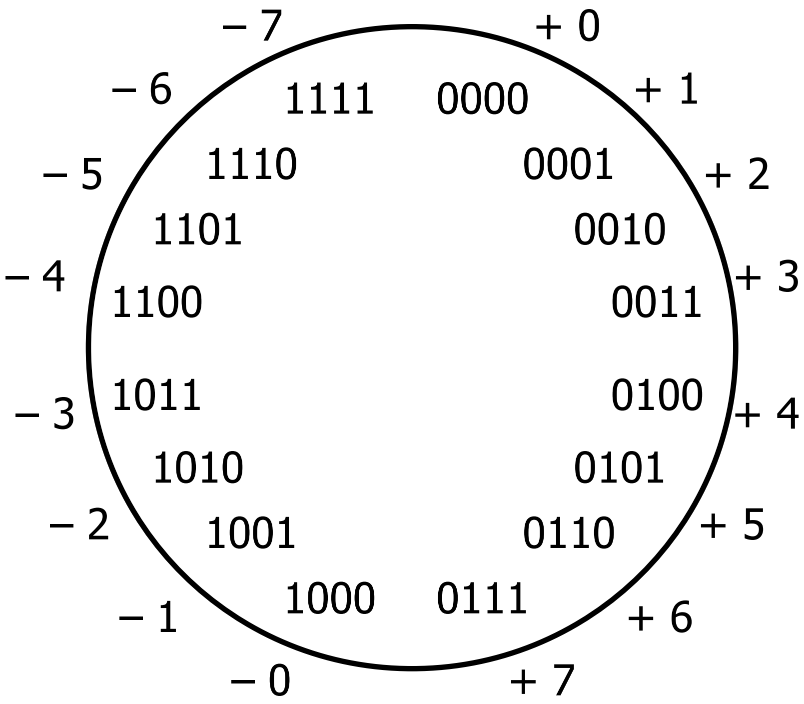
- **How should we represent -1 in binary?**
  - Possibility 1: 10000001_2
    Use the MSB for “+ or -”, and the other bits to give magnitude
    (Unfortunate side effect: there are two representations of 0!)



Sign-and-Magnitude Negatives

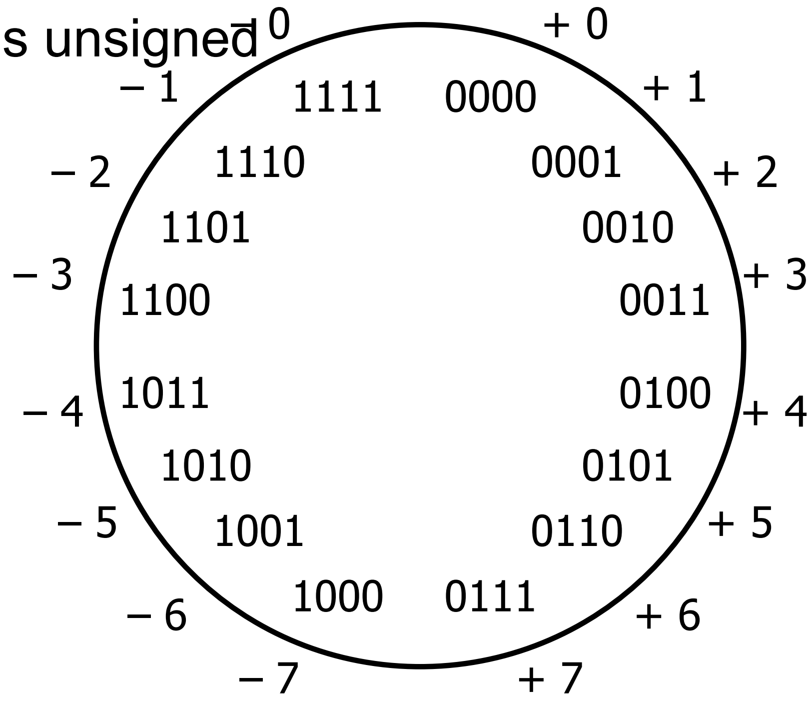
- **How should we represent -1 in binary?**
  - Possibility 1: 10000001_2
    Use the MSB for "+ or -", and the other bits to give magnitude
    Another problem: **math is cumbersome**

4 – 3 != 4 + (-3)



Ones' Complement Negatives

- **How should we represent -1 in binary?**
  - Possibility 2: 11111110_2
    Negative numbers: bitwise complements of positive numbers
    It would be handy if we could use the same hardware adder to add signed integers as unsigned



Ones' Complement Negatives

- **How should we represent -1 in binary?**
  - Possibility 2: 11111110_2
    Negative numbers: bitwise complements of positive numbers
    - Solves the arithmetic problem

Add		Invert, add, add carry		Invert and add	
4	0100	4	0100	– 4	1011
+ 3	+ 0011	– 3	+ 1100	+ 3	+ 0011
= 7	= 0111	= 1	1 0000	– 1	1110
		add carry:	+1		
			= 0001		

end-around carry

Ones' Complement Negatives

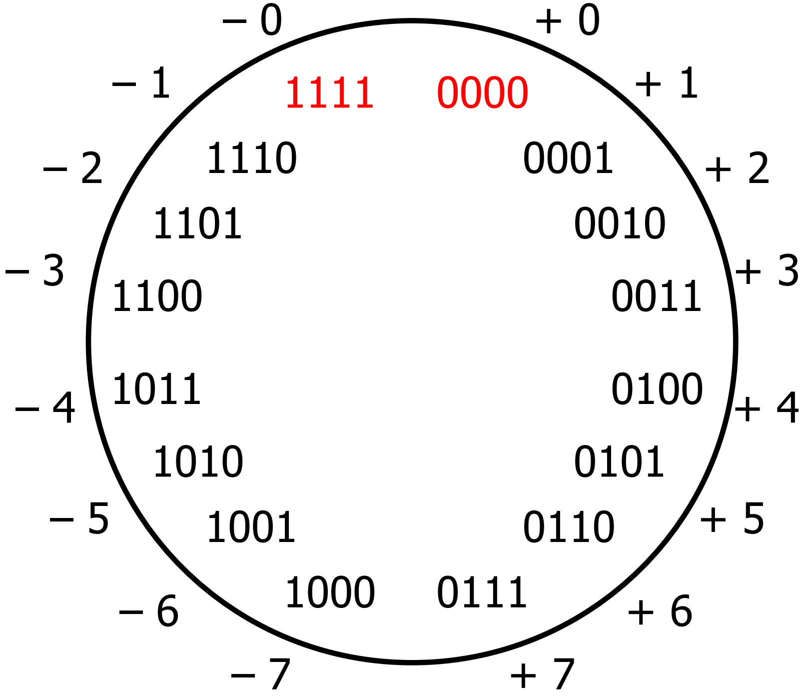
- **How should we represent -1 in binary?**
  - Possibility 2: 11111110_2
    Negative numbers: bitwise complements of positive numbers
    Use the same hardware adder to add signed integers as unsigned (but we have to keep track of the end-around carry bit)

Why does it work?

- The ones' complement of a 4-bit positive number *y* is $1111_2 - y$
  - $0111 \equiv 7_{10}$
  - $1111_2 - 0111_2 = 1000_2 \equiv -7_{10}$
- 1111_2 is 1 less than $10000_2 = 2^4 - 1$
  - $-y$ is represented by $(2^4 - 1) - y$

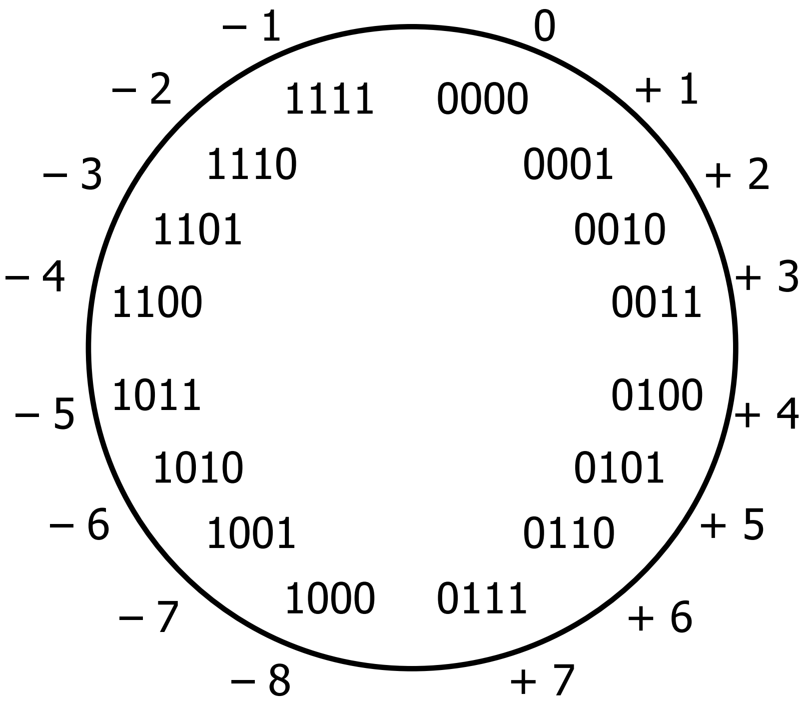
Ones' Complement Negatives

- **How should we represent -1 in binary?**
  - Possibility 2: 11111110_2
    Negative numbers: bitwise complements of positive numbers
    (But there are still two representations of 0!)



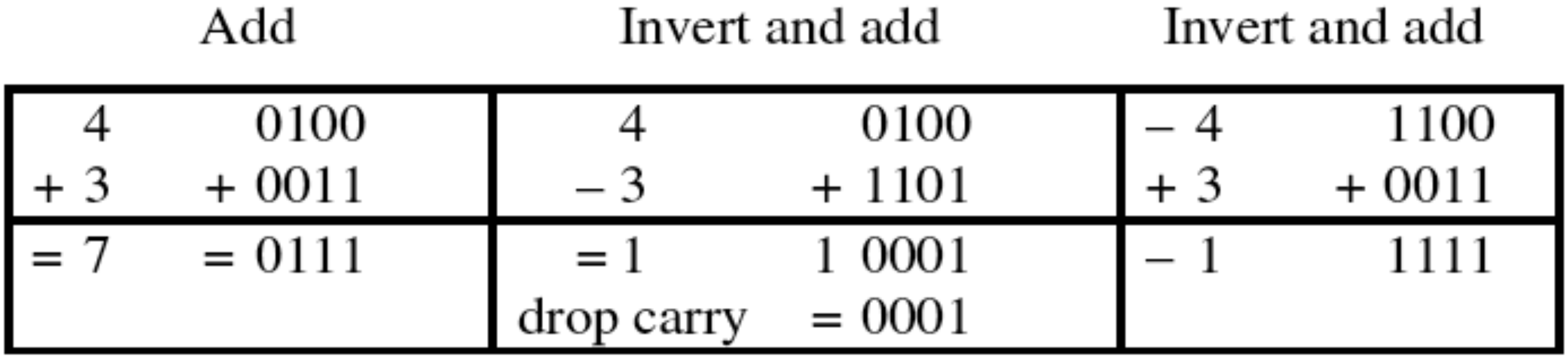
Two's Complement Negatives

- **How should we represent -1 in binary?**
  - Possibility 3: 11111111_2
    Bitwise complement **plus one**
    (Only one zero)



Two's Complement Negatives

- **How should we represent -1 in binary?**
  - Possibility 3: 11111111_2
    Bitwise complement **plus one**
    (Only one zero)
  - Simplifies arithmetic
    Use the same hardware adder to add signed integers as unsigned (simple addition; discard the highest carry bit)



Add		Invert and add		Invert and add	
4	0100	4	0100	– 4	1100
+ 3	+ 0011	– 3	+ 1101	+ 3	+ 0011
= 7	= 0111	= 1	1 0001	– 1	1111
		drop carry	= 0001		

Two's Complement Negatives

- **How should we represent -1 in binary?**
  - Two’s complement: Bitwise complement plus one

  Why does it work?

  - Recall: The ones’ complement of a b-bit positive number y is $(2^b – 1) – y$
  - Two’s complement adds one to the bitwise complement, thus, $-y$ is $2^b – y$
    - $–y$ and $2^b – y$ are equal mod 2^b (have the same remainder when divided by 2^b)
    - Ignoring carries is equivalent to doing arithmetic mod 2^b

Two's Complement Negatives

- **How should we represent -1 in binary?**
  - Two’s complement: Bitwise complement plus one
    - What should the 8-bit representation of -1 be?

```
 00000001
+????????
 00000000
```

(want whichever bit string gives right result)

```
 00000010     00000011
+????????    +????????
 00000000     00000000
```

Unsigned & Signed Numeric Values

X	Unsigned	Signed
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

- Both signed and unsigned integers have limits
  - If you compute a number that is too big, you wrap: 6 + 4 = ? 15U + 2U = ?
  - If you compute a number that is too small, you wrap: -7 – 3 = ? 0U – 2U = ?
  - Answers are only correct mod 2^b
- The CPU may be capable of “throwing an exception” for overflow on signed values
  - It won't for unsigned
- But C and Java just cruise along silently when overflow occurs...

Mapping Signed ↔ Unsigned

Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3		3
0100	4	↔ =	4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6		10
1011	-5	↔ +16	11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

Numeric Ranges

- **Unsigned Values**
  - *UMin* = 0
    000…0
  - *UMax* = $2^w - 1$
    111…1

Two's Complement Values

TMin = -2^{w-1}
100…0

TMax = $2^{w-1} - 1$
011…1

Other Values

Minus 1
111…1 0xFFFFFFFF (32 bits)

Values for *W* = 16

	Decimal	Hex	Binary
`UMax`	65535	`FF FF`	`11111111 11111111`
`TMax`	32767	`7F FF`	`01111111 11111111`
`TMin`	-32768	`80 00`	`10000000 00000000`
`-1`	-1	`FF FF`	`11111111 11111111`
`0`	0	`00 00`	`00000000 00000000`

Values for Different Word Sizes

	W			
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

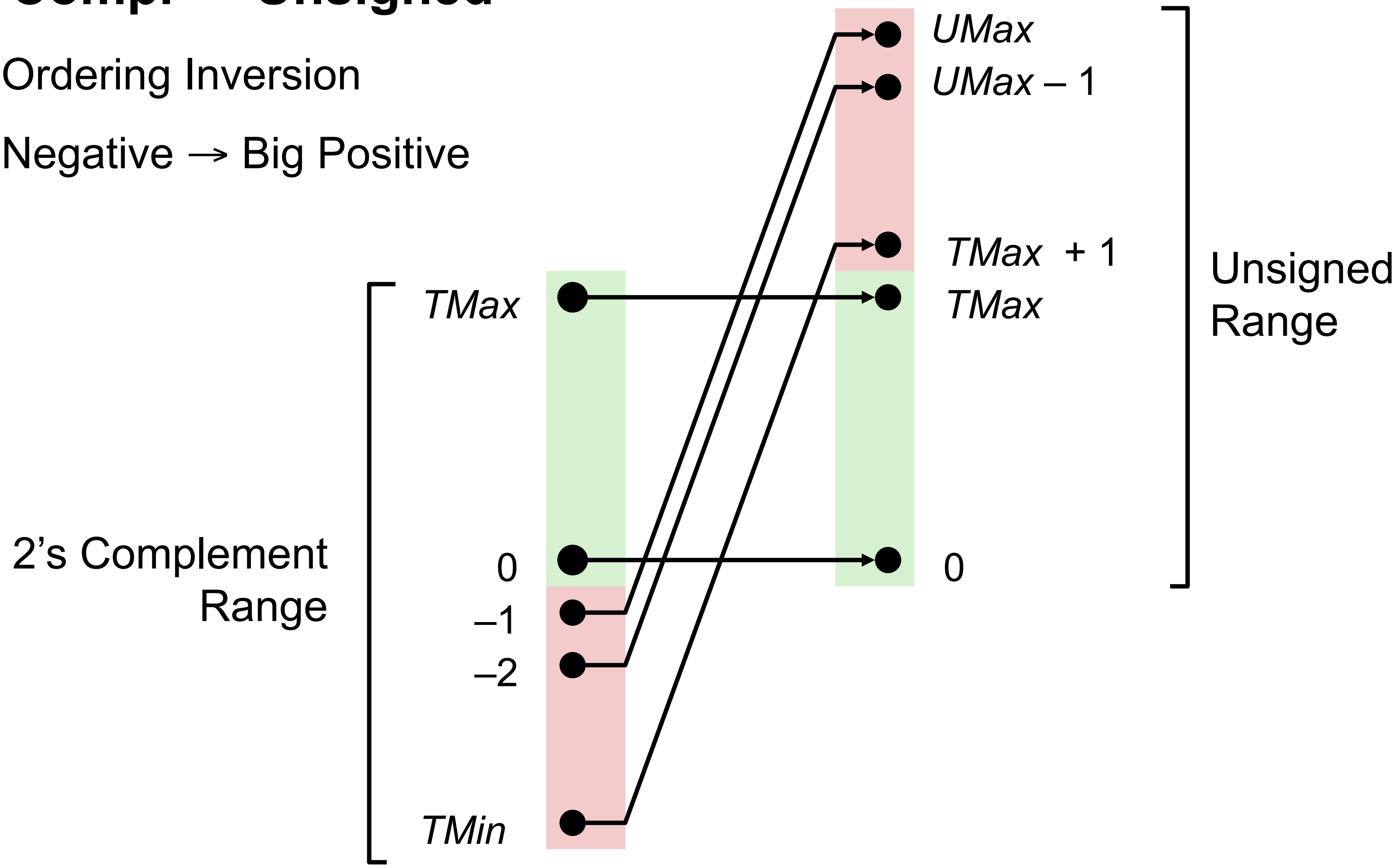
- **Observations**
  - |*TMin* | = *TMax* + 1
    - Asymmetric range
  - *UMax* = 2 * *TMax* + 1
- **C Programming**
  - #include <limits.h>
  - Declares constants, e.g.,
    - ULONG_MAX
    - LONG_MAX
    - LONG_MIN
  - Values platform specific

Conversion Visualized

2’s Comp. → Unsigned

Ordering Inversion

Negative → Big Positive



Signed vs. Unsigned in C

- **Constants**
  - By default are considered to be signed integers
  - Unsigned if have “U” as suffix
    - `0U, 4294967259U`
- **Casting**
    - `int tx, ty;`
    - `unsigned ux, uy;`
  - Explicit casting between signed & unsigned same as U2T and T2U
    - `tx = (int) ux;`
    - `uy = (unsigned) ty;`
  - Implicit casting also occurs via assignments and procedure calls
    - `tx = ux;`
    - `uy = ty;`

Casting Surprises

Expression Evaluation

If mix unsigned and signed in single expression,
signed values implicitly cast to unsigned

Including comparison operations **<, >, ==, <=, >=**

Examples for *W* = 32: **TMIN = -2,147,483,648** **TMAX = 2,147,483,647**

Constant$_1$	Constant$_2$	Relation	Evaluation
0	0U	==	unsigned
-1	0	<	signed
-1	0U	>	unsigned
2147483647	-2147483647-1	>	signed
2147483647U	-2147483647-1	<	unsigned
-1	-2	>	signed
(unsigned)-1	-2	>	unsigned
2147483647	2147483648U	<	unsigned
2147483647	(int) 2147483648U	>	signed

Shift Operations

Left shift: **`x << y`**

- Shift bit-vector `x` left by `y` positions
  - Throw away extra bits on left
  - Fill with 0s on right
- Multiply by 2**y

Right shift: **`x >> y`**

- Shift bit-vector `x` right by `y` positions
  - Throw away extra bits on right
- Logical shift (for unsigned)
  - Fill with 0s on left
- Arithmetic shift (for signed)
  - Replicate most significant bit on righ
  - Maintain sign of x
- Divide by 2**y
- *correct truncation (towards 0) requires some care with signed numbers*

Argument x	01100010
<< 3	00010*000*
Logical >> 2	*00*011000
Arithmetic >> 2	*00*011000

Argument x	10100010
<< 3	00010*000*
Logical >> 2	*00*101000
Arithmetic >> 2	*11*101000

Undefined behavior when y < 0 or y ≥ word_size

Using Shifts and Masks

Extract 2nd most significant byte of an integer

First shift: x >> (2 * 8)

Then mask: (x >> 16) & 0xFF

x	01100001 01100010 01100011 01100100
x >> 16	00000000 00000000 01100001 01100010
(x >> 16) & 0xFF	*00000000 00000000 00000000 11111111* 00000000 00000000 00000000 01100010

Extracting the sign bit

(x >> 31) & 1 - need the “& 1” to clear out all other bits except LSB

Conditionals as Boolean expressions (assuming x is 0 or 1 here)

if (x) a=y else a=z; which is the same as a = x ? y : z;

Can be re-written as: a = ((x << 31) >> 31) & y + (!x << 31) >> 31) & z

Sign Extension

Task:

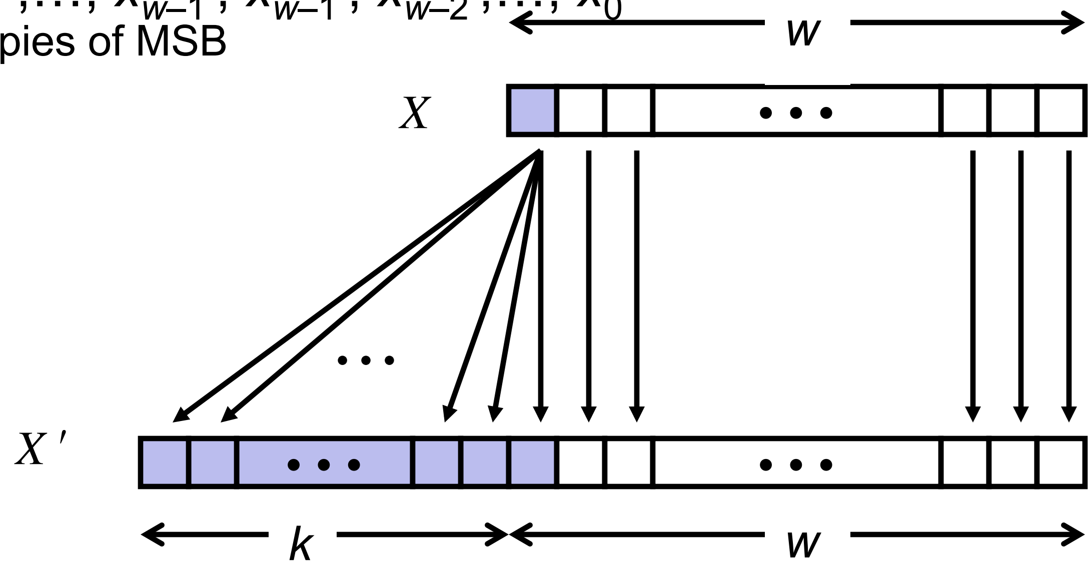
Given *w*-bit signed integer *x*

Convert it to *w*+*k*-bit integer with same value

Rule:

Make *k* copies of sign bit:

$$X' = \underbrace{x_{w-1},\ldots,x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2},\ldots,x_0$$



Sign Extension Example

```
short int x =  12345;
int       ix = (int) x;
short int y = -12345;
int       iy = (int) y;
```

	Decimal	Hex	Binary
x	12345	30 39	00110000 01101101
ix	12345	00 00 30 39	00000000 00000000 00110000 01101101
y	-12345	CF C7	11001111 11000111
iy	-12345	FF FF CF C7	11111111 11111111 11001111 11000111

Converting from smaller to larger integer data type

C automatically performs sign extension