Lecture 2:
The Magical Base-2

CSE 370, Autumn 2007
Benjamin Ylvisaker

Daily Quiz

- Have you added yourself to the class mailing list?
- Do it by 5:30 this afternoon to get a 4 on today's daily quiz
- Tell classmates who didn't make it to class on time at your own discretion

Administrivia

- Office hours

Monday	Ramkumar	???	lab
Tuesday	Josh	1:30-2:30	lab
Wednesday	Benjamin	1:30-2:30	210
Thursday	Benjamin	9:30-10:30	210
Friday	Nikhil	11:30-12:30	lab

Elementary Math Review

- Positional number notation
  - $2{,}104 = 2\times1{,}000 + 1\times100 + 0\times10 + 4\times1$
    $= 2\times10^3 + 1\times10^2 + 0\times10^1 + 4\times10^0$
- Generalize to arbitrary base b
  - $XYZ = X\times b^2 + Y\times b^1 + Z\times b^0$
    where X, Y and Z are digits with values in the range [0..b-1]

Bases of Interest

- In 370, we are interested in the following bases:
  - Binary {0,1}
  - Octal {0..7}
  - Decimal {0..9}
  - Hexadecimal {0..9,A..F}
    - A=10, B=11, C=12, D=13, E=14, F=15

Conversion to Decimal

- 1001101_2
  $= 1\times2^6 + 0\times2^5 + 0\times2^4 + 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$
  $= 1\times64 + 0\times32 + 0\times16 + 1\times8 + 1\times4 + 0\times2 + 1\times1$
  $= 64 + 8 + 4 + 1$
  $= 77$
- $92A70_{16}$
  $= 9\times16^4 + 2\times16^3 + 10\times16^2 + 7\times16^1 + 0\times16^0$
  $= 9\times65536 + 2\times4096 + 10\times256 + 7\times16 + 0\times1$
  $= 589824 + 8192 + 2560 + 112$
  $= 600688$

Arithmetic is the Same in All Bases

- $1001101_2 + 101011_2 = 1111000_2$; $32175_8 + 1622_8 = 34017_8$; $27AA32_{16} + 92A70_{16} = 30D4A2_{16}$

$$\begin{array}{r} 1001101_2 \\ +\ 101011_2 \\ \hline 1111000_2 \end{array} \quad \begin{array}{r} 32175_8 \\ +\ 1622_8 \\ \hline 34017_8 \end{array} \quad \begin{array}{r} 27AA32_{16} \\ +\ 92A70_{16} \\ \hline 30D4A2_{16} \end{array}$$

- Subtraction:

$$\begin{array}{r} 1001101_2 \\ -\ 101011_2 \\ \hline 100010_2 \end{array} \quad \begin{array}{r} 32175_8 \\ -\ 1622_8 \\ \hline 30353_8 \end{array} \quad \begin{array}{r} 27AA32_{16} \\ -\ 92A70_{16} \\ \hline 1E7FC2_{16} \end{array}$$

Multiplication, Too

-

$$\begin{array}{r} 1101101_2 \\ \times\ 101011_2 \\ \hline 1101101_2 \\ 1101101_2 \\ 0000000_2 \\ 1101101_2 \\ 0000000_2 \\ +1101101_2 \\ \hline 1001001001111_2 \end{array} \qquad \begin{array}{r} A3_{16} \\ \times 17_{16} \\ \hline 475_{16} \\ +A3_{16} \\ \hline EA5_{16} \end{array}$$

Division, Too

-
```
       1001   Remainder: 100
101)110001
   -101
      10
    - 0
     100
    -  0
     1001
    - 101
      100
```


Conversion to Binary by Successive Division

- $154_{10} \div 2_{10} = 77_{10}$ Remainder 0 — 10011010
  $77_{10} \div 2_{10} = 38_{10}$ Remainder 1
  $38_{10} \div 2_{10} = 19_{10}$ Remainder 0
  $19_{10} \div 2_{10} = 9_{10}$ Remainder 1
  $9_{10} \div 2_{10} = 4_{10}$ Remainder 1
  $4_{10} \div 2_{10} = 2_{10}$ Remainder 0
  $2_{10} \div 2_{10} = 1_{10}$ Remainder 0
  $1_{10} \div 2_{10} = 0_{10}$ Remainder 1

Read the result "up"

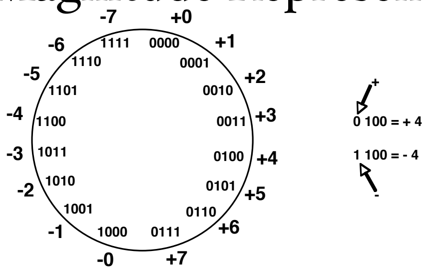
... and Back Again

- $10011010_2 \div 1010_2 = 1111_2$ Remainder 100_2
  $1111_2 \div 1010_2 = 1_2$ Remainder 101_2
  $1_2 \div 1010_2 = 0_2$ Remainder 1_2
- Converting from base B to C
  - Do divisions in base B
  - Divide by C

The Trouble with Negative Numbers

- The symbol "-" for negative can be used in any base, when doing arithmetic by hand
- Computers only have two symbols: 1, 0. No "-"
- Also, computers usually do arithmetic with numbers that are a fixed number of bits "wide" (like, 8, 16, 32, 64)

Sign/Magnitude Representation

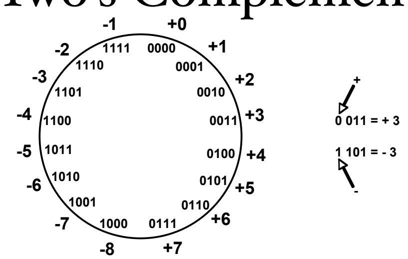


- High-order (left-most) bit is the sign. `0`=positive, `1`=negative
- Remaining bits are the magnitude
- With N bits, represent numbers between $-2^{N-1}+1$ and $2^{N-1}-1$
- Two representations of `0`!

Sign/Magnitude

- Pro: easy to read and write for humans
- Con: harder to do basic arithmetic correctly with a computer
- Result: rarely used

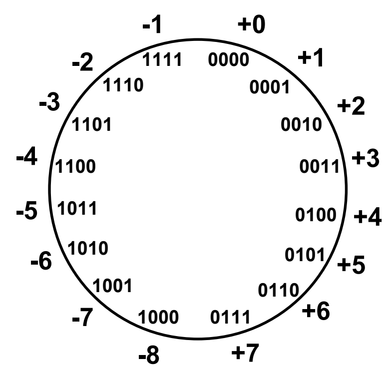
Two's Complement



- High-order (left-most) bit is the sign. `0`=positive, `1`=negative
- Remaining bits are the magnitude (encoded in a funny way)
- With N bits, represent numbers between -2^{N-1} and $2^{N-1}-1$
- Just one representations of `0`

Negation in 2's Complement

- Flip the bits and add 1



Addition in 2's Complement

```
•  0011 (3)          1101 (-3)
  +0101 (5)         +0101 (5)
   1000 (-8)         0010 (2)

   0011 (3)          1101 (-3)
  +1011 (-5)        +1011 (-5)
   1110 (-2)         1000 (-8)
```

- Subtraction is just addition with the second operand negated first

Later in the Course

- Efficient circuits for implementing arithmetic
- Detecting overflow/underflow
- Changing the width of numbers without changing the number

Fractional Numbers

- We might want to represent non-integral numbers
- Two popular approaches:
  - Fixed-point
  - Floating-point
- Not covered in 370

Thank You for Your Attention

- Lab 1 has changed slightly, I'll post an update soon (and send a mail to the class mailing list)
- Continue reading the book
- Continue/start homework 1
- Next time: the fundamentals of Boolean logic