Solutions for Midterm

## (Question 1 only!)

CSE 370
a)

| A | B | C | $\mathbf{D}$ | $\mathbf{E}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | $\mathbf{0}$ | $\mathbf{0}$ |
| 0 | 0 | 1 | $\mathbf{0}$ | $\mathbf{1}$ |
| 0 | 1 | 0 | $\mathbf{1}$ | $\mathbf{0}$ |
| 0 | 1 | 1 | $\mathbf{1}$ | $\mathbf{1}$ |
| 1 | 0 | 0 | $\mathbf{X}$ | $\mathbf{X}$ |
| 1 | 0 | 1 | $\mathbf{1}$ | $\mathbf{1}$ |
| 1 | 1 | 0 | $\mathbf{1}$ | $\mathbf{0}$ |
| 1 | 1 | 1 | $\mathbf{0}$ | $\mathbf{1}$ |

b)
$\mathrm{D}=\mathrm{S} \mathrm{m}(2,3,5,6)+\mathrm{d} 4=\mathrm{P} \mathrm{M}(0,1,7) * \mathrm{D} 4$
$\mathrm{E}=\mathrm{S} \mathrm{m}(1,3,5,7)+\mathrm{d} 4=\mathrm{P} \mathrm{M}(0,2,6) * \mathrm{M} 4$
c)

D(a,b,c)

| AB <br> C | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{X}$ |
| 1 | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |

## $\mathbf{E}(\mathbf{a}, \mathrm{b}, \mathbf{c})$

| AB <br> C | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{X}$ |
| 1 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |

Minimized Expressions
$\mathrm{D}=\mathbf{a} \mathbf{?} \mathbf{b}+\mathbf{a b} \boldsymbol{?}+\mathbf{b c} \boldsymbol{?}$ (SoP)
$=(\mathrm{a}+\mathrm{b})(\mathrm{a} ?+\mathrm{b} ?+\mathrm{c}$ ? $)(\mathrm{PoS})$
$\mathrm{E}=\mathrm{c}$ (both PoS and SoP)

Minimum PoS expression for $\mathrm{D}=$
$(a+b)(a ?+b ?+c ?)=(a+b+c)(a+b+c ?)(a ?+b ?+c ?)$ (unification)
$=\mathrm{P} \mathrm{M}(0,1,7)$ as required.
d)

The minimization done above does NOT contains a single-bit static-1 hazard. Note that the adjacent cells $110 \& 100$ even though not covered by the same prime implicant will still not result in a static-1 bit hazard as the transition is between a don?t-care and a 1.

One implementation, which will have static-1 hazard, would be
$\mathrm{D}=\mathrm{a} ? \mathrm{~b}+\mathrm{ab} ?+\mathrm{ac}$ ?
In which there is hazard between $100 \& 110$. If you try to remove this hazard by adding another prime implicant $\mathbf{a}$ ? $\mathbf{c}$ the hazard will surely go away but however this function ( $\mathrm{D}=\mathrm{a}$ ? $\mathrm{b}+\mathrm{ab}$ ? +ac ? +a ? c ) will no longer remain implementable using the PAL as even after using feedback, at most three product terms can be added using feedback once. (Feedback cannot be used twice else there would be no place left for E).


## $D=a ? b+a b ?+a ? c$

E=c
Implementation Using PLA

