Proving completeness

- You are given a set of operators. You are asked to show that you can implement any function using these operators. The simplest way to do this is to show that the set of operators you are given reduces reduces to a set of operators that we already know is complete, for example {NOT, AND, OR}.
- This means we have to show how to implement NOT, AND and OR using only the operators in the set you are given.

Example of completeness

- Show that { NOR } is complete.
- Solution:
  - Implement NOT:
    - A' = (A+A)' = A nor A
  - Implement AND:
    - AB = (A' + B')' = (A nor A) nor (B nor B)
  - Implement OR:
    - A + B = ((A + B)')' = (A nor B) nor (A nor B)

Simplification

- Some important rules for simplification (how do you prove these?):
  - AB + AB’ = A
  - A + AB = A
- Note that you can use the rules in either direction, to remove terms, or to add terms. Indeed, sometimes you need to add some terms in order to get to the simples solution.

Examples

- Simplify: ab’c + abc + a’bc

  ab’c + abc + a’bc

  = ab’c + abc + abc + a’bc = ac + bc
- Show that X + X’Y = X + Y

  X + X’Y

  = X(1 + Y) + X’Y

  = X + XY + X’Y

  = X + Y

Examples (cont'd)

- Simplify: $WX + XY + X'Z' + WY'Z'$

  $WX + XY + X'Z' + WY'Z'$

  $= WX + XY + X'Z' + WY'Z'X + WY'Z'X'$

  $= WX(1 + Y'Z') + XY + X'Z'(1 + WY')$

  $= WX + XY + X'Z'$

Examples (cont'd)

- Prove the consensus theorem, which says:

  $XY + X'Z + YZ = XY + X'Z$

- Solution:

  $XY + X'Z + YZ$

  $= XY + X'Z + (X + X')YZ$

  $= XY + X'Z + XYZ + X'YZ$

  $= XY + XYZ + X'Z + X'ZY$

  $= XY(1 + Z) + X'Z(1 + Y)$

  $= XY + X'Z$

Long example

- Simplify:

A’B’C’D’ + A’BC’D’ + A’BD + A’BC’D + ABCD + ACD’ + B’CD’
= A’C’D’(B’ + B) + A’BD(1 + C’) + ABCD + ACD’ + B’CD’
= A’C’D’ + A’BD + ABCD + ACD’ + B’CD’
= A’C’D’ + BD(A’ + AC) + ACD’ + B’CD’
= A’C’D’ + BD(A’ + C) + ACD’ + B’CD’ (Since X + X’Y = X + Y)
= A’C’D’ + A’BD + (BCD + ACD’) + B’CD’
= A’C’D’ + A’BD + (BCD + ACD’ + ABC) + B’CD’
(Added ABC by consensus)
= A’C’D’ + (A’BD + ABC + BCD) + (ABC + B’CD’ + ACD’)
= AC’D’ + A’BD + ABC + B’CD’

Karnaugh Maps

- The methods we saw were too cumbersome
- So enter more advanced methods…… Karnaugh Maps

Come to Tomm Lecture