Asymptotic Analysis

CSE 373
Data Structures & Algorithms
Ruth Anderson
Autumn 2012

Today's Outline

- **Announcements**
  - Assignment #1, due Thurs, Oct 4 at 11pm
  - Assignment #2, posted later this week, due Fri Oct 12 at BEGINNING of lecture
- **Algorithm Analysis**
  - How to compare two algorithms?
  - Analyzing code
  - Big-Oh

Comparing Two Algorithms…

What we want

- Rough Estimate
- Ignores Details

Big-O Analysis

- Ignores "details"

Gauging performance

- Uh, why not just run the program and time it?
  - Too much variability; not reliable:
    - Hardware: processor(s), memory, etc.
    - OS, version of Java, libraries, drivers
    - Programs running in the background
    - Implementation dependent
    - Choice of input
  - Timing doesn't really evaluate the *algorithm*; it evaluates an *implementation* in one very specific scenario

Comparing algorithms

When is one *algorithm* (not *implementation*) better than another?
- Various possible answers (clarity, security, …)
- But a big one is *performance*: for sufficiently large inputs, runs in less time (our focus) or less space

We will focus on large inputs (n) because probably any algorithm is "plenty good" for small inputs (if *n* is 10, probably anything is fast enough)

Answer will be *independent* of CPU speed, programming language, coding tricks, etc.

Answer is general and rigorous, complementary to "coding it up and timing it on some test cases"
- Can do analysis before coding!

Why Asymptotic Analysis?

- Most algorithms are fast for small *n*
  - Time difference too small to be noticeable
  - External things dominate (OS, disk I/O, …)
- BUT *n* is often large in practice
  - Databases, internet, graphics, …
- Time difference really shows up as *n* grows!

Analyzing code ("worst case")

Basic operations take "some amount of" constant time
- Arithmetic (fixed-width)
- Assignment
- Access one Java field **or array index**
- Etc.

(This is an *approximation*.)

Consecutive statements	Sum of times
Conditionals	Time of test plus slower branch
Loops	Sum of iterations
Calls	Time of call's body
Recursion	Solve *recurrence equation*

Example

2	3	5	16	37	50	73	75	126

Find an integer in a *sorted* array

```
// requires array is sorted
// returns whether k is in array
boolean find(int[]arr, int k){
   ???
}
```


Linear search

2	3	5	16	37	50	73	75	126

Find an integer in a *sorted* array

```
// requires array is sorted
// returns whether k is in array
boolean find(int[]arr, int k){
   for(int i=0; i < arr.length; ++i)
      if(arr[i] == k)
        return true;
   return false;
}
```

Best case:

Worst case:

Linear search

2	3	5	16	37	50	73	75	126

Find an integer in a *sorted* array

```
// requires array is sorted
// returns whether k is in array
boolean find(int[]arr, int k){
   for(int i=0; i < arr.length; ++i)
      if(arr[i] == k)
        return true;
   return false;
}
```

Best case: 6ish steps = $O(1)$
Worst case: 6ish*(arr.length)
= O(arr.length)

Binary search

2	3	5	16	37	50	73	75	126

Find an integer in a *sorted* array

```
// requires array is sorted
// returns whether k is in array
boolean find(int[]arr, int k){
    return help(arr,k,0,arr.length);
}
boolean help(int[]arr, int k, int lo, int hi) {
    int mid = (hi+lo)/2; //i.e., lo+(hi-lo)/2
    if(lo==hi)          return false;
    if(arr[mid]==k) return true;
    if(arr[mid]< k) return help(arr,k,mid+1,hi);
    else                return help(arr,k,lo,mid);
}
```


Binary search

Best case: 8ish steps = $O(1)$

Worst case: $T(n)$ = 10ish + $T(n/2)$ where n is **`hi-lo`**

- $O($**`log`** $n)$ where n is **`array.length`**
- Solve *recurrence equation* to know that…

```
// requires array is sorted
// returns whether k is in array
boolean find(int[]arr, int k){
    return help(arr,k,0,arr.length);
}
boolean help(int[]arr, int k, int lo, int hi) {
    int mid = (hi+lo)/2;
    if(lo==hi)          return false;
    if(arr[mid]==k) return true;
    if(arr[mid]< k) return help(arr,k,mid+1,hi);
    else                return help(arr,k,lo,mid);
}
```


Solving Recurrence Relations

1. Determine the recurrence relation. What is the base case?
   $T(n) = 10 + T(n/2)$ $T(1) = 13$ "ish"
2. "Expand" the original relation to find an equivalent general expression *in terms of the number of expansions*.
3. Find a closed-form expression by setting *the number of expansions* to a value which reduces the problem to a base case

Solving Recurrence Relations

1. Determine the recurrence relation. What is the base case?
   $T(n) = 10 + T(n/2)$ $T(1) = 13$
2. "Expand" the original relation to find an equivalent general expression *in terms of the number of expansions*.
   $T(n) = 10 + 10 + T(n/4)$
   $= 10 + 10 + 10 + T(n/8)$
   $= \ldots$
   $= 10k + T(n/(2^k))$
3. Find a closed-form expression by setting *the number of expansions* to a value which reduces the problem to a base case
   - $n/(2^k) = 1$ means $n = 2^k$ means $k = \log_2 n$
   - So $T(n) = 10 \log_2 n + 13$ (get to base case and do it)
   - So $T(n)$ is $O(\log n)$