CSE 370 - Winter 2008

## Homework 3 - Solutions

## Grading Breakdown:

1. CLD-II, Chapter 2, problem 2.31, parts a and b ( $1 / 2$ point for each SOP and POS Expression)3
2. CLD-II, Chapter 2, problem 2.35.
3

(1.5 points for each correct expression)
3. CLD-II, Chapter 2, problem 2.44.

6
(Truth table: $1 / 2$ point, Expressions: 4points, Conclusion $1 / 2$ point)
4. CLD-II, Chapter 3, problem 3.3, part a,b,c. 3
(1 point per part)
5. Design Problem 5
Total 20

## Additional Grading details:

Penalties:
-2 for non standard k-maps
Bonuses:
+2 for word processed and printed work
+1 for very neat handwritten work
+2 for deep insight shown on problems
+2 for a solution to a design or optimization problem that is $50 \%$ better than the average

Note: All bonuses and penalties are subject to a maximum score of 20 points and a minimum score of 0 .

1) $W(A, B, C)=\bar{A} B \bar{C}+\bar{A} B C+A \bar{B} \bar{C}+A \bar{B} C$


SoP: $\quad W=\bar{A} B+A \bar{B}$
PoS: $\quad W=(A+B)(\bar{A}+\bar{B})$

$$
X(A, B, C)=\bar{A} \bar{B} \bar{C}+\bar{A} B C+A \bar{B} \bar{C}+A B C
$$



SoP: $\quad X=\bar{B} \bar{C}+B C$
PoS: $\quad X=(\bar{B}+C)(B+\bar{C})$


SoP: $\quad X=\bar{A} \bar{B}+\bar{B} \bar{D}$
PoS: $\quad X=(\bar{B})(\bar{A}+\bar{D})$
2) See truth table on page 17


3) Truth table for increment-by-one

| $\boldsymbol{I}_{3}$ | $\boldsymbol{I}_{2}$ | $\boldsymbol{I}_{\mathbf{1}}$ | $\boldsymbol{I}_{0}$ | $\boldsymbol{O}_{3}$ | $\boldsymbol{O}_{2}$ | $\boldsymbol{O}_{1}$ | $\boldsymbol{O}_{o}$ |  | $\boldsymbol{I}_{3}$ | $\boldsymbol{I}_{2}$ | $\boldsymbol{I}_{1}$ | $\boldsymbol{I}_{o}$ | $\boldsymbol{O}_{3}$ | $\boldsymbol{O}_{2}$ | $\boldsymbol{O}_{1}$ | $\boldsymbol{O}_{o}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |  | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |  | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |  | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |  | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |  | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |  | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |  | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |

b) O 3 :


$O_{3}=I_{3} \bar{I}_{2}+I_{3} \bar{I}_{1}+I_{3} \bar{I}_{0}+\bar{I}_{3} I_{2} I_{1} I_{0}$
$O_{2}=I_{2} \bar{I}_{1}+I_{2} \bar{I}_{0}+\bar{I}_{2} I_{1} I_{0}$

O1:
$--I_{3}$
00

$O_{1}=I_{1} \bar{I}_{0}+\bar{I}_{1} I_{0}$
$O_{0}=\bar{I}_{0}$
c) Product of Sums

$$
\begin{aligned}
& O_{3}=\left(I_{3}+I_{2}\right)\left(I_{3}+I_{1}\right)\left(I_{3}+I_{0}\right)\left(\bar{I}_{3}+\bar{I}_{2}+\bar{I}_{1}+\bar{I}_{0}\right) \\
& O_{2}=\left(I_{2}+I_{1}\right)\left(I_{2}+I_{0}\right)\left(\bar{I}_{2}+\bar{I}_{1}+\bar{I}_{0}\right) \\
& O_{1}=\left(I_{1}+I_{0}\right)\left(\bar{I}_{1}+\bar{I}_{0}\right) \\
& O_{0}=\bar{I}_{0}
\end{aligned}
$$

Both implementations are equal in their number of literals.
4)
a)


$$
f=\bar{W} \bar{X}+W X+\bar{Y} Z+Y \bar{Z}
$$

b)


$$
f=A C+\bar{A} \bar{C}
$$

c)


$$
f=\bar{A} \bar{B}+\bar{B} \bar{C} \bar{D}+B \bar{C} D
$$

## 5. Design Problem

| Truth Table | * | B | N | $\Sigma$ | F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Min-term Number |  |  |  |  |  |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 1 |
| 3 | 0 | 0 | 1 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 1 | 0 | 0 |
| 7 | 0 | 1 | 1 | 1 | 1 |
| 8 | 1 | 0 | 0 | 0 | 1 |
| 9 | 1 | 0 | 0 | 1 | 1 |
| 10 | 1 | 0 | 1 | 0 | 1 |
| 11 | 1 | 0 | 1 | 1 | 1 |
| 12 | 1 | 1 | 0 | 0 | 0 |
| 13 | 1 | 1 | 0 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 1 |

\% A
$B=B$
$\mathrm{N}=\mathrm{C}$
$\Sigma=\mathrm{D}$

## K-Map:



## Basic Function:

$A B^{\prime}+B D+B^{\prime} C D^{\prime}$

Cost of basic implementation:
2 inverters $+22 \mathrm{i} / \mathrm{p}$ and gates $+13 \mathrm{i} / \mathrm{p}$ And gate +2 OR Gates
Cost: $4+30+35+30=99 \$$
Power: $8+20+15+20=63 \mathrm{uW}$
Area: $4+12+8+12=36$ sq generic units
Make 2 of the above. Substitute with a $2 \mathrm{i} / \mathrm{p}$ with $3 \mathrm{i} / \mathrm{p}$ And gate.

## Best Designs - For each metric

| Lowest Cost |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Student | Company | Function implemented | Inverters | $2 \mathrm{i} / \mathrm{p}$ and | $3 \mathrm{i} / \mathrm{p}$ and | $2 \mathrm{i} / \mathrm{p}$ or | xor | Cost | Power | Area | Product |
| Isac Myers | $\pm-\beta$ | $\left(B^{\prime}+D\right)\left[(A+B)+D^{\prime} C\right]$ | 2 | 2 | 0 | 3 | 0 | 79 | 58 | 34 |  |
|  | $\Sigma$ | $\left(B^{\prime}+D\right)\left[(A+B)+D^{\prime} C\right]$ | 5 | 1 | 0 | 4 | 0 | 85 | 70 | 40 |  |
|  |  |  |  |  |  |  |  | 164 | 128 | 74 | 1553408 |
| Lowest Power |  |  |  |  |  |  |  |  |  |  |  |
| Student |  | Function implemented | Inverters | $2 \mathrm{i} / \mathrm{p}$ and | $3 \mathrm{i} / \mathrm{p}$ and | $2 \mathrm{i} / \mathrm{p}$ or | xor | Cost | Power | Area | Product |
| Benjamin Lee | $\pm-\beta$ | $\left(B^{\prime}+D\right)\left[(A+B)+D^{\prime} C\right]$ | 2 | 2 | 0 | 3 | 0 | 79 | 58 | 34 |  |
|  | $\Sigma$ | $\left(B^{\prime}+D\right)\left[(A+B)+D^{\prime} C\right]$ | 2 | 1 | 1 | 3 | 0 | 99 | 63 | 36 |  |
|  | Total |  |  |  |  |  |  | 178 | 121 | 70 | 1507660 |
| Lowest Area |  |  |  |  |  |  |  |  |  |  |  |
| Student |  | Function implemented | Inverters | $2 \mathrm{i} / \mathrm{p}$ and | $3 \mathrm{i} / \mathrm{p}$ and | $2 \mathrm{i} / \mathrm{p}$ or | xor | Cost | Power | Area | Product |
| Benjamin Lee | $\pm-\beta$ | $\left(B^{\prime}+D\right)\left[(A+B)+D^{\prime} C\right]$ | 2 | 2 | 0 | 3 | 0 | 79 | 58 | 34 |  |
|  | $\Sigma$ | $\left(B^{\prime}+D\right)\left[(A+B)+D^{\prime} C\right]$ | 2 | 1 | 1 | 3 | 0 | 99 | 63 | 36 |  |
|  | Total |  |  |  |  |  |  | 178 | 121 | 70 | 1507660 |
| Best Balanced Design |  |  |  |  |  |  |  |  |  |  |  |
| Student |  | Function implemented | Inverters | $2 \mathrm{i} / \mathrm{p}$ and | $3 \mathrm{i} / \mathrm{p}$ and | $2 \mathrm{i} / \mathrm{p}$ or | xor | Cost | Power | Area |  |
| Benjamin Lee | $\pm-\beta$ | $\left(B^{\prime}+D\right)\left[(A+B)+D^{\prime} C\right]$ | 2 | 2 | 0 | 3 | 0 | 79 | 58 | 34 |  |
|  | $\Sigma$ | $\left(B^{\prime}+D\right)\left[(A+B)+D^{\prime} C\right]$ | 2 | 1 | 1 | 3 | 0 | 99 | 63 | 36 |  |
|  | Total |  | 4 | 3 | 1 | 6 | 0 | 178 | 121 | 70 | 1507660 |

