

Memory & Caches III

CSE 351 Autumn 2022

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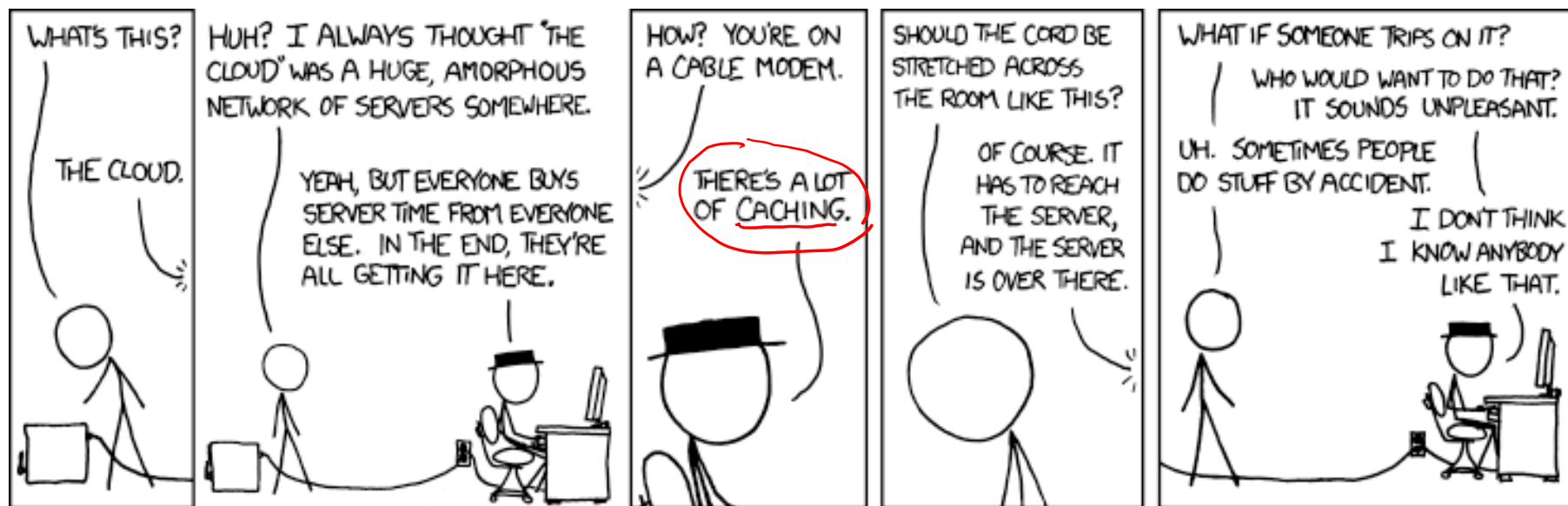
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Relevant Course Information

- ❖ Lab 3 due Friday (11/11)
- ❖ Lab 4 released Monday, due after Thanksgiving
 - Can do Part 1 after today; will need Lecture 19 to do Part 2
- ❖ hw17 due Wednesday (11/16)
 - Covers the major cache mechanics – BIG homework

Making memory accesses fast!

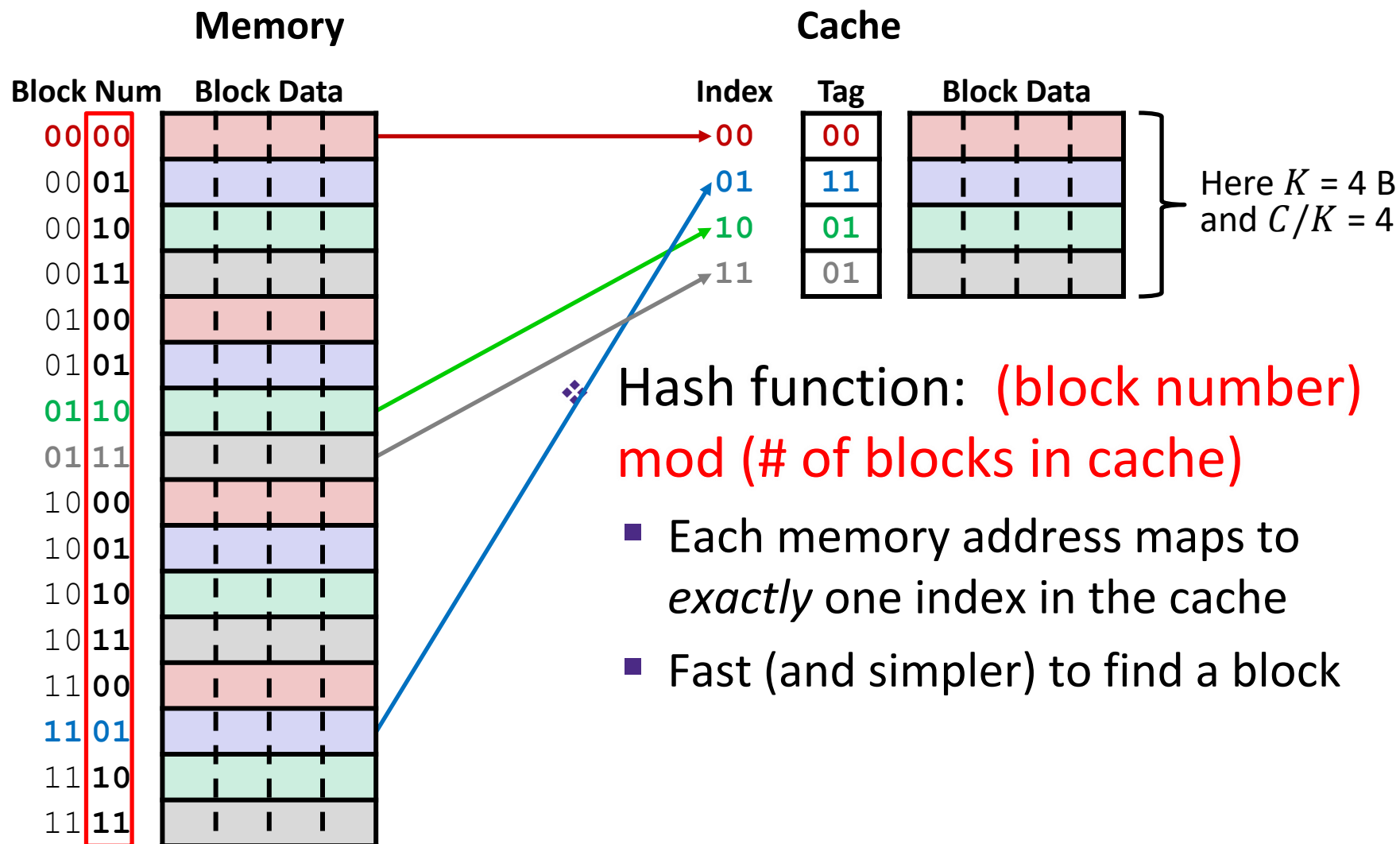
- ❖ Cache basics
- ❖ Principle of locality
- ❖ Memory hierarchies
- ❖ Cache organization
 - Direct-mapped (*sets*; index + tag)
 - **Associativity (*ways*)**
 - **Replacement policy**
 - Handling writes
- ❖ Program optimizations that consider caches

Reading Review

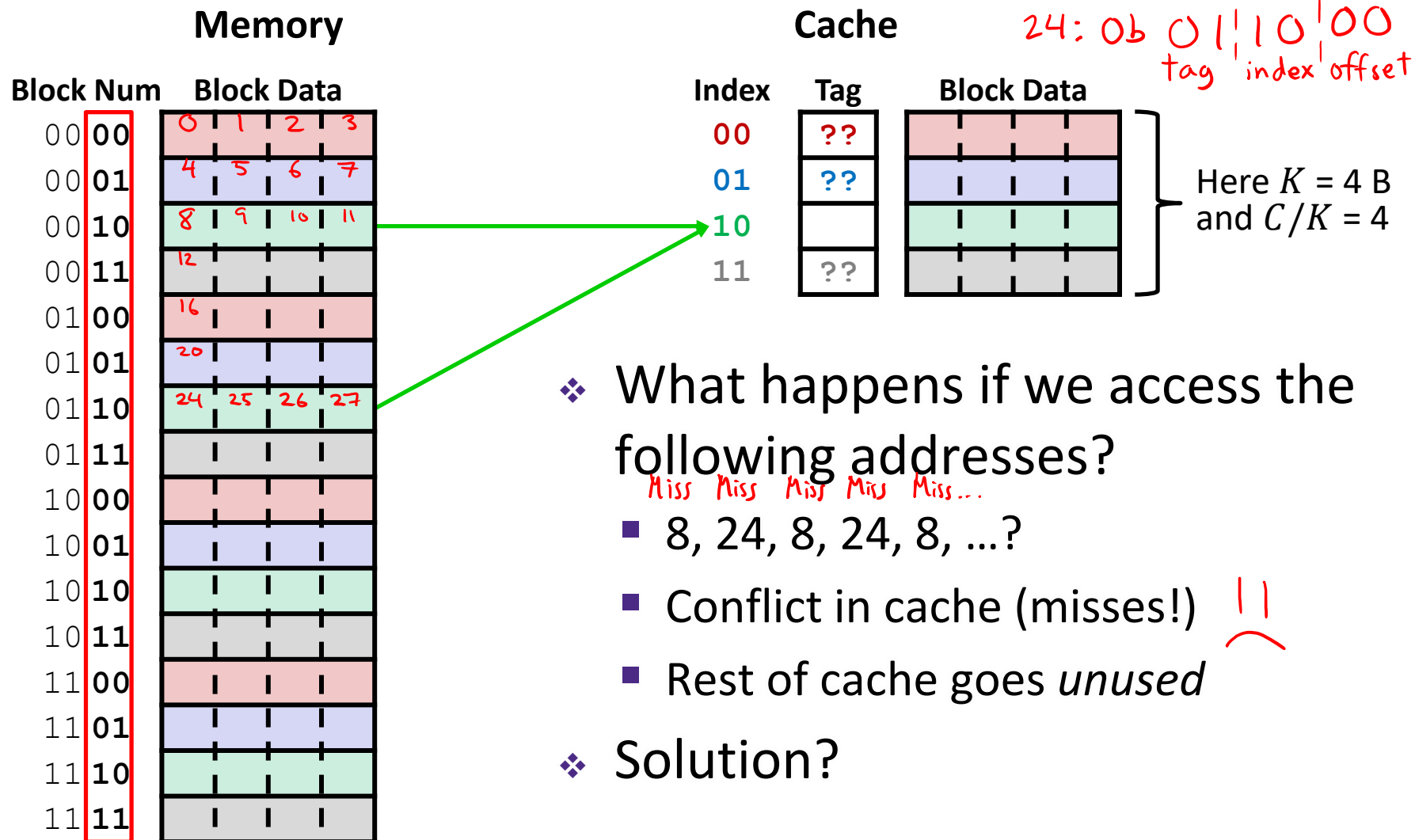
- ❖ Terminology:
 - Associativity: sets, fully-associative cache
 - Replacement policies: least recently used (LRU)
 - Cache line: cache block + management bits (valid, tag)
 - Cache misses: compulsory, conflict, capacity

- ❖ Questions from the Reading?

Review: Direct-Mapped Cache

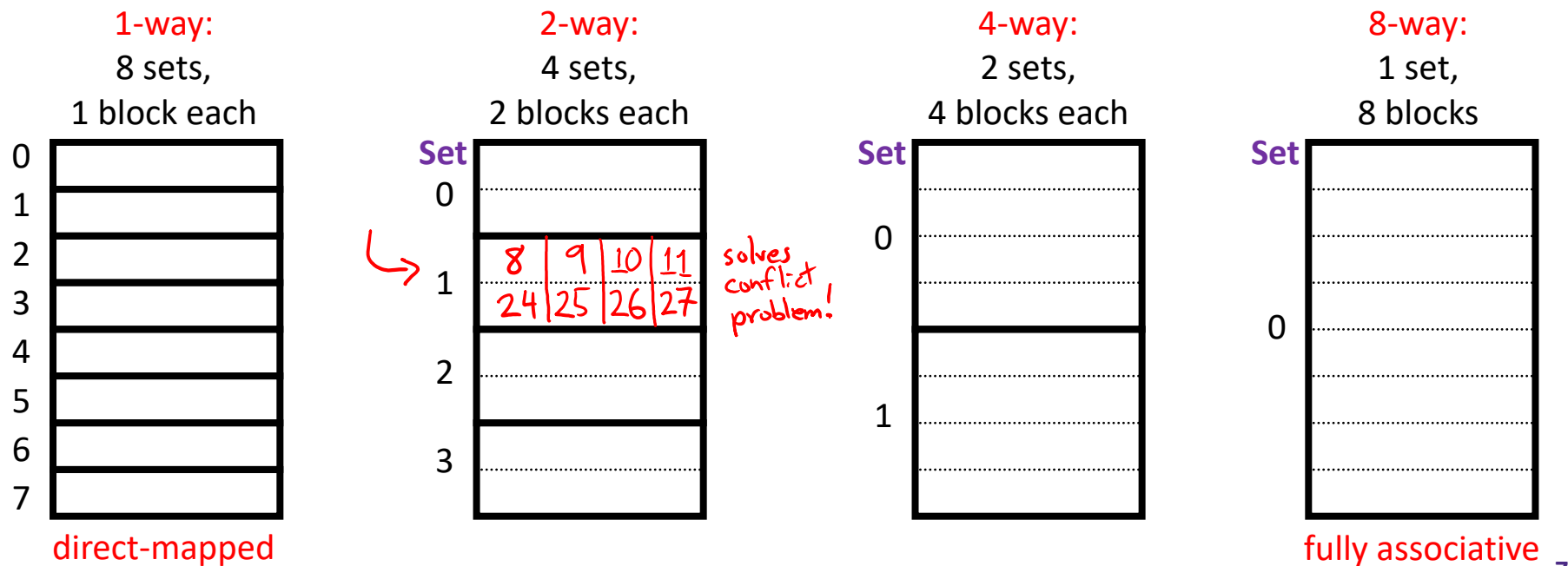


Direct-Mapped Cache Problem



Associativity

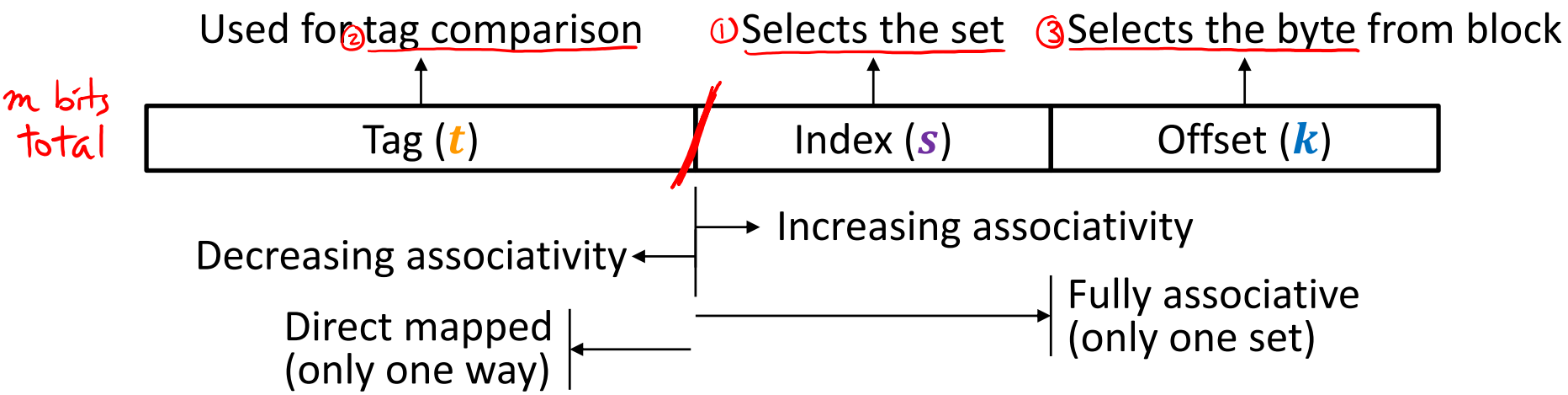
- ❖ What if we could store data in any place in the cache?
 - More complicated hardware = more power consumed, slower
- ❖ So we *combine* the two ideas:
 - Each address maps to exactly one **set**
 - Each set can store block in more than one **way**



Cache Organization (3)

Note: The textbook uses “b” for offset bits

- ❖ **Associativity (E):** # of ways for each set
 - Such a cache is called an “ E -way set associative cache”
 - We index into cache sets, of which there are $S = (C/K)/E$
 - Use lowest $\log_2(S) = s$ bits of block address
 - Direct-mapped: $E = 1$, so $s = \log_2(C/K)$ as we saw previously
 - Fully associative: $E = C/K$, so $s = 0$ bits

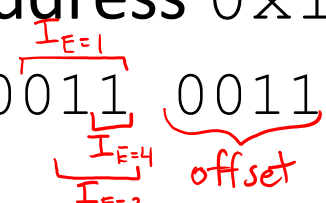


Example Placement

block size:	16 B
capacity:	8 blocks
address:	16 bits

❖ Where would data from address 0x1833 be placed?

■ Binary: 0b 0001 1000 0011 0011



$$t = m - s - k \quad s = \log_2(C/K/E) \quad k = \log_2(K)$$

m-bit address:

Tag (<i>t</i>)	Index (<i>s</i>)	Offset (<i>k</i>)
------------------	--------------------	---------------------

s = ?

Direct-mapped

Set	Tag	Data
(000)	0	
(001)	1	
(010)	2	
(011)	3	✓
(100)	4	
(101)	5	
(110)	6	
(111)	7	

s = ?

2-way set associative

Set	Tag	Data
(00)0		
(01)1		
(10)2		
(11)3	✓	✓

s = ?

4-way set associative

Set	Tag	Data
(0)0		
(1)1	✓	✓

Block Placement and Replacement

- ❖ Any empty block in the correct set may be used to store block
 - **Valid bit** for each cache block indicates if valid (1) or mystery (0) data
- ❖ If there are no empty blocks, which one should we replace?
 - No choice for direct-mapped caches
 - Caches typically use something close to *least recently used (LRU)* (hardware usually implements “not most recently used”)

Direct-mapped

Set	V	Tag	Data
0			
1			
2			
3			
4			
5			
6			
7			

2-way set associative

Set	V	Tag	Data
0			
1			
2			
3			

4-way set associative

Set	V	Tag	Data
0			
1			

Polling Questions

❖ We have a cache of size 2 KiB with block size of 128 B. If our cache has 2 sets, what is its associativity?

▪ Vote in Ed Lessons

A. 2

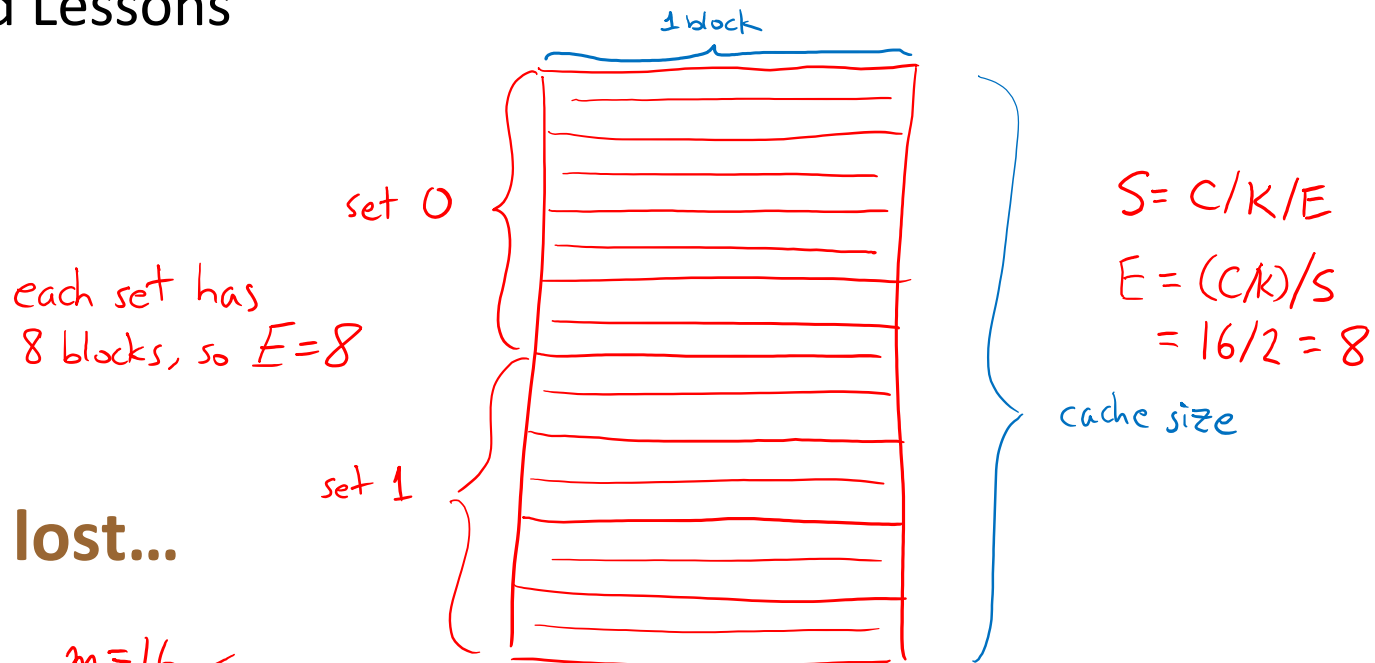
B. 4

C. 8

D. 16

E. We're lost...

$C = 2^{11} B$
 $K = 2^7 B$
 cache holds $C/K = 2^{11-7} = 2^4 = 16$ blocks

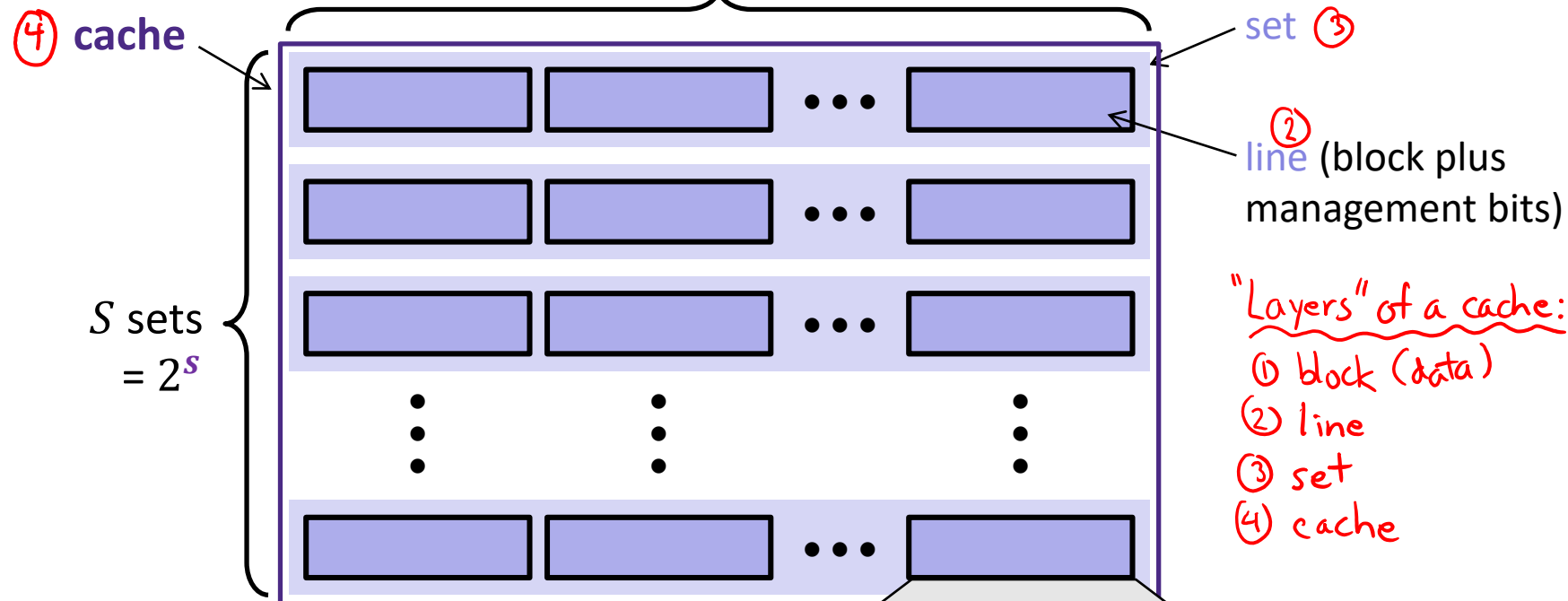


❖ If addresses are 16 bits wide, how wide is the Tag field? $k = \log_2(K) = 7$ bits, $s = \log_2(S) = 1$ bit, $t = m - s - k = 8$ bits

General Cache Organization (S, E, K)

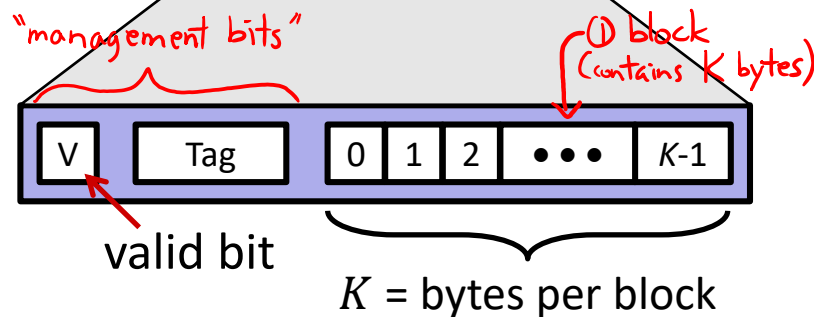
sets *associativity* *block size*

$E =$ blocks (or lines) per set



Cache size:

$C = K \times E \times S$ data bytes
(doesn't include V or Tag)



Notation Review

- ❖ We just introduced a lot of new variable names!
 - Please be mindful of block size notation when you look at past exam questions or are watching videos

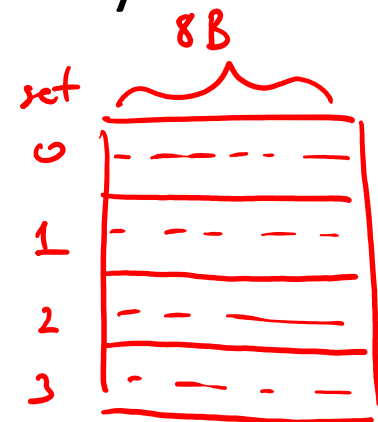
Parameter	Variable	Formulas
Block size	K (B in book)	$M = 2^m \leftrightarrow m = \log_2 M$ $S = 2^s \leftrightarrow s = \log_2 S$ $K = 2^k \leftrightarrow k = \log_2 K$ $C = K \times E \times S$ $s = \log_2(C/K/E)$ $m = t + s + k$
Cache size	C	
Associativity	E	
Number of Sets	S	
Address space	M	
Address width	m	
Tag field width	t	
Index field width	s	
Offset field width	k (b in book)	

Example Cache Parameters Problem

$\rightarrow 2^{10} \text{ B} \iff m = 10 \text{ bits}$ MP

- ❖ 1 KiB address space, 125 cycles to go to memory.

Fill in the following table:



C	Cache Size	64 B	2^6
K	Block Size	8 B	2^3
E	Associativity	2-way	2^1
HT	Hit Time	3 cycles	
MR	Miss Rate	20%	
	Tag Bits	5	
	Index Bits	2	$2^6 / 2^3 / 2^1$
	Offset Bits	3	
	AMAT	$3 + 0.2(125) = 28 \text{ clock cycles}$	

$t = m - s - k$

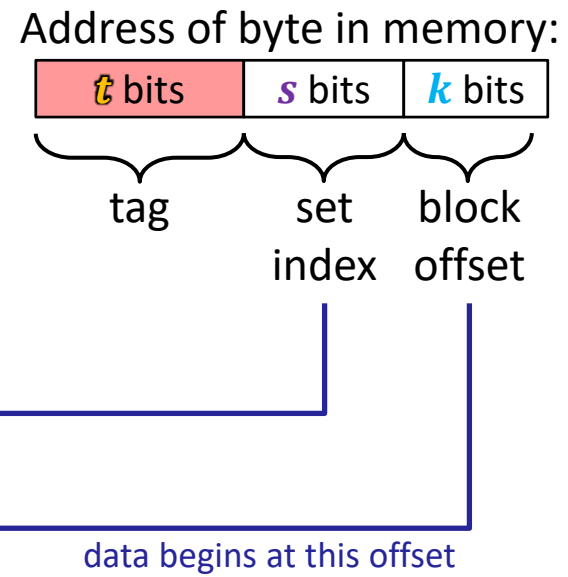
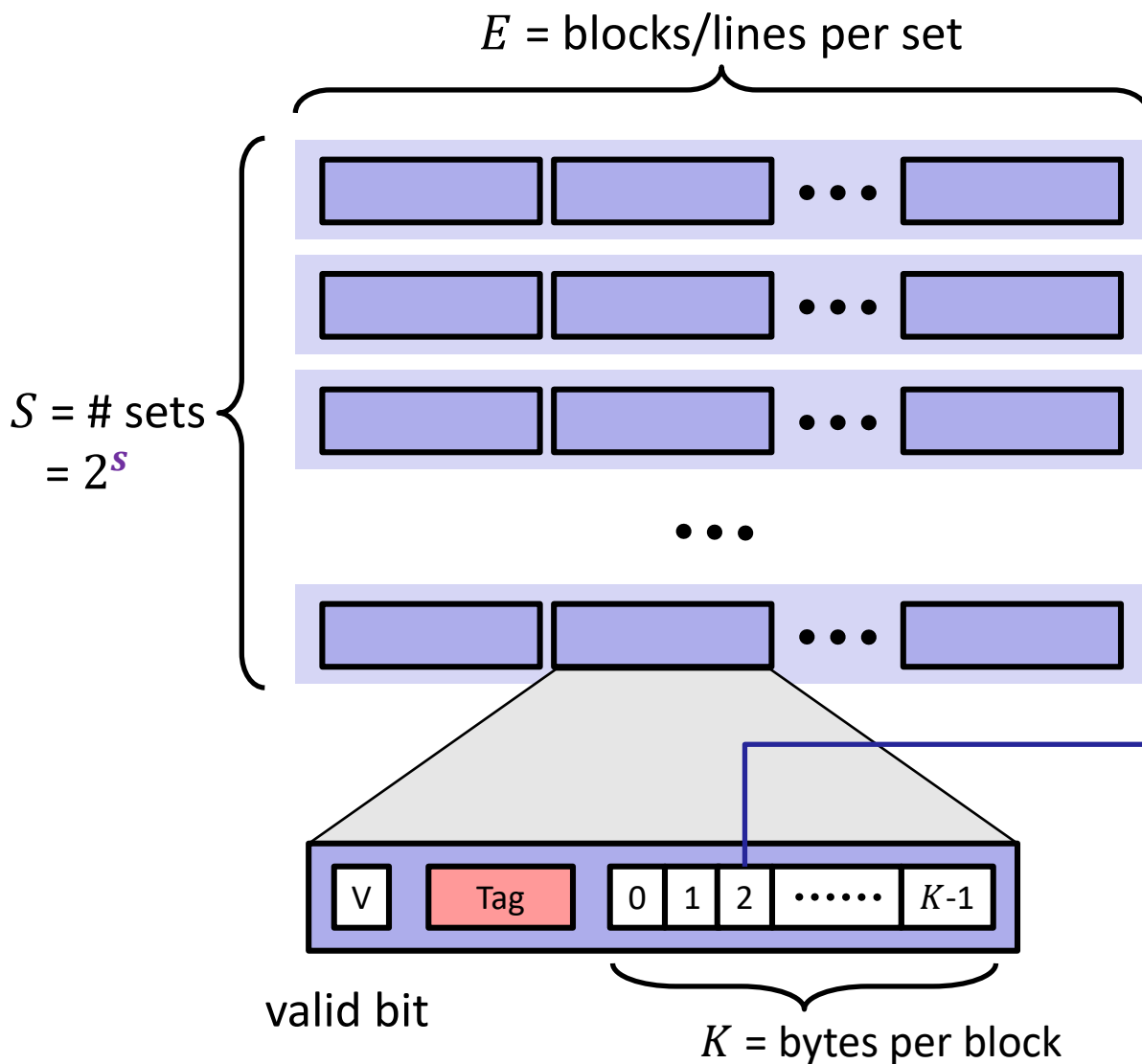
$s = \log_2(C/K/E)$

$k = \log_2(K)$

$AMAT = HT + MR * MP$

Cache Read

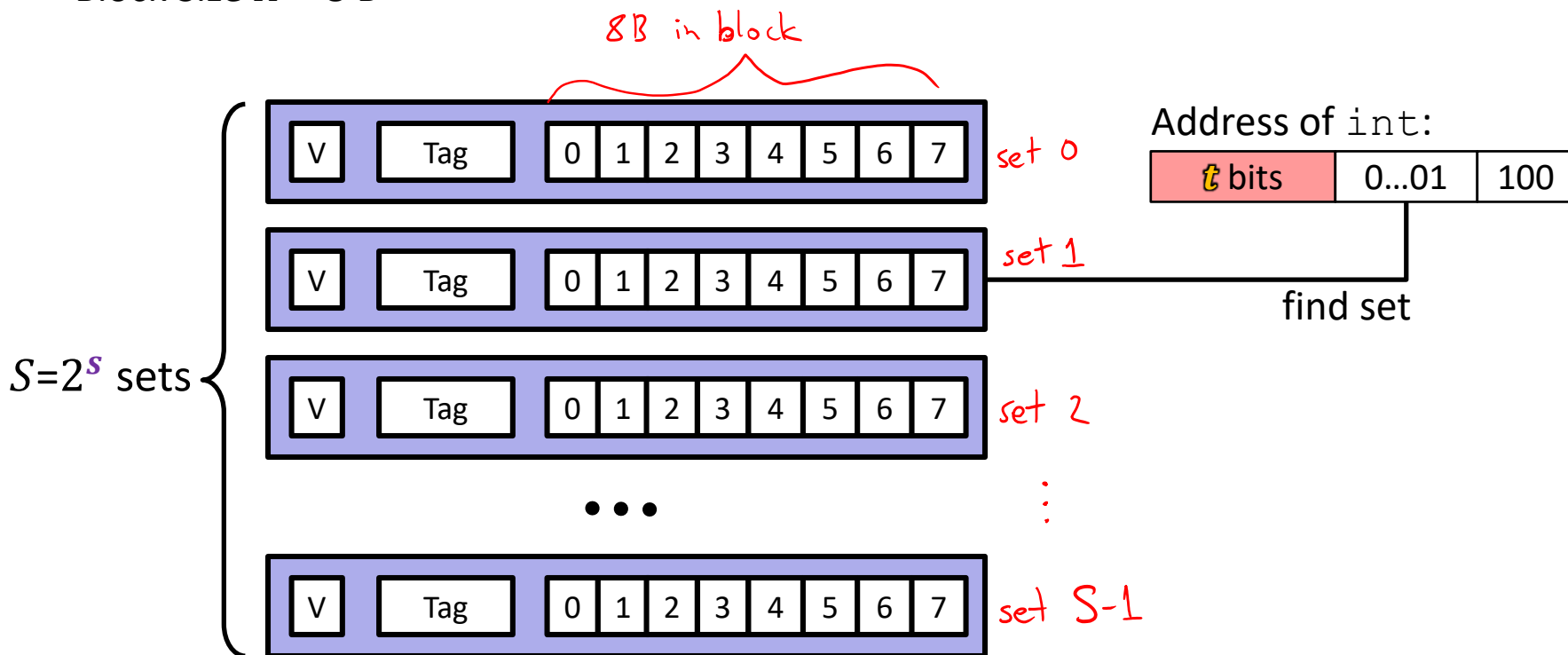
- 1) *Locate set*
- 2) *Check if any line in set is valid and has matching tag: hit*
- 3) *Locate data starting at offset*



Example: Direct-Mapped Cache ($E = 1$)

Direct-mapped: One line per set

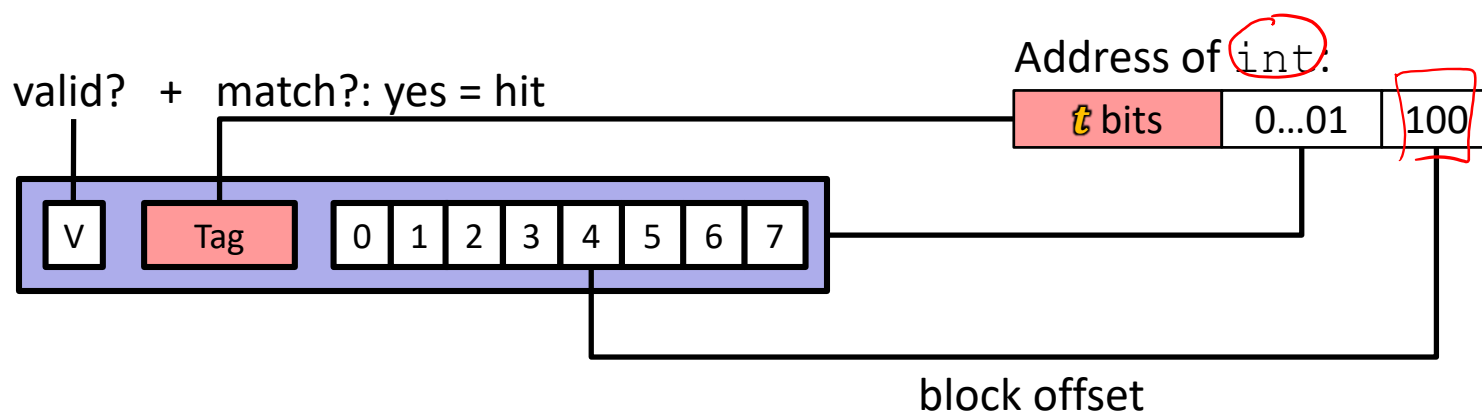
Block Size $K = 8$ B



Example: Direct-Mapped Cache ($E = 1$)

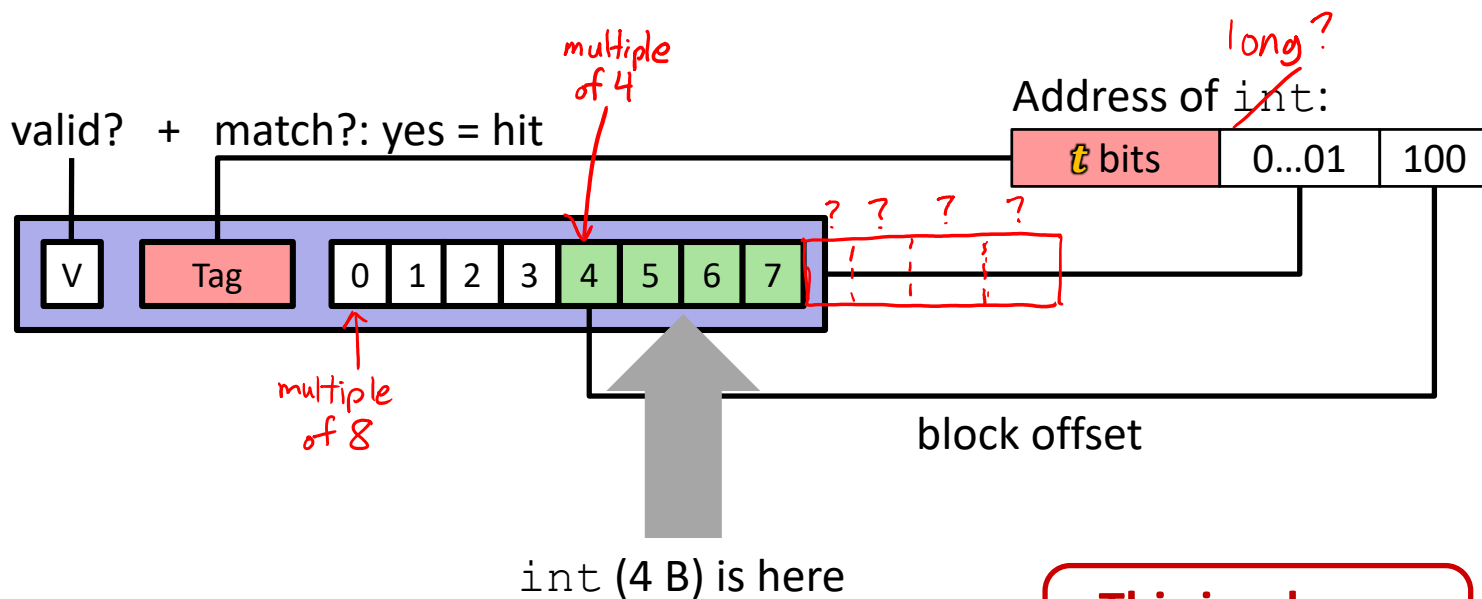
Direct-mapped: One line per set

Block Size $K = 8$ B



Example: Direct-Mapped Cache ($E = 1$)

Direct-mapped: One line per set
 Block Size $K = 8$ B



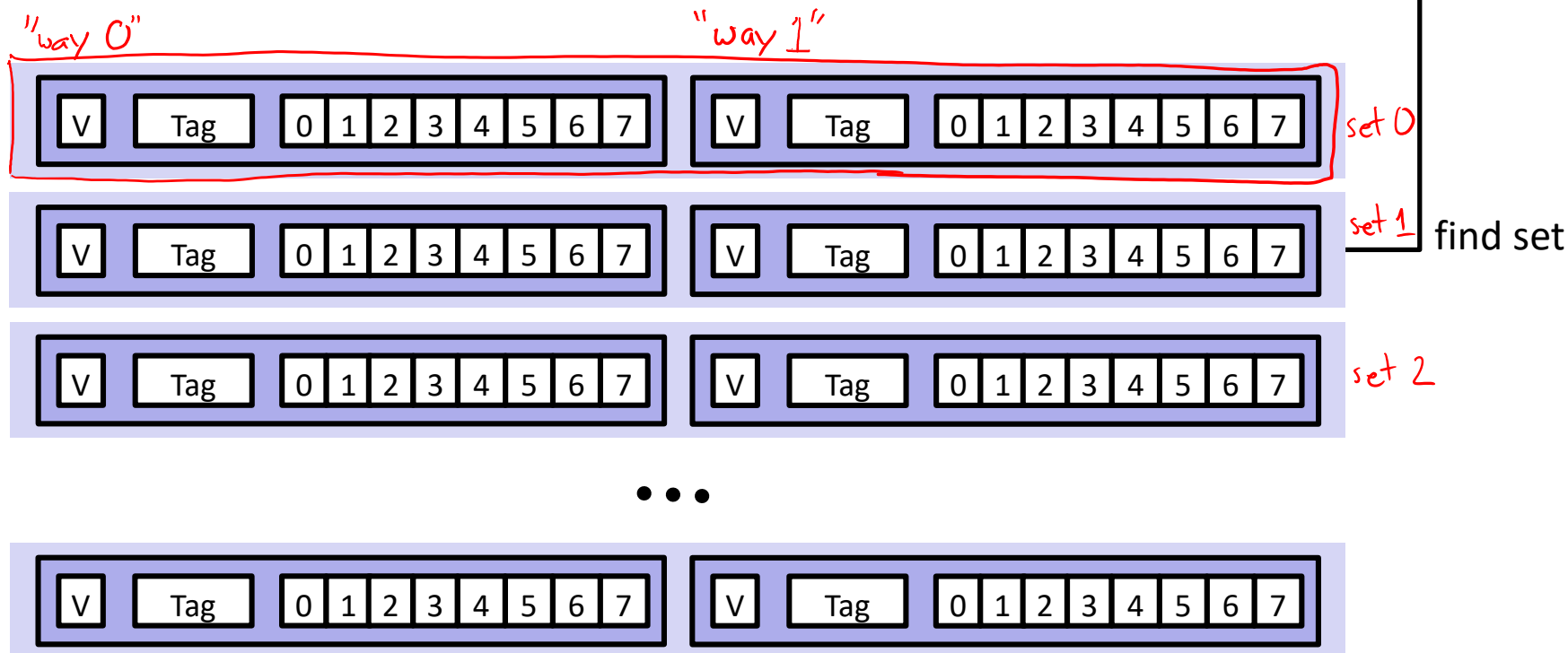
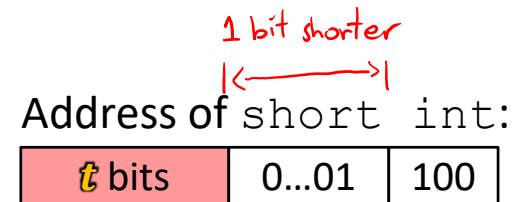
This is why we want alignment!

No match? Then old line gets evicted and replaced

no unnecessary extra cache accesses across block boundaries

Example: Set-Associative Cache ($E = 2$)

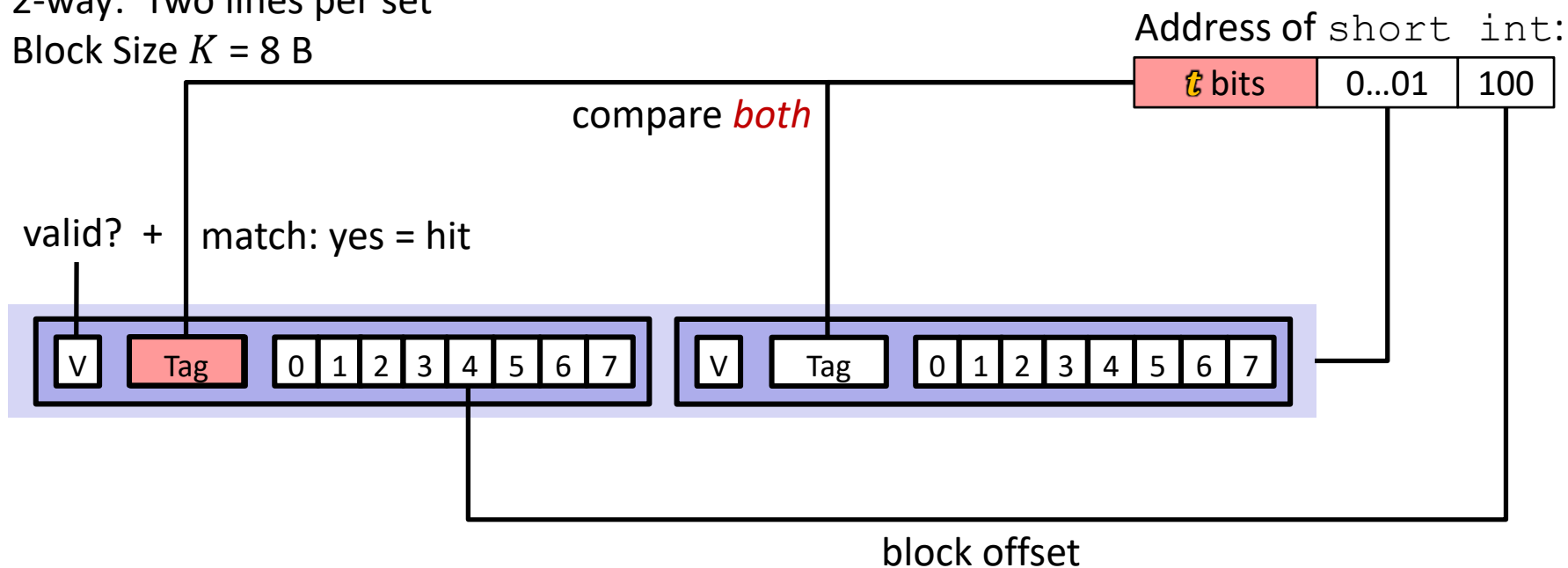
2-way: Two lines per set
 Block Size $K = 8$ B



Example: Set-Associative Cache ($E = 2$)

2-way: Two lines per set

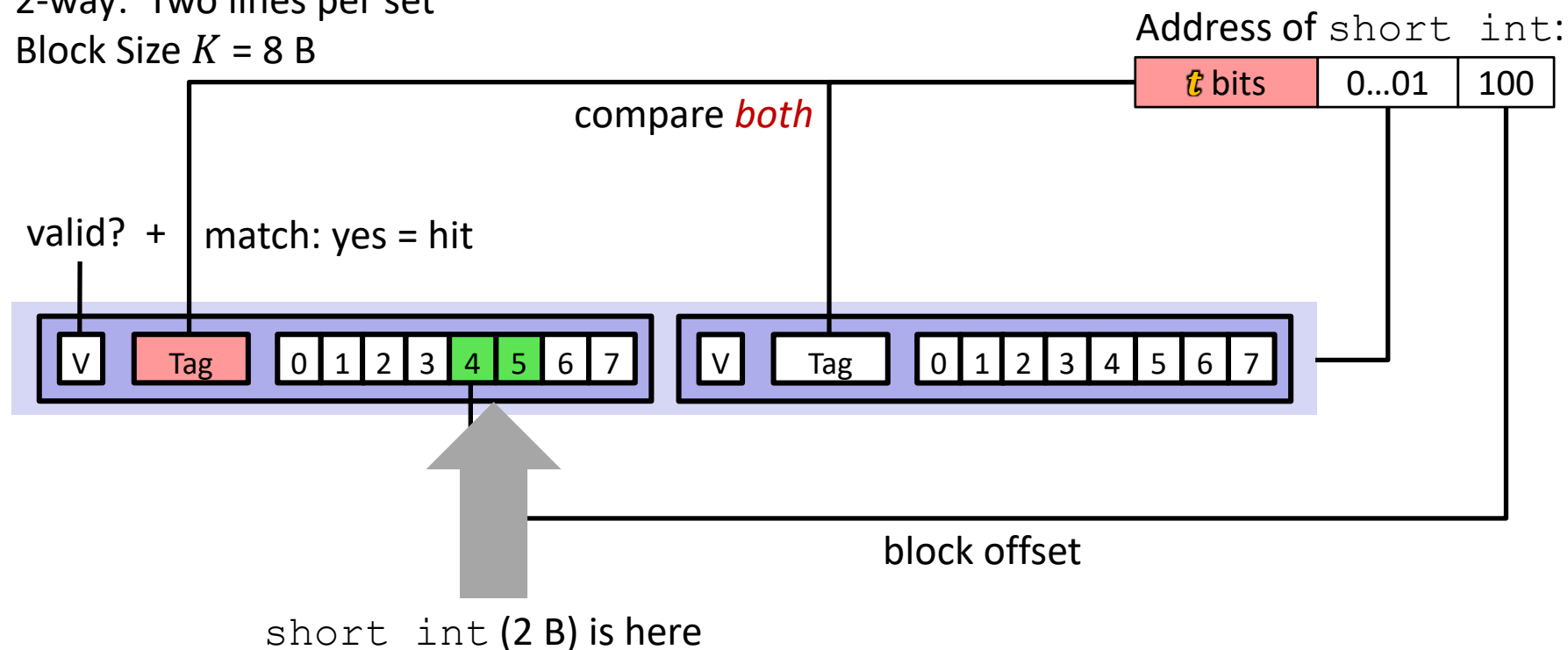
Block Size $K = 8$ B



Example: Set-Associative Cache ($E = 2$)

2-way: Two lines per set

Block Size $K = 8$ B



No match?

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), ...

Types of Cache Misses: 3 C's!

- ❖ **Compulsory** (cold) miss
 - Occurs on first access to a block
- ❖ **Conflict** miss
 - Conflict misses occur when the cache is large enough, but multiple data objects all map to the same slot
 - *e.g.*, referencing blocks 0, 8, 0, 8, ... could miss every time
 - Direct-mapped caches have more conflict misses than E -way set-associative (where $E > 1$)
- ❖ **Capacity** miss
 - Occurs when the set of active cache blocks (the *working set*) is larger than the cache (just won't fit, even if cache was *fully-associative*)
 - **Note:** *Fully-associative* only has Compulsory and Capacity misses

Example Code Analysis Problem

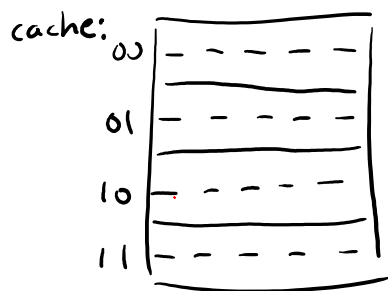
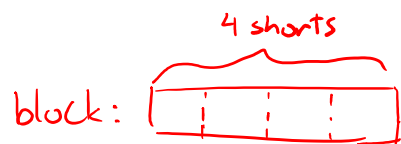
- Assuming the cache starts cold (all blocks invalid) and `sum`, `i`, and `j` are stored in registers, calculate the **miss rate**: 100%

t = 5 bits, s = 2 bits, k = 3 bits

- $m = 10$ bits, $C = 64$ B, $K = 8$ B, $E = 2$

2 bytes per element

```
#define SIZE 8
short ar[SIZE][SIZE], sum = 0; // &ar=0x200
for (int i = 0; i < SIZE; i++)
    for (int j = 0; j < SIZE; j++)
        sum += ar[j][i];
```



	tag	index	offset	
<code>ar[0][0]</code>	0b 10	0000	0000	→ M (1 st way)
<code>ar[1][0]</code>	0b 10	0001	0000	→ M (1 st way)
<code>ar[2][0]</code>	0b 10	0010	0000	→ M (2 nd way)
<code>ar[3][0]</code>	0b 10	0011	0000	→ M (2 nd way)
<code>ar[4][0]</code>	0b 10	0100	0000	→ M (replacement!)
⋮	⋮	⋮	⋮	
<code>ar[0][1]</code>	0b 10	0000	0010	→ M (conflict!)

matrix ar:

jumping by a row skips two block numbers (and sets)

A diagram showing a grid representing a matrix. A blue box highlights the element at row 0, column 1. An arrow points from the text below to this element.

Cache block holds 4 elements of a row of the matrix