

# Lecture 6: Boolean Cubes and Karnaugh Maps

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CSE 370, Autumn 2007  
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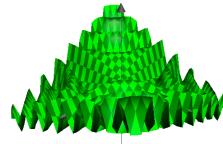
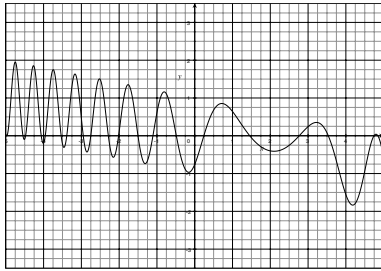
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## Where We Are

- Last lecture: 2-level implementations and canonical forms
- This lecture: Boolean cubes and K-maps
- Next lecture: K-map minimization
- Homework 1 back today or Wed. 2 due Wed.
- Read lab 2 before the start of your session

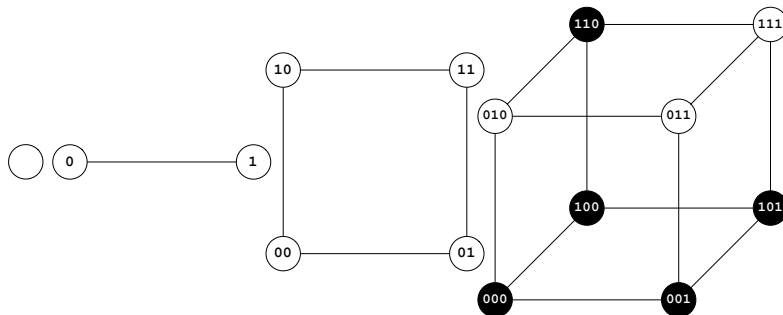
# Inspiration

- Visualization of real-valued functions

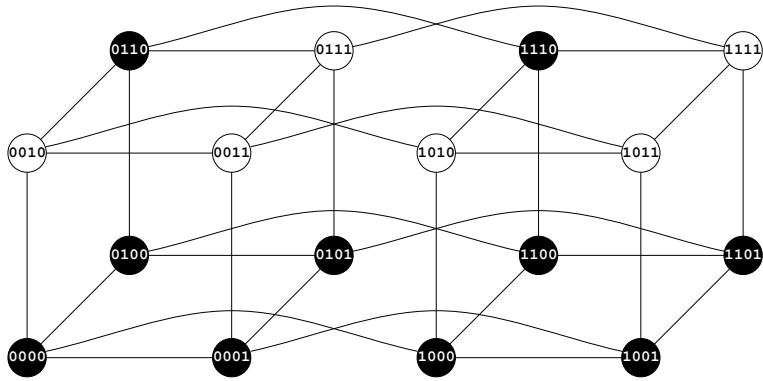


## Visualizing Boolean Functions

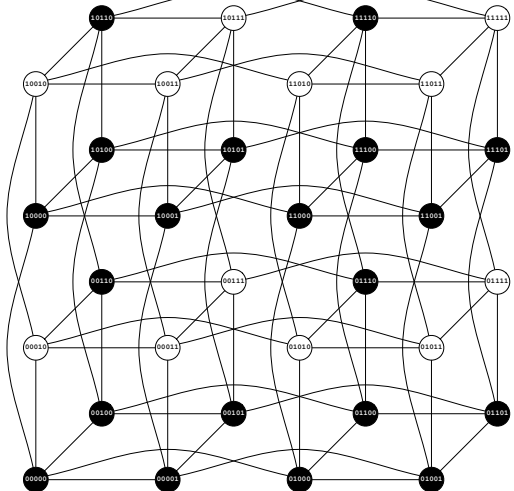
- Generally more input variables
- **Way** fewer possible values per variable



# Getting a Little Ridiculous

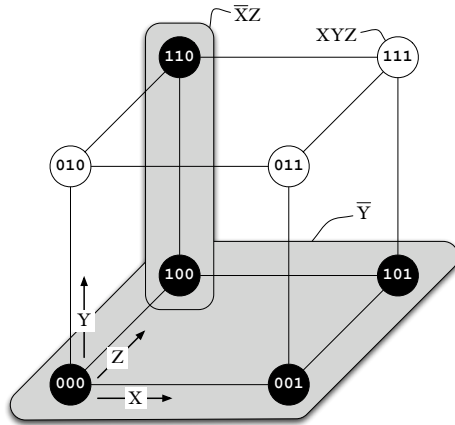


# Getting a Lot Ridiculous



# The Features of a Cube

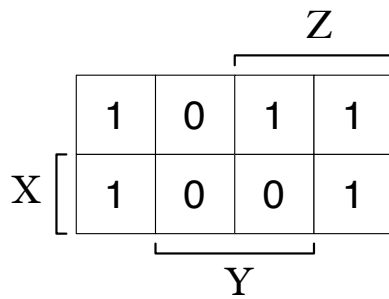
- | Z | Y | X | F |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |



# Karnaugh Maps

- Flattened Boolean cubes

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



## 4 Variable Example

- Inverse majority function

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0

A	B	C	D	F
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

A			
1	1	1	1
1	1	0	1
1	0	0	0
1	1	0	1
B			
D		C	

## Different Way to Draw a K-Map

- Inverse majority function

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0

A	B	C	D	F
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

A B				
C D	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	0	0	0
10	1	1	0	1

# Rectangles in K-Maps

- Always a power of 2 on a side
- Can “wrap around” the border
- Can only enclose all 1's or all 0's
  - 1's correspond to product terms
  - 0's correspond to sum terms

		A			
		B			
C	D	00	01	11	10
00		1	1	1	1
01		1	1	0	1
11		1	0	0	0
10		1	1	0	1

# Individual Terms

		A			
		B			
C	D	00	01	11	10
00		1	1	1	1
01		1	1	0	1
11		1	0	0	0
10		1	1	0	1

- $\neg C \neg D$  — points to the 1 in cell (00, 00)
- $\neg A \neg C$  — points to the 1 in cell (01, 00)
- $\neg A + \neg B + \neg D$  — points to the 1 in cell (01, 01)
- $\neg B \neg D$  — points to the 1 in cell (10, 00)

# Bad Rectangles

- Non-power of 2 width
- Non-rectangle
- Non-power of 2 height
- Encloses both 0's and 1's

		A	B			
	C	D	00	01	11	10
00	1	1	1	1	1	
01	1	1	0	1		
11	1	0	0	0		
10	1	1	0	1		

# Numbering the Squares

- Inverse majority function

A	B	C	D	#	A	B	C	D	#
0	0	0	0	0	1	0	0	0	8
0	0	0	1	1	1	0	0	1	9
0	0	1	0	2	1	0	1	0	10
0	0	1	1	3	1	0	1	1	11
0	1	0	0	4	1	1	0	0	12
0	1	0	1	5	1	1	0	1	13
0	1	1	0	6	1	1	1	0	14
0	1	1	1	7	1	1	1	1	15

		A			
		0	4	12	8
D		1	5	13	9
		3	7	15	10
		2	6	14	11
		B		C	

# Now You Try

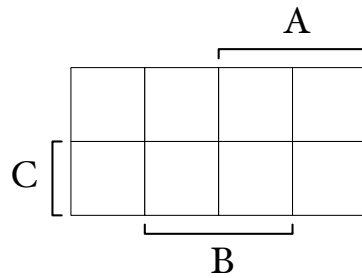
- Multiplexer

A	B	C	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

# Now You Try

- Multiplexer

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

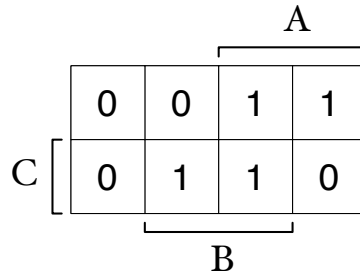




## Now You Try

- Multiplexer

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



## Thank You for Your Attention

- Pick up your quiz
- Read lab 2
- Continue homework 2
- Continue reading the book