Memory & Caches IV
CSE 351 Spring 2024

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The cache when you ask for something that was just evicted:

Playlist: [CSE 351 24Sp Lecture Tunes!]

Announcements, Reminders

- Happy Midterm madness!
- Mid-Quarter Survey on Canvas due tonight!
- HW 16 also due tonight! HW 17/18 due Friday (10 May).
- Lab 3 due Wednesday by 11:59 PM
- Lab 4 releasing on Wednesday-ish.
  - HW 19 helps you prepare for Lab 4 🫡

Reading Review

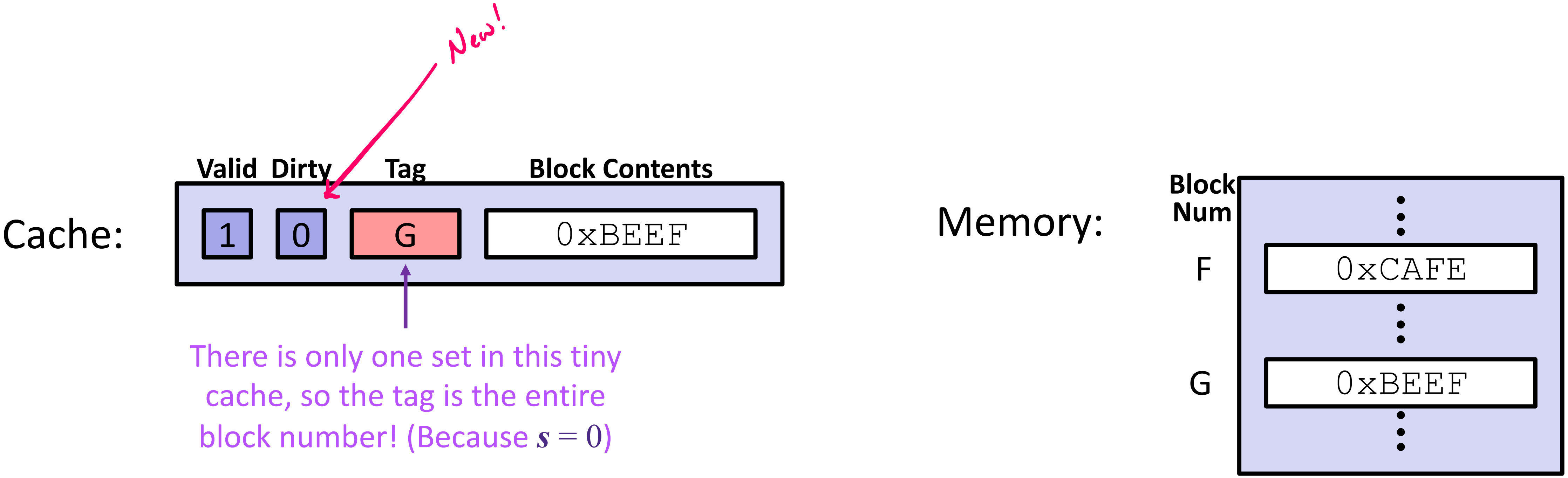
- Terminology:
  - Write-hit policies: write-back, write-through
  - Write-miss policies: write allocate, no-write allocate
  - Cache blocking

What about writes? (Review)

- Multiple copies of data may exist:
  - multiple levels of cache and main memory
- What to do on a write-hit (data already in cache)?
  - Write-through: write immediately to next level
  - Write-back: defer write to next level until line is evicted (replaced)
    - Must track which cache lines have been modified (using the "dirty bit")
- What to do on a write-miss (data not in cache)?
  - Write allocate: ("fetch on write") load into cache, then execute the write-hit policy
    - Good if more writes or reads to the location follow
  - No-write allocate: ("write around") just write immediately to next level
- Typical caches:
  - **Write-back + Write allocate, usually**
  - Write-through + No-write allocate, occasionally

Write-back, Write Allocate Example

Write-back: defer write to next level until line is evicted
Write-allocate: on a miss, bring the data into cache



There is only one set in this tiny cache, so the tag is the entire block number! (Because $s = 0$)

Note: We are making some unrealistic simplifications to keep this example simple and focus on the cache policies!

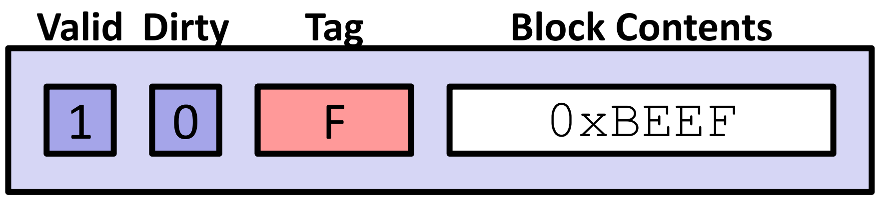
Write-back, Write Allocate Example

Write-back: defer write to next level until line is evicted
Write-allocate: on a miss, bring the data into cache

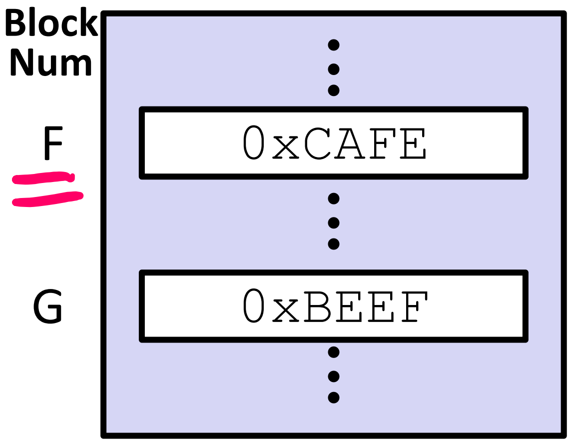
1) `mov $0xFACE, (F)` ← Not valid x86, assume we mean an address associated with this block num

Write Miss

Cache:



Memory:



Step 1: Bring **F** into cache

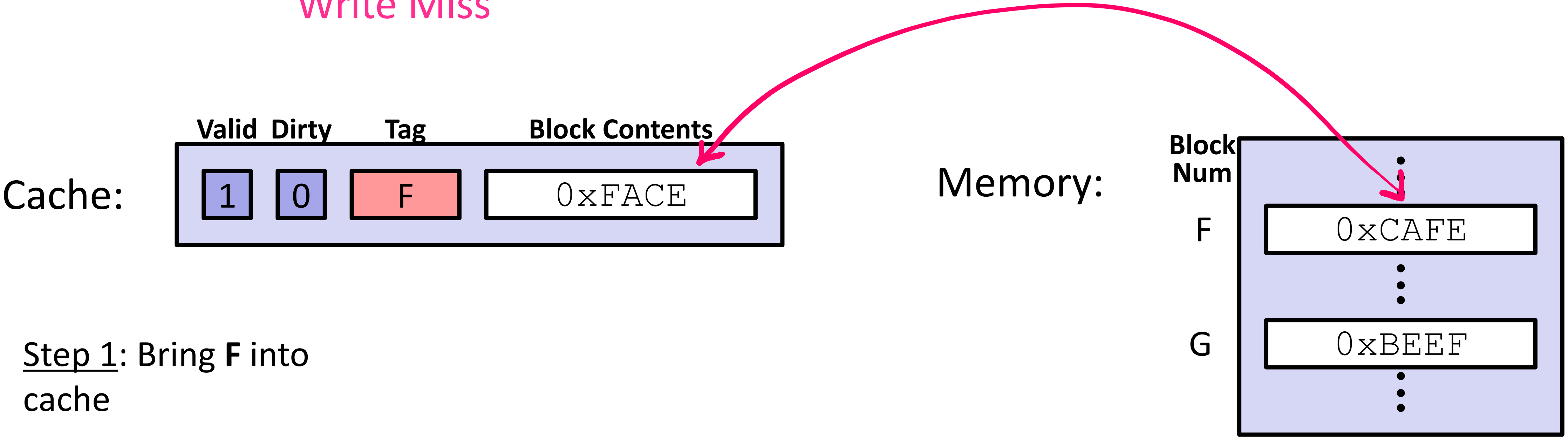
Write-back, Write Allocate Example

Write-back: defer write to next level until line is evicted
Write-allocate: on a miss, bring the data into cache

1) `mov $0xFACE, (F)`

Write Miss

Difference!



Step 1: Bring **F** into cache

Step 2: Write `0xFACE` to cache only and set the dirty bit. Why? Look at the values!

Write-back, Write Allocate Example

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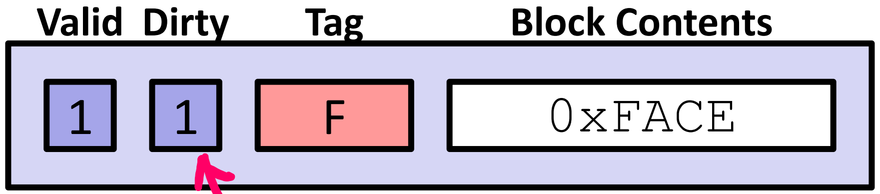
Write-allocate: on a miss, bring the data into cache

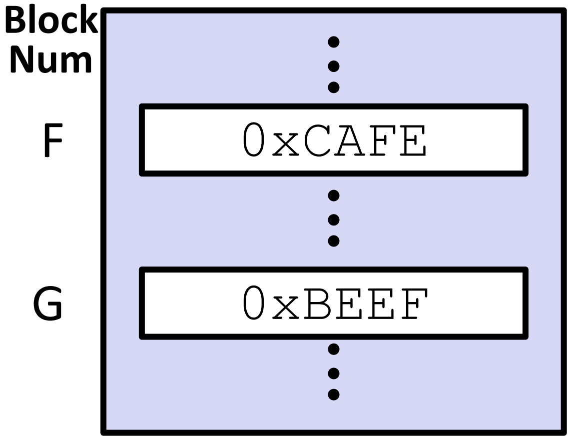
1) `mov $0xFACE, (F)`

Write Miss

Cache:



Memory:



This is set to 1 because the contents differ.

Step 1: Bring **F** into cache

Step 2: Write `0xFACE` to cache only and set the dirty bit. Why? Look at the values!

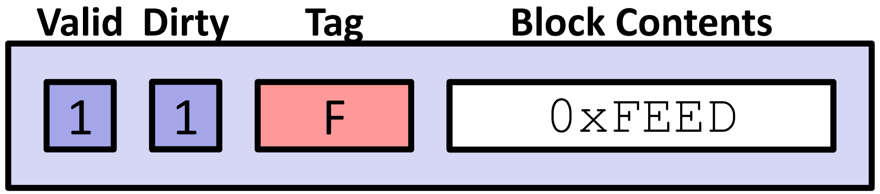
Write-back, Write Allocate Example

Write-back: defer write to next level until line is evicted
Write-allocate: on a miss, bring the data into cache

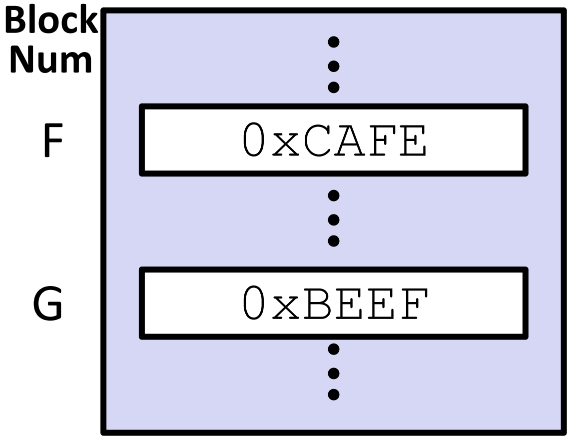
1) `mov $0xFACE, (F)` 2) `mov $0xFEED, (F)`

Write Miss Write Hit

Cache:



Memory:



Step: Write `0xFEED` to cache only (and set the dirty bit)

Redondant? Yes, but just do it. It's protocol!

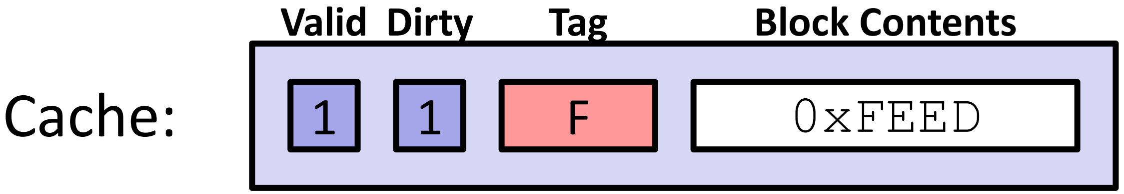
Write-back: defer write to next level until line is evicted
Write-allocate: on a miss, bring the data into cache

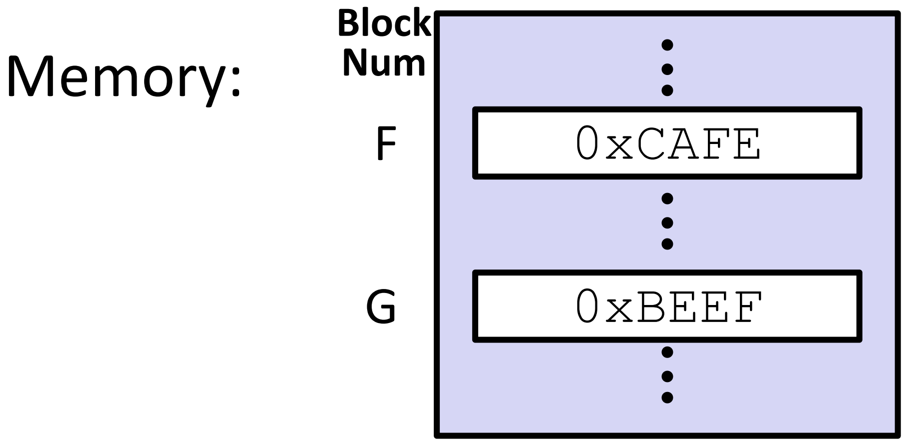
Write-back, Write Allocate Example

1) `mov $0xFACE, (F)` Write Miss

2) `mov $0xFEED, (F)` Write Hit

3) `mov (G), %ax` Read Miss





<u>Step 1</u>: Write **F** back to memory since it is dirty

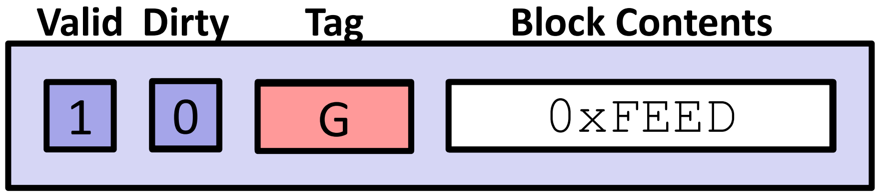
Write-back: defer write to next level until line is evicted
Write-allocate: on a miss, bring the data into cache

Write-back, Write Allocate Example

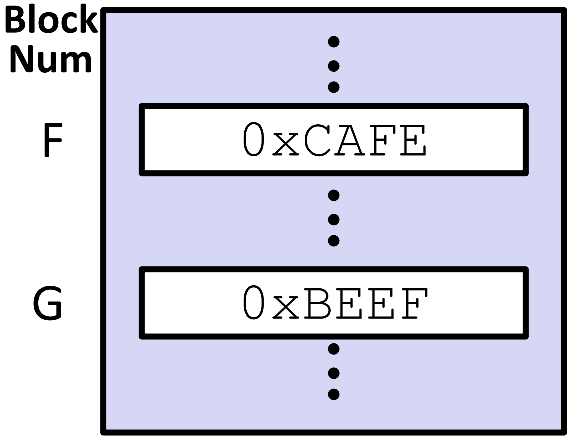
1) `mov $0xFACE, (F)` 2) `mov $0xFEED, (F)` 3) `mov (G), %ax`

Write Miss — Write Hit — Read Miss

Cache:



Memory:



Step 1: Write **F** back to memory since it is dirty

Step 2: Bring **G** into the cache so that we can copy it into `%ax`

Cache Simulator

- Want to play around with cache parameters and policies? Check out our cache simulator!
  - https://courses.cs.washington.edu/courses/cse351/cachesim/
- Way to use:
  - Take advantage of "explain mode" and navigable history to test your own hypotheses and answer your own questions
  - Self-guided Cache Sim Demo posted along with Section 7
  - Will be used in HW19 – Lab 4 Preparation

Polling Question

- Which of the following cache statements is FALSE?

A. A write-through cache will always match data with the memory hierarchy level below it

B. We can reduce compulsory misses by ~~decreasing~~ our block size

to make it true: increasing, not decreasing

C. A write-back cache will save time for code with good temporal locality on writes

D. We can reduce conflict misses by increasing associativity

E. We're lost…

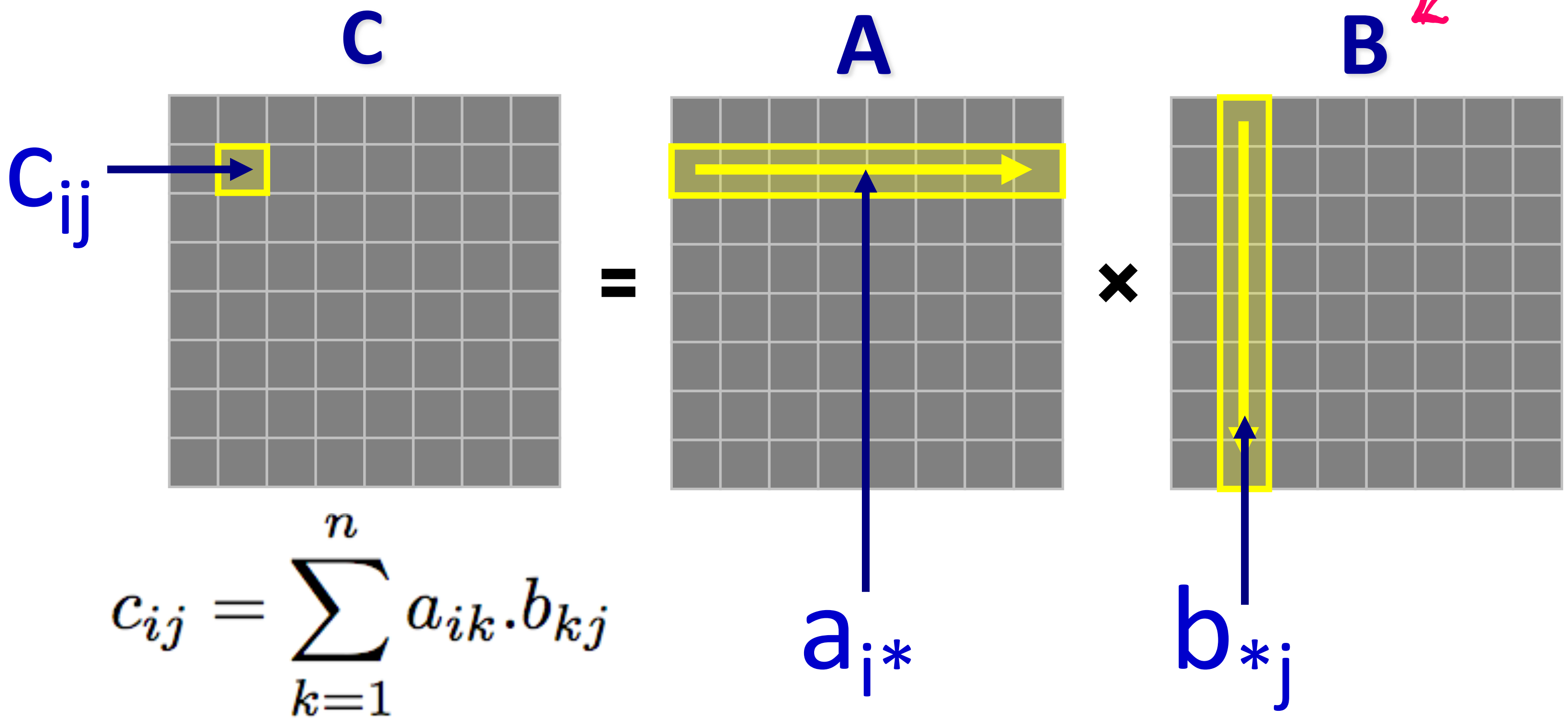
Optimizations for the Memory Hierarchy

- Write code that has locality!
  - Spatial: access data contiguously
  - Temporal: make sure access to the same data is not too far apart in time

- How can you achieve locality?
  - Adjust memory accesses in *code* (software) to improve miss rate (MR)
    - Requires knowledge of **both** how caches work as well as your system's parameters
  - Proper choice of algorithm
  - Loop transformations

Example: Matrix Multiplication (Why?)

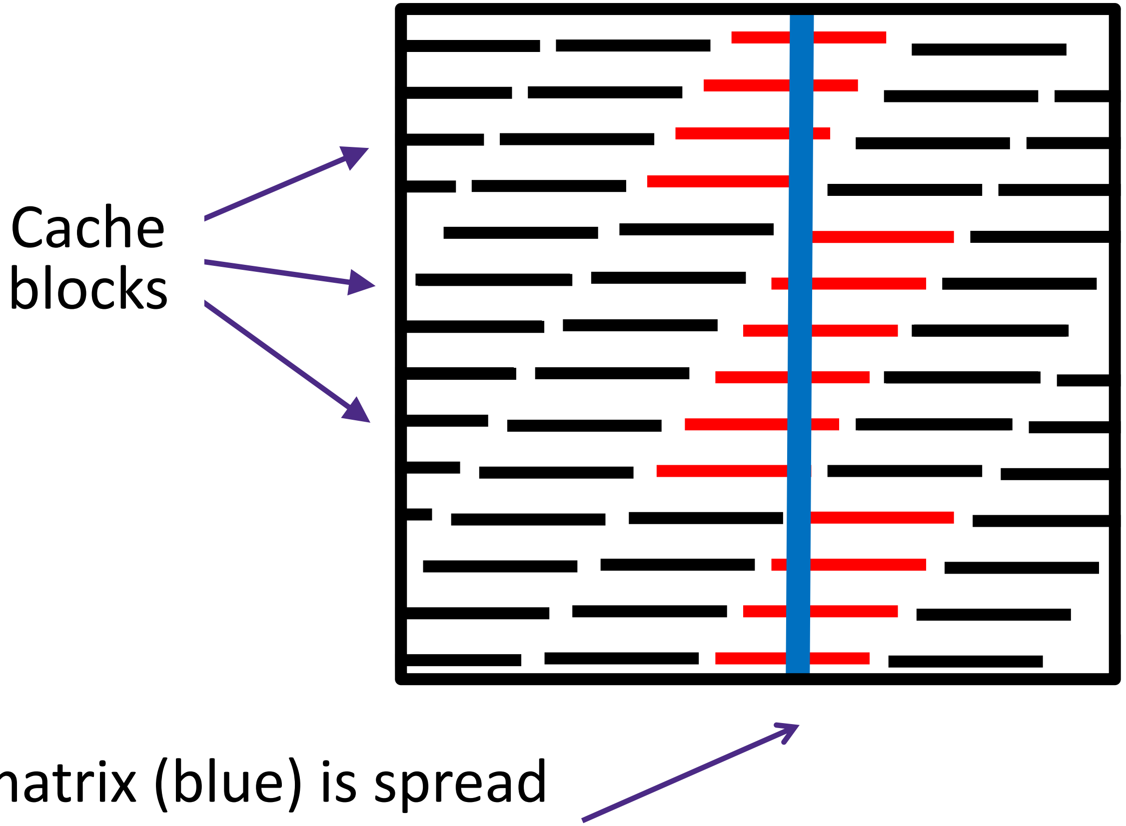
Sigh, our column-major problem child.



$$c_{ij} = \sum_{k=1}^{n} a_{ik} . b_{kj}$$

Matrices in Memory

- How do cache blocks fit into this scheme?
  - Row major matrix in memory:



column of matrix (blue) is spread among cache blocks shown in red

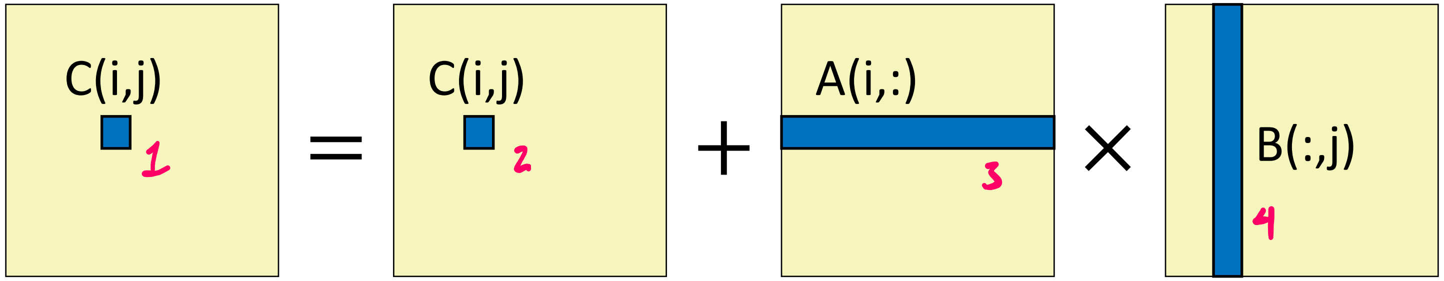
Aaand, this is why..

Naïve Matrix Multiply

```
# move along rows of A
for (i = 0; i < n; i++)
  # move along columns of B
  for (j = 0; j < n; j++)
    # EACH k loop reads row of A, col of B
    # Also read & write c(i,j) n times
    for (k = 0; k < n; k++)
      c[i*n+j] += a[i*n+k] * b[k*n+j];
```

Something to think about: How many memory accesses in this line?

4



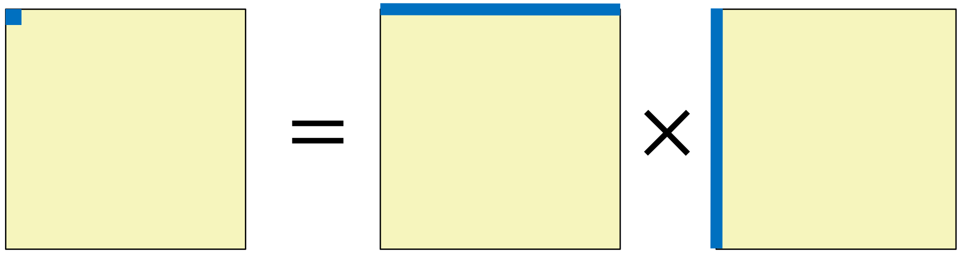
Cache Miss Analysis (Naïve)

Ignoring matrix `c`

- Scenario Parameters:
  - Square matrix ($n \times n$), elements are `doubles`
  - Cache block size K = 64 B = 8 `doubles`
  - Cache size $C \ll n$ (much smaller than n)

- Each iteration:
  - $\frac{n}{8} + n = \frac{9n}{8}$ misses

matrix: A B



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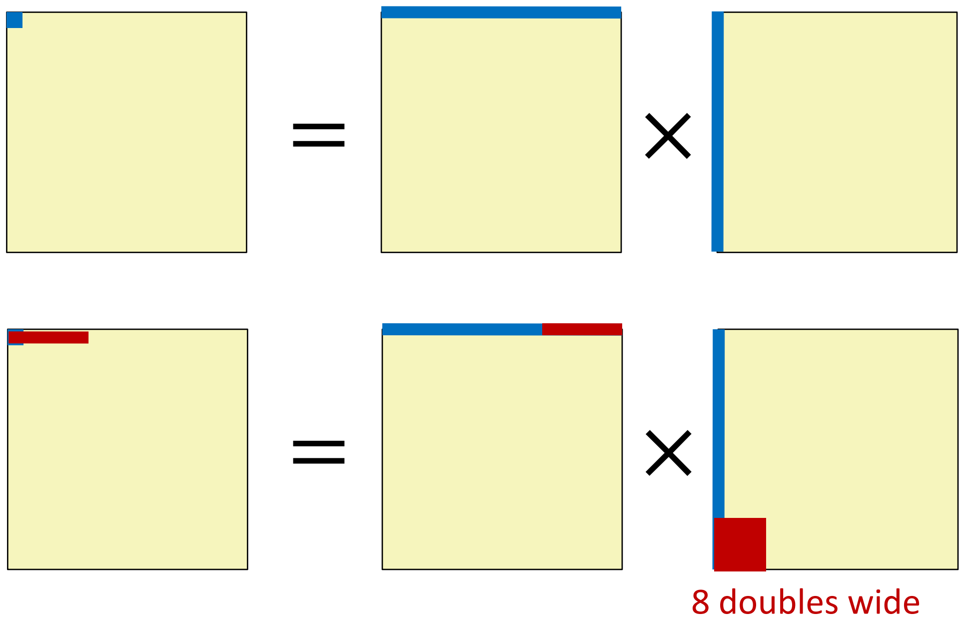
  - Afterwards in cache: (schematic)



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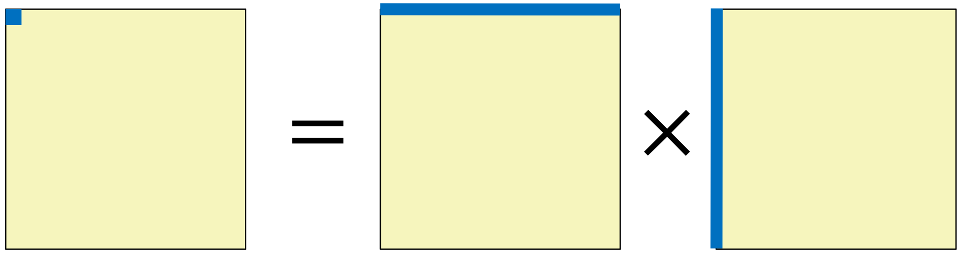


- Total misses: $\frac{9n}{8} \times n^2 = \frac{9}{8} n^3$

once per element in the $n \times n$ product matrix

Linear Algebra to the Rescue (1)

Ignoring matrix `c`

- Can get the same result of a matrix multiplication by splitting the matrices into smaller submatrices
(matrix "blocks")

- For example, multiply two 4×4 matrices:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \text{ with } B \text{ defined similarly.}$$

$$AB = \begin{bmatrix} (A_{11}B_{11} + A_{12}B_{21}) & (A_{11}B_{12} + A_{12}B_{22}) \\ (A_{21}B_{11} + A_{22}B_{21}) & (A_{21}B_{12} + A_{22}B_{22}) \end{bmatrix}$$

Linear Algebra to the Rescue (2)

Ignoring matrix C

C_{11}	C_{12}	C_{13}	C_{14}
C_{21}	C_{22}	C_{23}	C_{24}
C_{31}	C_{32}	C_{43}	C_{34}
C_{41}	C_{42}	C_{43}	C_{44}

A_{11}	A_{12}	A_{13}	A_{14}
A_{21}	A_{22}	A_{23}	A_{24}
A_{31}	A_{32}	A_{33}	A_{34}
A_{41}	A_{42}	A_{43}	A_{144}

B_{11}	B_{12}	B_{13}	B_{14}
B_{21}	B_{22}	B_{23}	B_{24}
B_{32}	B_{32}	B_{33}	B_{34}
B_{41}	B_{42}	B_{43}	B_{44}

Matrices of size $n \times n$, split into 4 blocks of size r (n=$4r$)

$$C_{22} = A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32} + A_{24}B_{42} = \sum_k A_{2k}*B_{k2}$$

Multiplication operates on small "block" matrices

- Choose size so that they fit in the cache!
- This technique called "*cache blocking*"

Blocked Matrix Multiply

- Blocked version of the naïve algorithm (wtf???):

```
# move by rxr BLOCKS now
for (i = 0; i < n; i += r)
  for (j = 0; j < n; j += r)
    for (k = 0; k < n; k += r)
      # block matrix multiplication
      for (ib = i; ib < i+r; ib++)
        for (jb = j; jb < j+r; jb++)
          for (kb = k; kb < k+r; kb++)
            c[ib*n+jb] += a[ib*n+kb]*b[kb*n+jb];
```

  - r = block matrix size (assume r divides n evenly)

Cache Miss Analysis (Blocked)

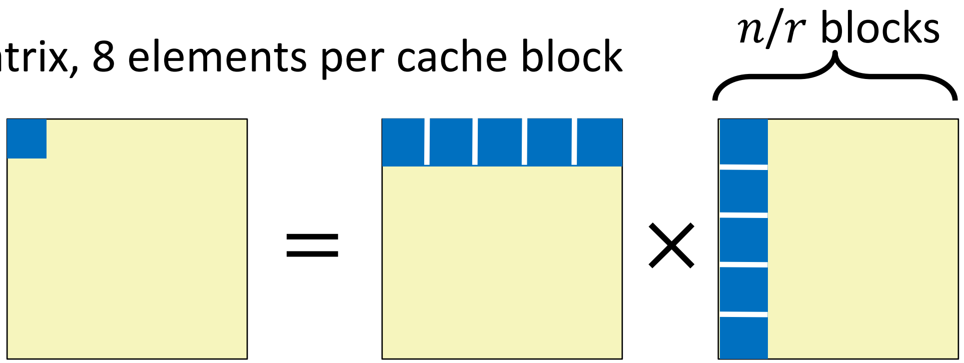
Ignoring matrix c

- Scenario Parameters:
  - Cache block size K = 64 B = 8 doubles
  - Cache size $C \ll n$ (much smaller than n)
  - Three blocks ■ ($r \times r$) fit into cache: $3r^2 < C$

r^2 elements per sub-matrix, 8 elements per cache block

- Each block iteration:
  - $r^2/8$ misses per block
  - $\frac{2n}{r} \times \frac{r^2}{8} = \frac{nr}{4}$

n/r blocks in row and in column



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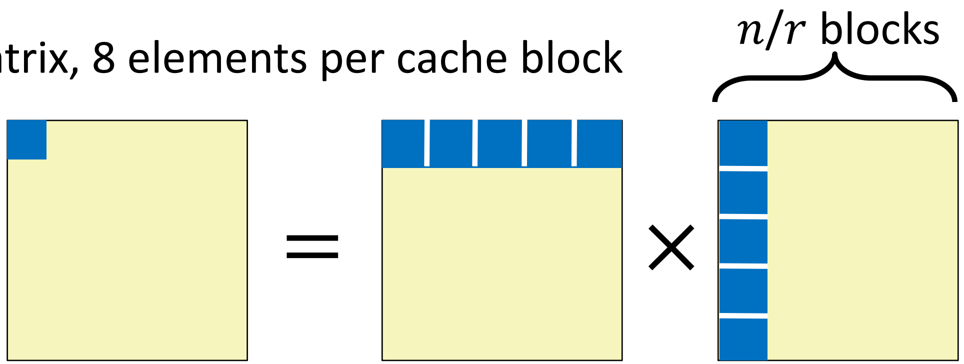
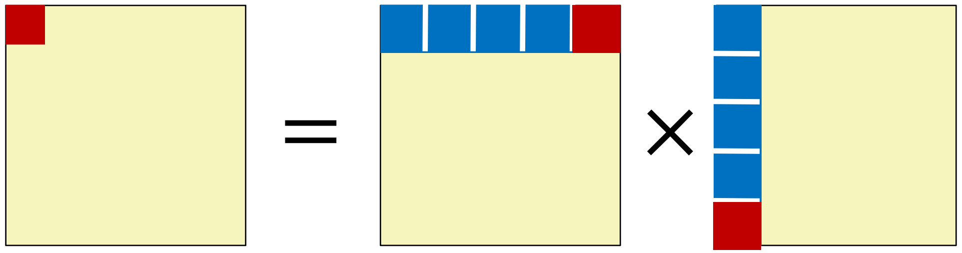


n/r blocks in row and in column

  - Afterwards in cache (schematic)



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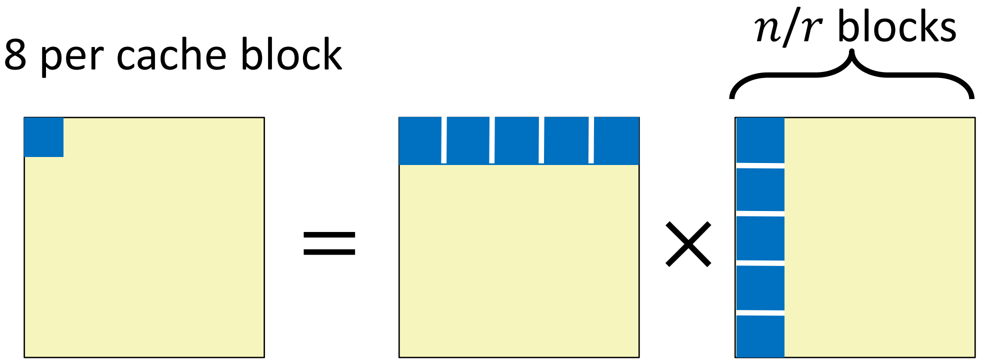


n/r blocks in row and column

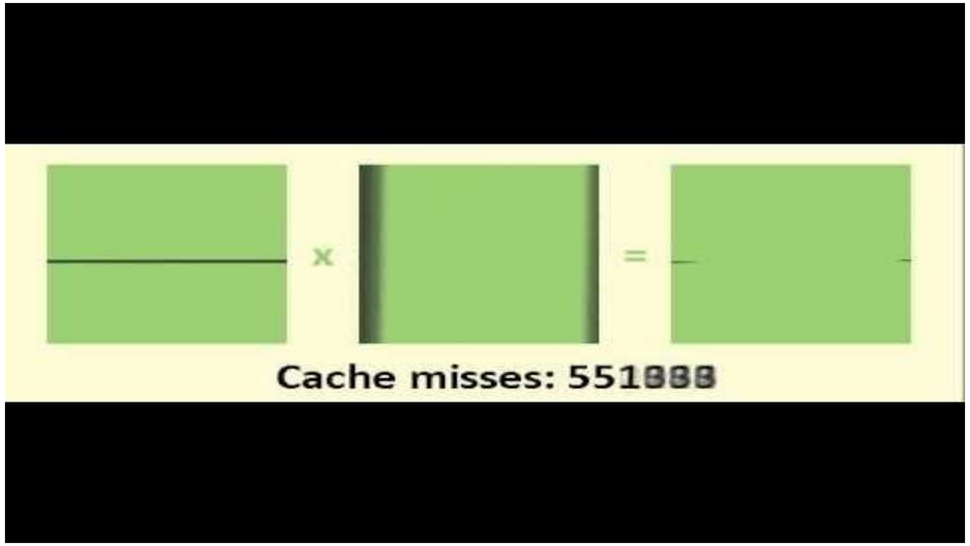
- Total misses:
  - $\frac{nr}{4} \times \left(\frac{n}{r}\right)^2 = \frac{n^3}{4r}$

number of blocks in product matrix

Compare this to $\frac{9}{8}n^3$!!!

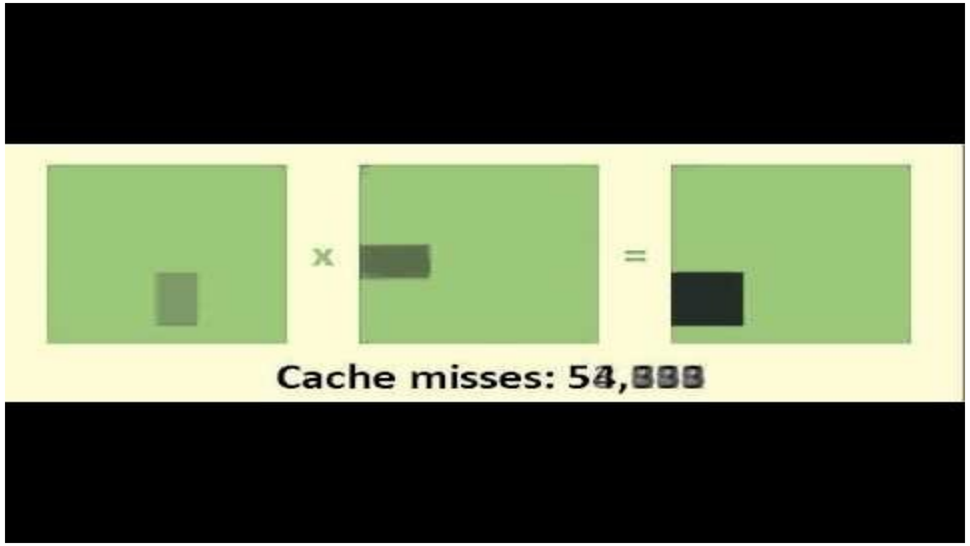
Matrix Multiply Visualization

Naïve:



≈ 1,020,000
cache misses

Blocked:



≈ 90,000
cache misses

Here n = 100, C = 32 KiB, r = 30

Cache-Friendly Code

- Programmer can optimize for cache performance
  - How data structures are organized
  - How data are accessed
    - Nested loop structure
    - Blocking is a general technique
- All systems favor "cache-friendly code"
  - Getting absolute optimum performance is very platform specific
    - Cache size, cache block size, associativity, etc.
  - Can get most of the advantage with generic coding rules
    - Keep working set reasonably small (temporal locality)
    - Use small strides (spatial locality)
    - Focus on inner loop code