

CSE 370 Fall 99
Solution for Assignment 2
10/12/99

1.a)

Reading the pMOS (top) part directly

$$F = (A' + B')(B' + C')(A + C')$$

Also to recheck you should read the nMOS part directly

$$F' = AB + BC + A'C$$

And check that these expressions are negation of each other.

$$\begin{aligned} (F')' &= (AB + BC + A'C)' = (AB)'(BC)'(A'C)' \\ &= (A' + B')(B' + C')(A + C') \\ &\quad \text{(DeMorgans Law all the way!)} \end{aligned}$$

b)

Applying the consensus Theorem on F' results in following simplification on F'

$$F' = AB + A'C$$

A more elaborate proof

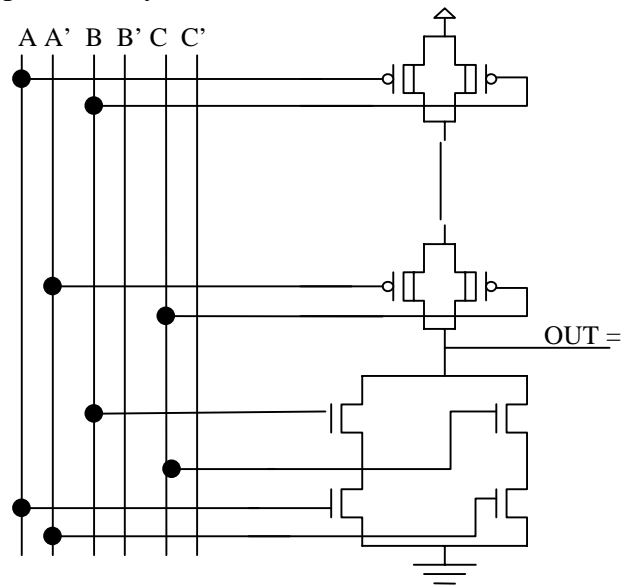
$$\begin{aligned} F' &= AB + A'C + BC \\ &= AB + A'C + BC(A + A') \\ &= AB + ABC + A'C + A'BC \\ &= AB(1 + C) + A'C(1 + B) \\ &= AB + A'C \end{aligned}$$

c)

part a) : 12 transistors

part b): 8 transistors

so four transistors saved .Hence $8\mu\text{m}^2$ area saved.



1. a)

To prove that NAND is complete show that NOT, AND & OR gates can be implemented using the NAND gates.

$$A' = (A + A)' = A \text{ nand } A$$

$$AB = ((AB)')' = (A \text{ nand } B)' = ((A \text{ nand } B) \text{ nand } (A \text{ nand } B))$$

$$A + B = ((A + B)')' = (A' \text{ nand } B')' = (A' \text{ nand } B') = ((A \text{ nand } A) \text{ nand } (B \text{ nand } B))$$

b)

$$F = A'B + B'C$$

$$= (A \text{ nand } A)B + (B \text{ nand } B)C \quad (\text{using the not expansion})$$

$$= X + Y \quad (\text{Let})$$

so using the expansion for AND gate in terms of nand

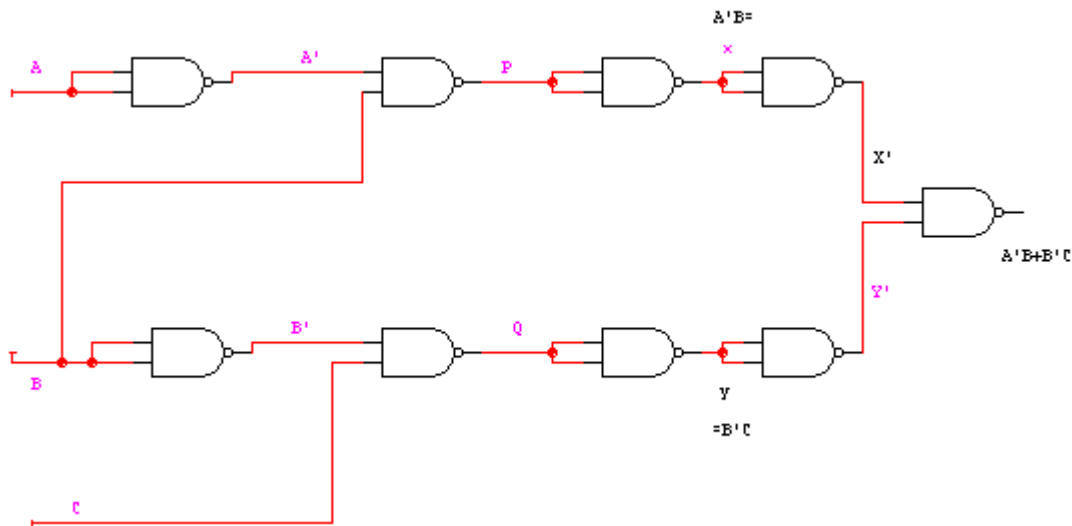
$$X = (A \text{ nand } A)B = [(A \text{ nand } A) \text{ nand } B] \text{ nand } [(A \text{ nand } A) \text{ nand } B]$$

$$Y = (B \text{ nand } B)C = [(B \text{ nand } B) \text{ nand } C] \text{ nand } [(B \text{ nand } B) \text{ nand } C]$$

And

$$F = X + Y = (X \text{ nand } X) \text{ nand } (Y \text{ nand } Y)$$

So we get the following schematic.



The opportunity for minimization is evident by the diagram. Note that from P to X' there are two NOT gates (implemented using NAND). So they are redundant and hence can be removed. Same for Q and Y'. So the inputs P and Q can *directly* be used as inputs for the final (rightmost) NAND gate.

5.

a) The truth table for function P is shown below.

A	B	C	P(A,B,C)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Reading directly from the truth table:

$$P(A,B,C) = A'B'C + A'BC' + AB'C' + ABC$$

b) XOR can be considered as a two-input parity function. So obv.

$$P(A,B,C) = (A \text{ xor } B) \text{ xor } C$$

More formally, proof goes as follows

First of all note that

$$A \text{ xor } B = A'B + AB'$$

And hence

$$(A \text{ xor } B)' = A'B' + AB$$

Now for the proof

$$\begin{aligned} P(A,B,C) &= A'B'C + A'BC' + AB'C' + ABC \\ &= C(A'B' + AB) + C'(A'B + AB') \\ &= C(A \text{ xor } B)' + C'(A \text{ xor } B) \\ &= C \text{ xor } (A \text{ xor } B). \end{aligned}$$

c)

The function P (the wire P) is 1 when there is an odd number of 1's in A, B and C. This implies that together P, A, B and C should **always** have an even number of 1's if transmitted correctly i.e. They should always have their parity as 0 in case there is no error.

A	B	C	P	E(A,B,C,P)
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

d) Using the same intuitive argument as above we get

$$E(A,B,C,P) = (A \text{ xor } B) \text{ xor } (C \text{ xor } P)$$

A more formal proof would be on the same lines as in part b) above basically grouping

Terms with $C'D'$, $C'D$, CD' , CD together first then basis of D and D' and finally as a whole one big term. The schematic is drawn below
NOTE: XOR is associative.

