# Lecture 7

- Logistics
  - HW2 due Wednesday --- Friday?
  - Lab3 this week
  - 4/25 lecture canceled
- Last lecture
  - Logic simplification
    - Boolean cubes
    - Karnaugh maps
- Today
  - Continuing on K-maps with examples (do this later)
  - Don't cares
  - k-maps for POS minimization



# Incompletely specified functions: Don't cares

- Functions of n inputs have $2^n$ possible configurations
  - Some combinations may be unused
  - Call unused combinations "don't cares"
  - Exploit don't cares during logic minimization
  - Don't care ≠ no output
- Example: A BCD increment-by-1
  - Function F computes the next number in a BCD sequence
    - If the input is $0010_2$, the output is $0011_2$
  - BCD encodes decimal digits 0–9 as $0000_2$–$1001_2$
    - Don't care about binary numbers $1010_2$–$1111_2$

# Truth table for a BCD increment-by-1

| INPUTS | | | | OUTPUTS | | | |
|---|---|---|---|---|---|---|---|
| A | B | C | D | W | X | Y | Z |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | X | X | X | X |
| 1 | 0 | 1 | 1 | X | X | X | X |
| 1 | 1 | 0 | 0 | X | X | X | X |
| 1 | 1 | 0 | 1 | X | X | X | X |
| 1 | 1 | 1 | 0 | X | X | X | X |
| 1 | 1 | 1 | 1 | X | X | X | X |

off-set for W: m0–m6, m9

on-set for W: m7 and m8

Don't care set for W:
We **don't care**
about the output values

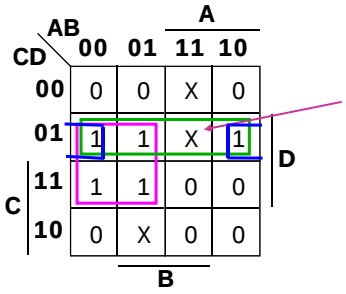


# Notation

- Don't cares in canonical forms
  - Three distinct logical sets: {on}, {off}, {don't care}
- Canonical representations of a BCD increment-by-1
  - Minterm expansion
    - W = m7+m8+d10+d11+d12+d13+d14+d15
      = $\Sigma$m(7,8) + d(10,11,12,13,14,15)
  - Maxterm expansion
    - W = M0•M1•M2•M3•M4•M5•M6•M9•D10•D11•D12•D13•D14•D15
      = $\Pi$M(0,1,2,3,4,5,6,9) • D(10,11,12,13,14,15)
- In K-maps, can treat 'don't cares' as 0s or 1s
  - Depending on which is more advantageous

# Example: with don't cares

- F(A,B,C,D) = Σm(1,3,5,7,9) + d(6,12,13)
  - F = A'D + B'C'D *without using* don't cares
  - F = A'D + C'D *using* don't cares

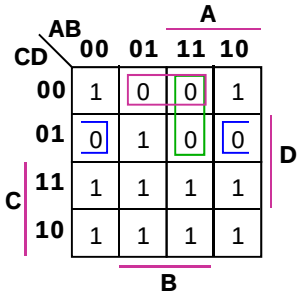



Assign X == "1"
⇒ allows a 2-cube rather than a 1-cube



# POS minimization using k-maps

- Using k-maps for POS minimization
  - Encircle the zeros in the map
  - Interpret indices complementary to SOP form




F = (B'+C+D)(B+C+D')(A'+B'+C)

Check using de Morgan's on SOP

F' = BC'D'+B'C'D+ABC'

(F')' = (BC'D'+B'C'D+ABC')'

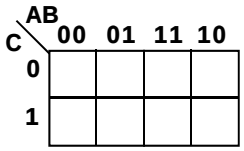
(F')' = (BC'D')'+(B'C'D)'+(ABC')'

F = (B'+C+D)(B+C+D')(A'+B'+C)

# K-maps minimization examples

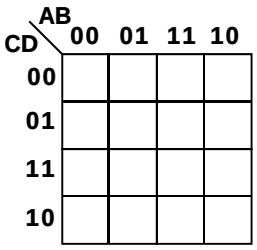
F(A,B,C) = Σm(0,3,6,7)

F(A,B,C) =
F'(A,B,C) =



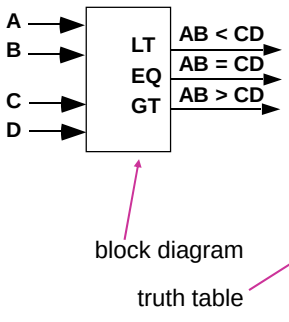

F(A,B,C,D) = Σm(0,3,7,8,11,15)

F(A,B,C,D) =
F'(A,B,C,D) =






# Design example: a two-bit comparator




| A | B | C | D | LT | EQ | GT |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| | | 0 | 1 | 1 | 0 | 0 |
| | | 1 | 0 | 1 | 0 | 0 |
| | | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| | | 0 | 1 | 0 | 1 | 0 |
| | | 1 | 0 | 1 | 0 | 0 |
| | | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| | | 0 | 1 | 0 | 0 | 1 |
| | | 1 | 0 | 0 | 1 | 0 |
| | | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| | | 0 | 1 | 0 | 0 | 1 |
| | | 1 | 0 | 0 | 0 | 1 |
| | | 1 | 1 | 0 | 1 | 0 |

Need a 4-variable Karnaugh map
for each of the 3 output functions

# Design example: a two-bit comparator (con't)

**K-map for LT**

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 |
| 01 | 1 | 0 | 0 | 0 |
| 11 | 1 | 1 | 0 | 1 |
| 10 | 1 | 1 | 0 | 0 |

C, D, A, B

**K-map for EQ**

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 0 |
| 01 | 0 | 1 | 0 | 0 |
| 11 | 0 | 0 | 1 | 0 |
| 10 | 0 | 0 | 0 | 1 |

C, D, A, B

**K-map for GT**

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 1 | 1 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 1 | 0 |

C, D, A, B

LT = A'B'D+A'C+B'CD

GT = BC'D'+AC'+ABD'

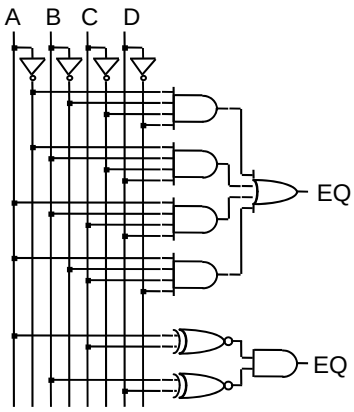
EQ = A'B'C'D'+A'BC'D+ABCD+AB'CD'

= (A xnor C)•(B xnor D)



# Design example: a two-bit comparator (con't)

- Two ways to implement EQ:

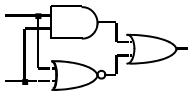



Option 1:
EQ = A'B'C'D'+A'BC'D+ABCD+AB'CD'

5 gates but they require lots of inputs

Option 2
EQ = (A xnor C) •(B xnor D)

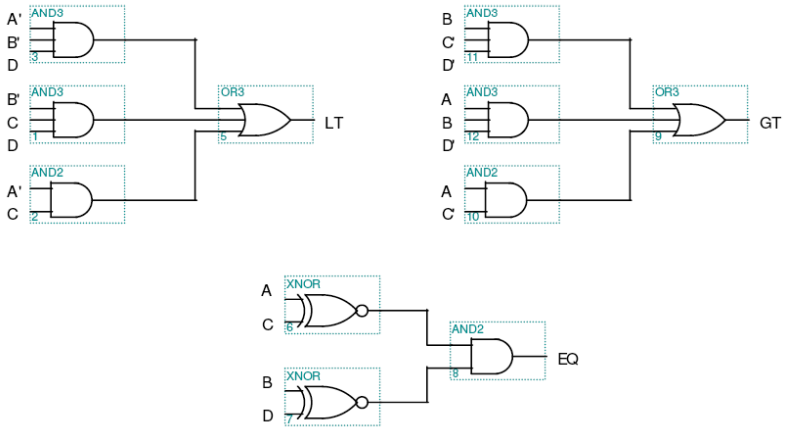
XNOR is constructed from 3 simple gates



7 gates but they all have 2 inputs each

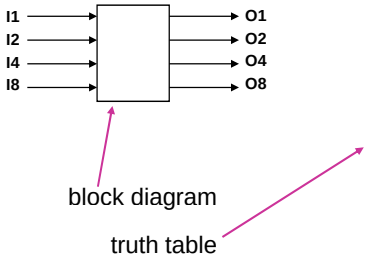
# Design example: a two-bit comparator (con't)

## Circuit schematics






# Design example: BCD increment by 1




| I8 | I4 | I2 | I1 | O8 | O4 | O2 | O1 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | X | X | X | X |
| 1 | 0 | 1 | 1 | X | X | X | X |
| 1 | 1 | 0 | 0 | X | X | X | X |
| 1 | 1 | 0 | 1 | X | X | X | X |
| 1 | 1 | 1 | 0 | X | X | X | X |
| 1 | 1 | 1 | 1 | X | X | X | X |

Need a 4-variable Karnaugh map for each of the 4 output functions

# Design example: BCD increment by 1 (con't)

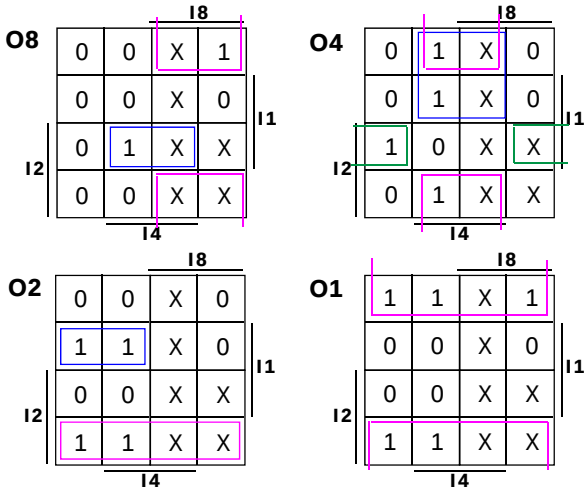
$O_8 = I_4I_2I_1 + I_8I_1$'

$O_4 = I_4I_2' + I_4I_1' + I_4'I_2I_1$

$O_2 = I_8'I_2'I_1 + I_2I_1$'

$O_1 = I_1'$

We greatly simplify the logic by using the don't cares

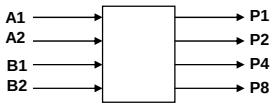





# Design example: BCD increment by 1 (con't)

- Draw the circuit schematic
  - $O_8 = I_4I_2I_1 + I_8I_1'$
  - $O_4 = I_4I_2' + I_4I_1' + I_4'I_2I_1$
  - $O_2 = I_8'I_2'I_1 + I_2I_1'$
  - $O_1 = I_1'$

# Design example: a two-bit multiplier




block diagram

truth table

| A2 | A1 | B2 | B1 | P8 | P4 | P2 | P1 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 1 | 0 | 0 | 0 | 0 |
| | | 1 | 0 | 0 | 0 | 0 | 0 |
| | | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 1 | 0 | 0 | 0 | 1 |
| | | 1 | 0 | 0 | 0 | 1 | 0 |
| | | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 1 | 0 | 0 | 1 | 0 |
| | | 1 | 0 | 0 | 1 | 0 | 0 |
| | | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 1 | 0 | 0 | 1 | 1 |
| | | 1 | 0 | 0 | 1 | 1 | 0 |
| | | 1 | 1 | 1 | 0 | 0 | 1 |

Need a 4-variable Karnaugh map for each of the 4 output functions



# Two-bit multiplier (cont'd)

$P_8 = A_2A_1B_2B_1$

| B2B1 \ A2A1 | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 1 | 0 |
| 10 | 0 | 0 | 0 | 0 |

$P_4 = A_2B_2B_1' + A_2A_1'B_2$

| B2B1 \ A2A1 | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 1 |
| 10 | 0 | 0 | 1 | 1 |

$P_2 = A_2'A_1B_2 + A_1B_2B_1' + A_2B_2'B_1 + A_2A_1'B_1$

| B2B1 \ A2A1 | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | 0 | 1 | 0 | 1 |
| 10 | 0 | 1 | 1 | 0 |

$P_1 = A_1B_1$

| B2B1 \ A2A1 | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 1 | 1 | 0 |
| 11 | 0 | 1 | 1 | 0 |
| 10 | 0 | 0 | 0 | 0 |