Sequential logic examples

- Finite state machine concept
  - FSMs are the decision making logic of digital designs
  - partitioning designs into datapath and control elements
  - when inputs are sampled and outputs asserted
- Basic design approach: a 4-step design process
- Implementation examples and case studies
  - finite-string pattern recognizer
  - complex counter
  - traffic light controller
  - door combination lock

General FSM design procedure

- (1) Determine inputs and outputs
- (2) Determine possible states of machine
  - – state minimization
- (3) Encode states and outputs into a binary code
  - – state assignment or state encoding
  - – output encoding
  - – possibly input encoding (if under our control)
- (4) Realize logic to implement functions for states and outputs
  - – combinational logic implementation and optimization
  - – choices made in steps 2 and 3 can have large effect on resulting logic

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Finite string pattern recognizer (step 1)

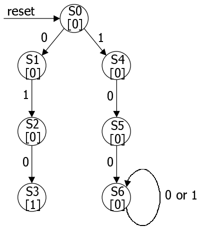
- Finite string pattern recognizer
  - one input (X) and one output (Z)
  - output is asserted whenever the input sequence …010… has been observed, as long as the sequence 100 has never been seen
- Step 1: understanding the problem statement
  - sample input/output behavior:

X: 0 0 1 0 1 0 1 0 0 1 0 …
Z: 0 0 0 1 0 1 0 1 0 0 0 …

X: 1 1 0 1 1 0 1 0 0 1 0 …
Z: 0 0 0 0 0 0 0 1 0 0 0 …

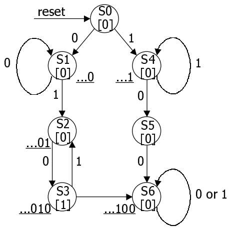
Finite string pattern recognizer (step 2)

- Step 2: draw state diagram
  - for the strings that must be recognized, i.e., 010 and 100
  - a Moore implementation



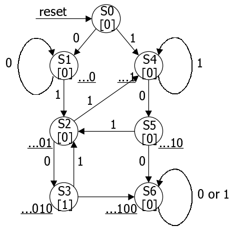
Finite string pattern recognizer (step 2, cont'd)

- Exit conditions from state S3: have recognized …010
  - if next input is 0 then have …0100 = ...100 (state S6)
  - if next input is 1 then have …0101 = …01 (state S2)
- Exit conditions from S1: recognizes strings of form …0 (no 1 seen)
  - loop back to S1 if input is 0
- Exit conditions from S4: recognizes strings of form …1 (no 0 seen)
  - loop back to S4 if input is 1



Finite string pattern recognizer (step 2, cont'd)

- S2 and S5 still have incomplete transitions
  - S2 = …01; If next input is 1, then string could be prefix of (01)1(00) S4 handles just this case
  - S5 = …10; If next input is 1, then string could be prefix of (10)1(0) S2 handles just this case
- Reuse states as much as possible
  - look for same meaning
  - state minimization leads to smaller number of bits to represent states
- Once all states have a complete set of transitions we have a final state diagram



Finite string pattern recognizer (step 3)

- Verilog description including state assignment (or state encoding)

```
module string (clk, X, rst, Q0, Q1, Q2, Z);
input clk, X, rst;
output Q0, Q1, Q2, Z;

reg state[0:2];
`define S0 = [0,0,0]; //reset state
`define S1 = [0,0,1]; //strings ending in   ...0
`define S2 = [0,1,0]; //strings ending in  ...01
`define S3 = [0,1,1]; //strings ending in ...010
`define S4 = [1,0,0]; //strings ending in   ...1
`define S5 = [1,0,1]; //strings ending in  ...10
`define S6 = [1,1,0]; //strings ending in ...100

assign Q0 = state[0];
assign Q1 = state[1];
assign Q2 = state[2];
assign Z = (state == `S3);
```

```
always @(posedge clk) begin
  if rst state = `S0;
  else
    case (state)
      `S0: if (X) state = `S4 else state = `S1;
      `S1: if (X) state = `S2 else state = `S1;
      `S2: if (X) state = `S4 else state = `S3;
      `S3: if (X) state = `S2 else state = `S6;
      `S4: if (X) state = `S4 else state = `S5;
      `S5: if (X) state = `S2 else state = `S6;
      `S6: state = `S6;
      default: begin
        $display ("invalid state reached");
        state = 3'bxxx;
    endcase

end

endmodule
```

Finite string pattern recognizer

- Review of process
  - understanding problem
    - write down sample inputs and outputs to understand specification
  - derive a state diagram
    - write down sequences of states and transitions for sequences to be recognized
  - minimize number of states
    - add missing transitions; reuse states as much as possible
  - state assignment or encoding
    - encode states with unique patterns
  - simulate realization
    - verify I/O behavior of your state diagram to ensure it matches specification

Complex counter

- A synchronous 3-bit counter has a mode control M
  - when M = 0, the counter counts up in the binary sequence
  - when M = 1, the counter advances through the Gray code sequence

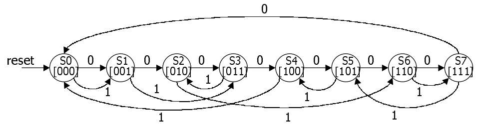
binary: 000, 001, 010, 011, 100, 101, 110, 111
Gray: 000, 001, 011, 010, 110, 111, 101, 100

- Valid I/O behavior (partial)

Mode Input M	Current State	Next State
0	000	001
0	001	010
1	010	110
1	110	111
1	111	101
0	101	110
0	110	111

Complex counter (state diagram)

- Deriving state diagram
  - one state for each output combination
  - add appropriate arcs for the mode control



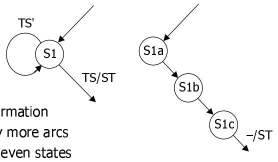
Complex counter (state encoding)

- Verilog description including state encoding

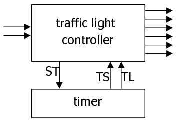
```
module string (clk, M, rst, Z0, Z1, Z2);
input clk, X, rst;
output Z0, Z1, Z2;

reg state[0:2];
`define S0 = [0,0,0];
`define S1 = [0,0,1];
`define S2 = [0,1,0];
`define S3 = [0,1,1];
`define S4 = [1,0,0];
`define S5 = [1,0,1];
`define S6 = [1,1,0];
`define S7 = [1,1,1];

assign Z0 = state[0];
assign Z1 = state[1];
assign Z2 = state[2];
```

```
always @(posedge clk) begin
  if rst state = `S0;
  else
    case (state)
      `S0: state = `S1;
      `S1: if (M) state = `S3 else state = `S2;
      `S2: if (M) state = `S6 else state = `S3;
      `S3: if (M) state = `S2 else state = `S4;
      `S4: if (M) state = `S0 else state = `S5;
      `S5: if (M) state = `S4 else state = `S6;
      `S5: if (M) state = `S7 else state = `S7;
      `S5: if (M) state = `S5 else state = `S0;
 endcase

end

endmodule
```

Traffic light controller as two communicating FSMs

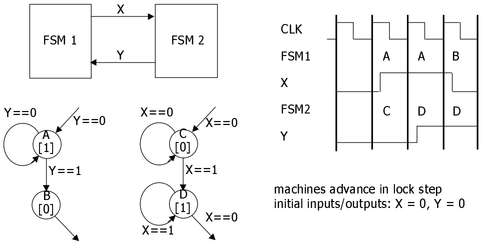


- Without separate timer
  - S0 would require 7 states
  - S1 would require 3 states
  - S2 would require 7 states
  - S3 would require 3 states
  - S1 and S3 have simple transformation
  - S0 and S2 would require many more arcs
    - C could change in any of seven states
- By factoring out timer
  - greatly reduce number of states
    - 4 instead of 20
  - counter only requires seven or eight states
    - 12 total instead of 20



Communicating finite state machines

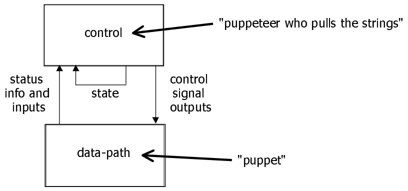
- One machine's output is another machine's input



machines advance in lock step
initial inputs/outputs: X = 0, Y = 0

Data-path and control

- Digital hardware systems = data-path + control
  - datapath: registers, counters, combinational functional units (e.g., ALU), communication (e.g., busses)
  - control: FSM generating sequences of control signals that instructs datapath what to do next



Digital combinational lock

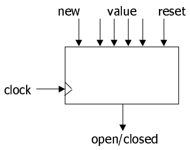
- Door combination lock:
  - punch in 3 values in sequence and the door opens; if there is an error the lock must be reset; once the door opens the lock must be reset
  - inputs: sequence of input values, reset
  - outputs: door open/close
  - memory: must remember combination or always have it available
  - open questions: how do you set the internal combination?
    - stored in registers (how loaded?)
    - hardwired via switches set by user

Implementation in software

```
integer combination_lock ( ) {
    integer v1, v2, v3;
    integer error = 0;
    static integer c[3] = 3, 4, 2;

    while (!new_value( ));
    v1 = read_value( );
    if (v1 != c[1]) then error = 1;

    while (!new_value( ));
    v2 = read_value( );
    if (v2 != c[2]) then error = 1;

    while (!new_value( ));
    v3 = read_value( );
    if (v2 != c[3]) then error = 1;

    if (error == 1) then return(0); else return (1);
}
```

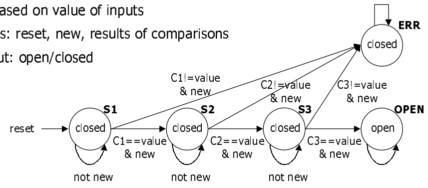
Determining details of the specification

- How many bits per input value?
- How many values in sequence?
- How do we know a new input value is entered?
- What are the states and state transitions of the system?



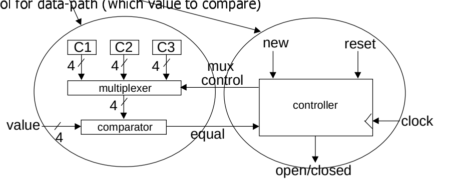
Digital combination lock state diagram

- States: 5 states
  - represent point in execution of machine
  - each state has outputs
- Transitions: 6 from state to state, 5 self transitions, 1 global
  - changes of state occur when clock says its ok
  - based on value of inputs
- Inputs: reset, new, results of comparisons
- Output: open/closed



Data-path and control structure

- Data-path
  - storage registers for combination values
  - multiplexer
  - comparator
- Control
  - finite-state machine controller
  - control for data-path (which value to compare)



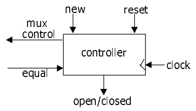
State table for combination lock

- Finite-state machine
  - refine state diagram to take internal structure into account
  - state table ready for encoding

reset	new	equal	state	next state	mux	open/closed
1	–	–	–	S1	C1	closed
0	0	–	S1	S1	C1	closed
0	1	0	S1	ERR	–	closed
0	1	1	S1	S2	C2	closed
...						
0	1	1	S3	OPEN	–	open
...						

Encodings for combination lock

- Encode state table
  - state can be: S1, S2, S3, OPEN, or ERR
    - needs at least 3 bits to encode: 000, 001, 010, 011, 100
    - and as many as 5: 00001, 00010, 00100, 01000, 10000
    - choose 4 bits: 0001, 0010, 0100, 1000, 0000
  - output mux can be: C1, C2, or C3
    - needs 2 to 3 bits to encode
    - choose 3 bits: 001, 010, 100
  - output open/closed can be: open or closed
    - needs 1 or 2 bits to encode
    - choose 1 bit: 1, 0

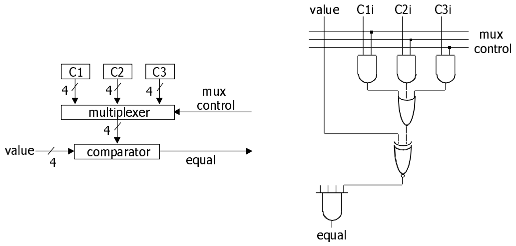


reset	new	equal	state	next state	mux	open/closed
1	–	–	–	0001	001	0
0	0	–	0001	0001	001	0
0	1	0	0001	0000	–	0
0	1	1	0001	0010	010	0
...						
0	1	1	0100	1000	–	1
...						

mux is identical to last 3 bits of state
open/closed is identical to first bit of state
therefore, we do not even need to implement
FFs to hold state, just use outputs

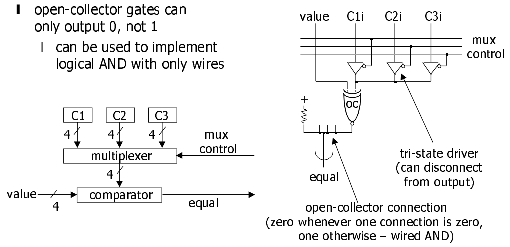
Data-path implementation for combination lock

- Multiplexer
  - easy to implement as combinational logic when few inputs
  - logic can easily get too big for most PLDs



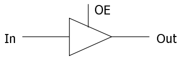
Data-path implementation (cont'd)

- Tri-state logic
  - utilize a third output state: "no connection" or "float"
  - connect outputs together as long as only one is "enabled"
  - open-collector gates can only output 0, not 1
    - can be used to implement logical AND with only wires

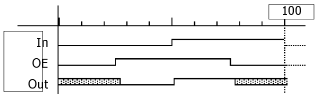


Tri-state gates

- The third value
  - logic values: "0", "1"
  - don't care: "X" (must be 0 or 1 in real circuit!)
  - third value or state: "Z" — high impedance, infinite R, no connection
- Tri-state gates
  - additional input – output enable (OE)
  - output values are 0, 1, and Z
  - when OE is high, the gate functions normally
  - when OE is low, the gate is disconnected from wire at output
  - allows more than one gate to be connected to the same output wire
    - as long as only one has its output enabled at any one time (otherwise, sparks could fly)

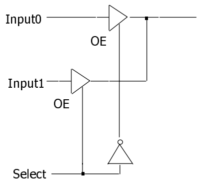


	In	OE	Out
non-inverting	X	0	Z
tri-state	0	1	0
buffer	1	1	1



Tri-state and multiplexing

- When using tri-state logic
  - (1) make sure never more than one "driver" for a wire at any one time (pulling high and low at the same time can severely damage circuits)
  - (2) make sure to only use value on wire when its being driven (using a floating value may cause failures)
- Using tri-state gates to implement an economical multiplexer



when Select is high
Input1 is connected to F

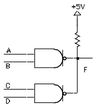
when Select is low
Input0 is connected to F

this is essentially a 2:1 mux

Open-collector gates and wired-AND

- Open collector: another way to connect gate outputs to the same wire
  - gate only has the ability to pull its output low
  - it cannot actively drive the wire high (default – pulled high through resistor)
- Wired-AND can be implemented with open collector logic
  - if A and B are "1", output is actively pulled low
  - if C and D are "1", output is actively pulled low
  - if one gate output is low and the other high, then low wins
  - if both gate outputs are "1", the wire value "floats", pulled high by resistor
    - low to high transition usually slower than it would have been with a gate pulling high
  - hence, the two NAND functions are ANDed together

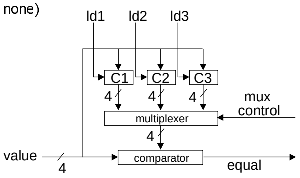
open-collector
NAND gates



with ouputs wired together
using "wired-AND"
to form (AB)'(CD)'

Digital combination lock (new data-path)

- Decrease number of inputs
- Remove 3 code digits as inputs
  - use code registers
  - make them loadable from value
  - need 3 load signal inputs (net gain in input (4*3)–3=9)
    - could be done with 2 signals and decoder (ld1, ld2, ld3, load none)



Section summary

- FSM design
  - understanding the problem
  - generating state diagram
  - implementation using synthesis tools
  - iteration on design/specification to improve qualities of mapping
  - communicating state machines
- Four case studies
  - understand I/O behavior
  - draw diagrams
  - enumerate states for the "goal"
  - expand with error conditions
  - reuse states whenever possible