## **Homework 1 – Solutions**

1) a) 8

b) 108

e) 512

f) 83

h) 4096

i) 942

2) a) 110101 i) F0

3) b) 100 000 111 111

f) 1000 1111 1100

4) a) 101101 f) 100101001

5) a) 11111 f) 10 1011

6) Select 2 Bits to encode the suit (00 = ♦, 01 = ♥, 10 = ♣, 11 = ♠) and 4 Bits to encode the value (1 = Ace, ..., 13 = King, 0, 14, 15 are invalid codes)

Encoding 1:  $s_1s_0|v_3v_2v_1v_0$ , Encoding 2:  $v_3v_2v_1v_0|s_1s_0$ 

$$C_{\textit{Jack},\textit{Diamond}} = \overline{s_1} \, \overline{s_2} \, v_3 \, \overline{v_2} \, v_1 \, v_0$$

$$C_{Seven.*} = \overline{V_3} V_2 V_1 V_0$$

$$C_{*.Hearts} = \overline{s_2} \, s_1 \wedge \overline{(\overline{v_3} \, \overline{v_2} \, \overline{v_1} \, \overline{v_0})} \wedge \overline{(v_3 \, v_2 \, v_1)}$$

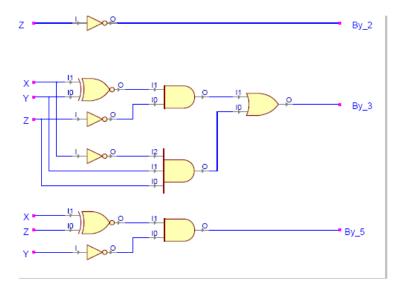
## \*For Encoding 2 bits are just reordered

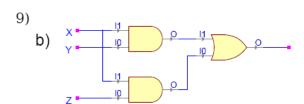
Several other correct solutions exist for this problem.

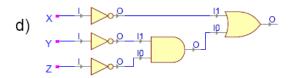
7)

$\boldsymbol{\mathit{C}}$	$\boldsymbol{B}$	$\boldsymbol{A}$	$Z_{a}$	$Z_{b}$	$Z_{c}$
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	0	1
0	1	1	0	1	1
1	0	0	0	0	1
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	1	1

8)  $By_2 = \overline{Z}$   $By_3 = \overline{X}\overline{Y}\overline{Z} \lor \overline{X}YZ \lor XY\overline{Z} = (\overline{X}\overline{Y} \lor XY)\overline{Z} \lor \overline{X}YZ = (X \equiv Y)\overline{Z} \lor \overline{X}YZ$  $By_5 = \overline{X}\overline{Y}\overline{Z} \lor X\overline{Y}Z = \overline{Y}(\overline{X}\overline{Z} \lor XZ) = \overline{Y}(X \equiv Z)$ 







10)b) Show that  $X(X \lor Y) = X$ 

$$X(X \lor Y) = XX \lor XY = X \lor XY = X1 \lor XY = X(1 \lor Y) = X1 = X$$
 q.e.d

d) Show that  $(X \lor Y)(\overline{X} \lor Z) = XZ \lor \overline{X} Y$ 

$$\begin{array}{l} (X\vee Y)(\overline{X}\vee Z) \!=\! X\overline{X}\vee XZ\vee Y\overline{X}\vee YZ \! = \! 0\vee XZ\vee Y\overline{X}\vee YZ \! = \! XZ\vee Y\overline{X}\vee YZ\\ =\! XZ\!\vee Y\overline{X}\vee YZ1 \! =\! XZ\!\vee Y\overline{X}\vee YZ(X\!\vee \overline{X}) \! =\! XZ\!\vee Y\overline{X}\vee YZX\vee YZ\overline{X}\\ =\! XZ(1\vee Y)\vee \overline{X}Y(1\vee Z) \! =\! XZ\!\vee \overline{X}Y \text{ q.e.d} \end{array}$$

11)b)

$\boldsymbol{A}$	$\boldsymbol{B}$	$A \vee \overline{B}$	$(A \vee \overline{B})B$	AB
0	0	1	0	0
0	1	0	0	0
1	0	1	0	0
1	1	1	1	1

The LHS and RHS columns are equal  $\sqrt{\phantom{a}}$ 

c)

$\boldsymbol{A}$	$\boldsymbol{B}$	$\boldsymbol{\mathit{C}}$	$A \vee B$	$\overline{A} \lor C$	AC	$\overline{A}B$	$(A \lor B)(\overline{A} \lor C)$	$AC \lor \overline{A}B$
0	0	0	0	1	0	0	0	0
0	0	1	0	1	0	0	0	0
0	1	0	1	1	0	1	1	1
0	1	1	1	1	0	1	1	1
1	0	0	1	0	0	0	0	0
1	0	1	1	1	1	0	1	1
1	1	0	1	0	0	0	0	0
1	1	1	1	1	1	0	1	1

The LHS and RHS columns are equal  $\sqrt{\phantom{a}}$ 

12)e) 
$$\overline{(X \lor Y)Z} = \overline{(X \lor Y)} \lor \overline{Z} = (\overline{X} \land \overline{Y}) \lor \overline{Z}$$

f) 
$$\overline{X} \lor \overline{YZ} = \overline{X} \land \overline{YZ} = \overline{X} \land YZ$$

## **Grading Breakdown:**

1.	Half point per sub-division.	3
2.	1 point per sub-division.	1
<b>3.</b>	Not Graded	X
4.	Not Graded	X
5.	1 point per sub-division.	1
6.	6 points. Encodings – 3points. Specific Codes – 3points	6
7.	Not Graded	X
8.	Not Graded	$\mathbf{X}$
9.	4 points. 2 points pre sub-division.	4
10	5	
11.	. Not Graded	X
12	. Not Graded	X

Total 20