

CSE 370 Fall 99
Solution for Assignment 3
10/18/99

2.

Leap	A	B	C	D	28	29	30	31
-	0	0	0	0	X	X	X	X
-	0	0	0	1	0	0	0	1
0	0	0	1	0	1	0	0	0
1	0	0	1	0	0	1	0	0
-	0	0	1	1	0	0	0	1
-	0	1	0	0	0	0	1	0
-	0	1	0	1	0	0	0	1
-	0	1	1	0	0	0	1	0
-	0	1	1	1	0	0	0	1
-	1	0	0	0	0	0	0	1
-	1	0	0	1	0	0	1	0
-	1	0	1	0	0	0	0	1
-	1	0	1	1	0	0	1	0
-	1	1	0	0	0	0	0	1
-	1	1	0	1	X	X	X	X
-	1	1	1	0	X	X	X	X
-	1	1	1	1	X	X	X	X

- a) $28(\text{leap}, a, b, c, d) = m_2 + \sum d(0, 13, 14, 15, 16, 29, 30, 31)$
 $29(\text{leap}, a, b, c, d) = m_{18} + \sum d(0, 13, 14, 15, 16, 29, 30, 31)$
 $30(a, b, c, d) = \sum m(4, 6, 9, 11) + \sum d(0, 13, 14, 15)$
 $31(a, b, c, d) = \sum m(1, 3, 5, 7, 8, 10, 12) + \sum d(0, 13, 14, 15)$

b)

Reading from the schematic

$$28 = (\text{leap})' a' b' c d' = m_2$$

which trivially matches with a) by putting all the don't-cares to '0'

$$29 = (\text{leap}) a' b' c d' = m_{18}$$

ditto

$$31 = a \text{ XOR } d = a' d + a d' = (a' d + a d')(b' c' + b' c + b c' + b c)$$

$$= (a' b' c' d + a' b' c d + a' b c' d + a' b c d + a b' c' d' + a b' c d' + a b c' d' + a b c d')$$

$$= \sum m(1, 3, 5, 7, 8, 10, 12, 14)$$

which is same as 31 above if *only* d14 is taken as '1' (so add a term m14) and rest as '0'.

$$30 = (a \text{ XOR } d)' (a' b' c d)' = (a' d + a d')' (a' b' c d)'$$

solution 1

$$a \text{ XOR } d = \sum m(1, 3, 5, 7, 8, 10, 12, 14)$$

$$\text{so } (a \text{ XOR } d)' = \sum m(0, 2, 4, 6, 9, 11, 13, 15)$$

(all the terms NOT present on the previous line)
 similarly $(a'b'cd')' = \Sigma m(0,1,3,4,5,6,7,8,9,10,11,12,13,14,15)$

so their AND will have only THOSE minterms which are common (why ??)

so as implemented

$$30(a,b,c,d) = \Sigma m(0,4,6,9,11,13,15)$$

which is same as a) if we let $d(0,13,15) = 1$.

solution 2

basically similar as above

$$X' \text{ and } Y' = [(X' \text{ and } Y')]' = (X + Y)'$$

So putting X as “a XOR d” and Y as “a'b'cd”

$$30(a,b,c,d) = \text{NOT}(a'b'cd' + a \text{ XOR } d) = \text{NOT}[m_2 + \Sigma m(1,3,5,7,8,10,12,14)]$$

$$= \text{NOT}(\Sigma m(1,2,3,5,7,8,10,12,14)) = \Sigma m(0,4,6,9,11,13,15)$$

which is same as above in *solution 1*

3.

R0	R1	C0	C1	S0	S1
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	1	1
0	1	1	0	0	1
0	1	1	1	1	0
1	0	0	0	1	1
1	0	0	1	0	0
1	0	1	0	1	0
1	0	1	1	0	1
1	1	0	0	1	1
1	1	0	1	1	1
1	1	1	0	1	1
1	1	1	1	1	1

$$S0 = \Sigma m(5,7,8,10,12,13,14,15)$$

$$= \Pi M(0,1,2,3,4,6,9,11)$$

$$S1 = \Sigma m(5,6,8,11,12,13,14,15)$$

$$= \Pi M(0,1,2,3,4,7,9,10)$$

b)

$$S0 = \Sigma m(5,7,13,15) + \Sigma m(8,10,12,14)$$

$$= \Sigma_m(-1-1)+\Sigma_m(1--0) \quad (\text{using the Unifying Theorem})$$

$$= R1C1+R0C1'$$

$$S1= \Sigma_m(5,13)+\Sigma_m(6,14)+\Sigma_m(8,12)+\Sigma_m(11,15)$$

$$= \Sigma_m(-101)+\Sigma_m(-110)+\Sigma_m(1-00)+\Sigma_m(1-11) \quad (\text{using the Unifying Theorem})$$

$$= R1C0'C1+R1C0C1'+R0C0'C1'+R0C0C1$$

$$= R1(C0'C1+C0C1') + R0(C0'C1+C0C1)$$

$$= R1(C0 \text{ xor } C1) + R0(C0 \text{ xor } C1)' \quad (\text{optional})$$

4.

$$\text{a) } f(X,Y)=XY+XY'$$

$$=X(Y+Y')$$

$$=X*1$$

$$=X$$

$$\text{b) } f(X,Y)=(X+Y)(X+Y')$$

$$=XX+XY'+YX+YY'$$

$$=X+XY'+XY+0$$

$$=X+X(Y+Y')$$

$$=X+X$$

$$=X$$

$$\text{c) } f(X,Y,Z)=YZ'+X'YZ+XYZ$$

$$=YZ'+YZ(X+X')$$

$$=YZ'+YZ$$

$$=Y(Z+Z')$$

$$=Y$$

$$\text{d) } f(X,Y,Z)=(X+Y)(X'+Y+Z)(X'+Y+Z')$$

Let $A=X'+Y$. Now,

$$(X'+Y+Z)(X'+Y+Z') = (A+Z)(A+Z')$$

$$=AA+AZ'+ZA+ZZ'$$

$$=A+A(Z+Z')+0$$

$$=A+A$$

$$=A$$

So

$$F(X,Y,Z)=(X+Y)A$$

$$=(X+Y)(X'+Y)$$

$$=XX'+XY+YX'+YY$$

$$=0+Y(X+X')+Y$$

$$=Y+Y$$

$$=Y$$

$$\text{e) } f(X,Y,Z,W)=X+XYZ+X'YZ+X'Y+WX+W'X$$

$$\begin{aligned}
&=X(1+YZ)+X'Y(Z+1)+X(W+W') \\
&=X+X'Y+X \\
&=X+XY+X'Y+X \\
&=X+X+Y(X+X') \\
&=X+Y
\end{aligned}$$

5.

$$\begin{aligned}
\text{a) } \Sigma_m(0,1,2,7,8,9,10,15) &= \\
&=A'B'C'D'+A'B'C'D+A'B'CD'+A'BCD+AB'C'D'+AB'C'D+AB'CD'+ABCD
\end{aligned}$$

$$\begin{aligned}
\text{b) } \Pi M(3,4,5,6,11,12,13,14) &= \\
&=(A+B+C'+D')(A+B'+C+D)(A+B'+C+D')(A+B'+C'+D) \\
&\quad (A'+B+C'+D')(A'+B'+C+D)(A'+B'+C+D')(A'+B'+C'+D)
\end{aligned}$$

$$\begin{aligned}
\text{c) } f' &= \Sigma_m(3,4,5,6,11,12,13,14) \\
&=A'B'CD+A'BC'D'+A'BC'D+A'BCD'+AB'CD+ABC'D'+ABC'D+ABCD'
\end{aligned}$$

$$\begin{aligned}
\text{d) } f' &= \Pi M(0,1,2,7,8,9,10,15) \\
&=(A+B+C+D)(A+B+C+D')(A+B+C'+D)(A+B'+C'+D') \\
&\quad (A'+B+C+D)(A'+B+C+D')(A'+B+C'+D)(A'+B'+C'+D')
\end{aligned}$$