## CSE 370 Homework 3 Solutions

## 1. Static Hazards

a) $F=A B+A^{\prime} C^{\prime}$

Kmap of this function shown here; add logic for blue box to eliminate static 1 hazard


Static 1 Hazards:

$$
(A, B, C)=(1,1,0)->(0,1,0)
$$

New circuit is: $F=A B+A^{\prime} C^{\prime}+B C^{\prime}$

b) $F=A^{\prime} C+B^{\prime} C^{\prime}+A B D$

Kmap for this function is shown below; red lines are already implemented in function, blue terms must be added to eliminate the 3 static 1 hazards


## Static 1 Hazards:

$$
\begin{aligned}
& (A, B, C, D)=(0,0,1,1)->(0,0,0,1) \\
& (A, B, C, D)=(0,0,1,0)->(0,0,0,0) \\
& (A, B, C, D)=(1,1,1,1)->(0,1,1,1) \\
& (A, B, C, D)=(1,1,0,1)->(1,0,0,1)
\end{aligned}
$$

New circuit is: $F=A^{\prime} C+B^{\prime} C^{\prime}+A B D+A^{\prime} B^{\prime}+A^{\prime} B D+A C^{\prime} D$


Note: OR gates should be 16 -input OR gate, but visio only has 5-input OR gates
c) $\quad \mathrm{F}=(\mathrm{W}+\mathrm{X}+\mathrm{Y})\left(\mathrm{X}^{\prime}+\mathrm{Z}^{\prime}\right)$

Kmap for this function is shown below; red lines are already implemented in function, blue term must be added to eliminate the static 0 hazard

| $w x$ |  | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 0 | 0 |  |  |
| 01 |  | 0 | 0 |  |
| 11 |  | 0 | 0 |  |
| 10 |  |  |  |  |

## Static 0 Hazards:

$(W, X, Y, Z)=(0,0,0,1)->(0,1,0,1)$
New circuit is:


## 2. Timing Diagrams

The non-oscillating steady state for this circuit can easily be found by assuming the node to the right of $S$ is a 1 and tracing the resulting path. From $S$, you can find that $B$ is a 1 , and C is therefore a 0 . D is then known to be a 1 , and at the other end of the loop, A is of course the same value as the node to the right of S , a 1 .


Note: for this timing diagram, assume that the steady state is at Ops, and the switch is changed to the down position at $\mathrm{t}=10 \mathrm{~ns}$. Also, assume $\mathrm{T}_{\mathrm{pd}}=10 \mathrm{~ns}$.
3. 2 bit adder

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 0 |

$X=A C+A B D+B C D$
$Y=A^{\prime} B^{\prime} C+A^{\prime} C D^{\prime}+A^{\prime} B C^{\prime} D+A B^{\prime} C^{\prime}+A C^{\prime} D^{\prime}+A B C D$
$Z=B D^{\prime}+B^{\prime} D=B \oplus D$
To implement this circuit, you can use an 8 to 1 multiplexer for each output.
Since some outputs ( Y and Z ) depend on more than 3 inputs (the number of select bits for an 8 to 1 mux ), you can use the other input for the 2 bit adder as an input to the multiplexer. Many implementations are possible, one is shown here:


Note: $0^{\text {th }}$ position for muxes is at top. A is most significant select bit.
4. Full Adder using multiplexers
a) using 28 -to-1 multiplexers


b) using 2 4-to-1 multiplexers

c) see solution for d)
d) using 5 2-to-1 MUXes


## 5. Decoders

a) $\quad \mathrm{f}(\mathrm{P}, \mathrm{Q}, \mathrm{R})=\overline{(P Q+R)}$ using 3:8 decoder

b) $\quad f(P, Q, R, S, T)=(P+Q) S+(R+T) S^{\prime}$ using 2 2:4 decoders


Note: Only 16 -input OR gate is necessary, but visio only has up to 5 inputs, so the OR gate is drawn this way

## 6. Implementation Methods

Note: Function can be reduced to F = AC + AD
a) 8:1 mux

b) 4:16 decoder


Note: Once again, visio does not have 6-input OR gates, so a tree of OR gates is used.
c) PLA-like structure

7. Rotate - assume this is supposed to implement a rotate right function (left is also acceptable), inputs are $A[7: 0]$ and $R[2: 0]$, outputs are $B[7: 0]$


