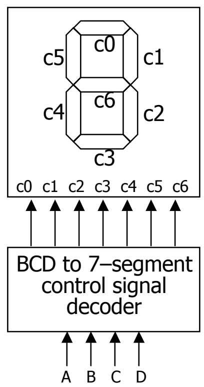
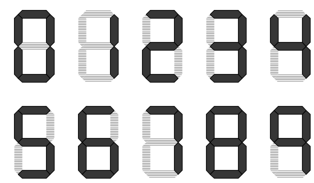
BCD to 7-segment display controller

- Understanding the problem
  - input is a 4 bit bcd digit (A, B, C, D)
  - output is the control signals for the display (7 outputs C0 – C6)
- Block diagram





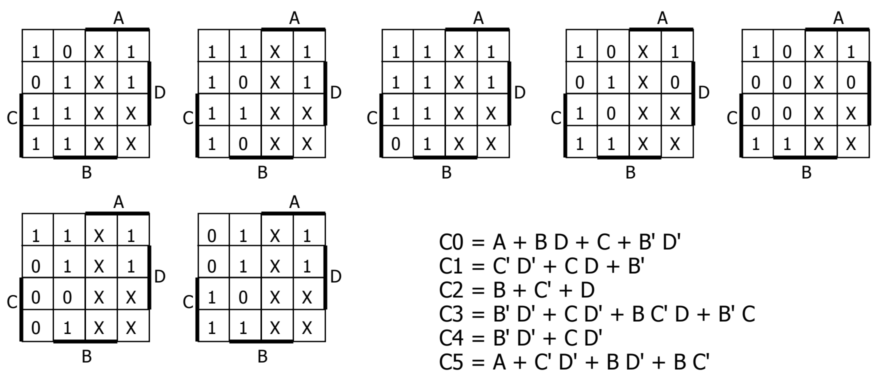
Formalize the problem

- Truth table
  - show don't cares
- Choose implementation target
  - if ROM, we are done
  - don't cares imply PAL/PLA may be attractive
- Follow implementation procedure
  - minimization using K-maps

A	B	C	D	C0	C1	C2	C3	C4	C5	C6
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	0	0	1	1
1	0	1	–	–	–	–	–	–	–	–
1	1	–	–	–	–	–	–	–	–	–

Implementation as minimized sum-of-products

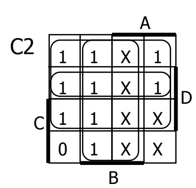
⌘ 15 unique product terms when minimized individually

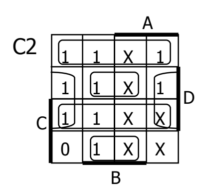


C0 = A + B D + C + B' D'
C1 = C' D' + C D + B'
C2 = B + C' + D
C3 = B' D' + C D' + B C' D + B' C
C4 = B' D' + C D'
C5 = A + C' D' + B D' + B C'
C6 = A + C D' + B C' + B' C

Implementation as minimized S-o-P (cont'd)

⌘ Can do better
- 9 unique product terms (instead of 15)
- share terms among outputs
- each output not necessarily in minimized form

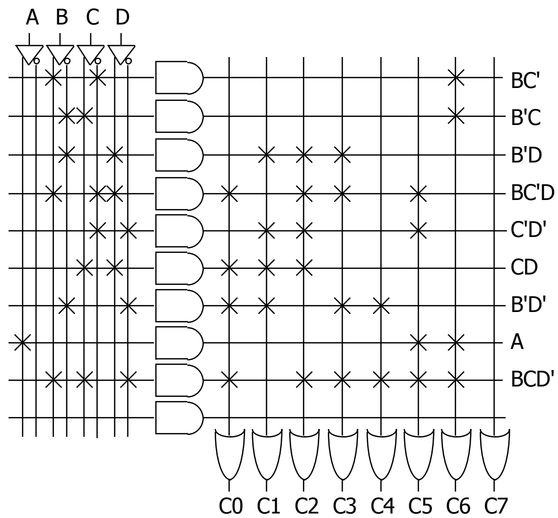




C0 = A + B D + C + B' D'
C1 = C' D' + C D + B'
C2 = B + C' + D
C3 = B' D' + C D' + B C' D + B' C
C4 = B' D' + C D'
C5 = A + C' D' + B D' + B C'
C6 = A + C D' + B C' + B' C

C0 = B C' D + C D + B' D' + B C D' + A
C1 = B' D + C' D' + C D + B' D'
C2 = B' D + B C' D + C' D' + C D + B C D'
C3 = B C' D + B' D + B' D' + B C D'
C4 = B' D' + B C D'
C5 = B C' D + C' D' + A + B C D'
C6 = B' C + B C' + B C D' + A

PLA implementation



PAL implementation

- Limit of 4 product terms per output
  - decomposition of functions with larger number of terms
  - do not share terms in PAL anyway
    (although there are some with some shared terms)

    C2 = B + C' + D

    C2 = B' D + B C' D + C' D' + C D + B C D'

    C2 = B' D + B C' D + C' D' + W
    W = C D + B C D'
    ← need another input and another output
  - decompose into multi-level logic (hopefully with CAD support)
    - find common sub-expressions among functions

      C0 = C3 + A' B X' + A D Y
      C1 = Y + A' C5' + C' D' C6
      C2 = C5 + A' B' D + A' C D
      C3 = C4 + B D C5 + A' B' X'
      C4 = D' Y + A' C D'
      C5 = C' C4 + A Y + A' B X
      C6 = A C4 + C C5 + C4' C5 + A' B' C

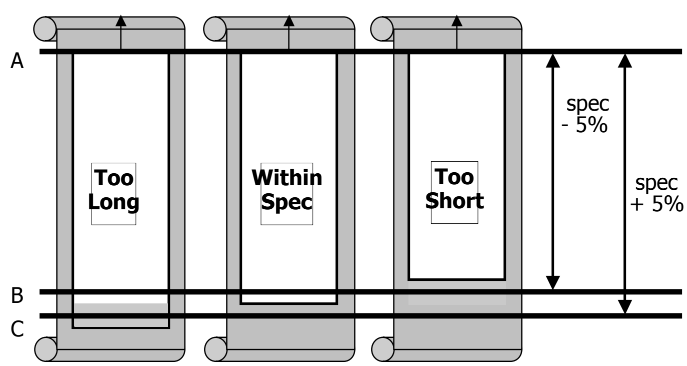
      X = C' + D'
      Y = B' C'

Production line control

- Rods of varying length (+/-10%) travel on conveyor belt
  - mechanical arm pushes rods within spec (+/-5%) to one side
  - second arm pushes rods too long to other side
  - rods that are too short stay on belt
  - 3 light barriers (light source + photocell) as sensors
  - design combinational logic to activate the arms
- Understanding the problem
  - inputs are three sensors
  - outputs are two arm control signals
  - assume sensor reads "1" when tripped, "0" otherwise
  - call sensors A, B, C

Sketch of problem

- Position of sensors
  - A to B distance = specification – 5%
  - A to C distance = specification + 5%



Formalize the problem

- Truth table
  - show don't cares

A	B	C	Function
0	0	0	do nothing
0	0	1	do nothing
0	1	0	do nothing
0	1	1	do nothing
1	0	0	too short
1	0	1	don't care
1	1	0	in spec
1	1	1	too long

logic implementation now straightforward
just use three 3-input AND gates

"too short" = AB'C'
(only first sensor tripped)

"in spec" = A B C'
(first two sensors tripped)

"too long" = A B C
(all three sensors tripped)