Binary Search Trees

CSE 373
Data Structures & Algorithms
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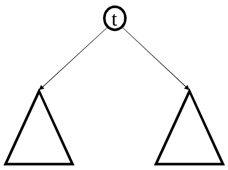
Autumn 2012

Today's Outline

- **Announcements**
  - Assignment #2 due Fri, Oct 12 at the BEGINNING of lecture
- **Today's Topics:**
  - **Asymptotic Analysis**
  - **Binary Search Trees**

Tree Calculations

Recall: height is max number of edges from root to a leaf



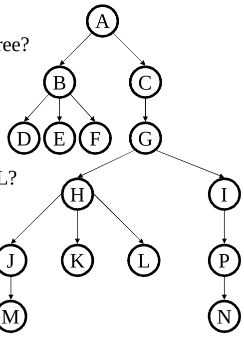
Find the height of the tree...

runtime:

Tree Calculations Example

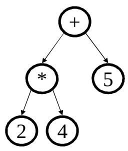
What is the **height** of this tree?

What is the **depth** of node L?



More Recursive Tree Calculations: Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree



(an expression tree)

Three types:

- Pre-order: Root, left subtree, right subtree
- In-order: Left subtree, root, right subtree
- Post-order: Left subtree, right subtree, root

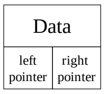
Traversals

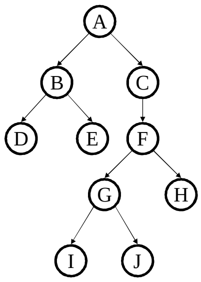
```
void traverse(BNode t){
  if (t != NULL)
    traverse (t.left);
    print t.element;
    traverse (t.right);
  }
}
```

Which one is this?

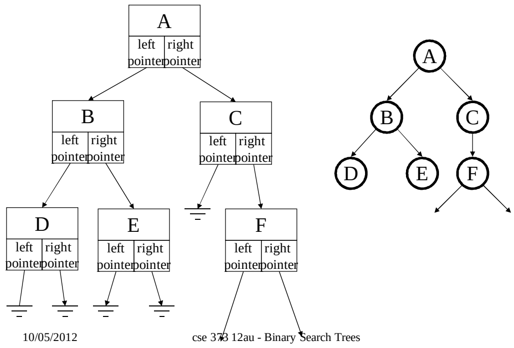
Binary Trees

- Binary tree is
  - a root
  - left subtree *(maybe empty)*
  - right subtree *(maybe empty)*
- Representation:

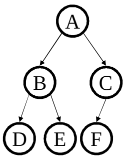




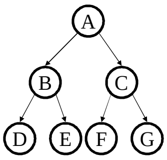
Binary Tree: Representation



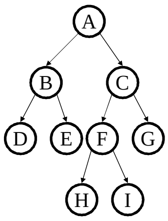
Binary Tree: Special Cases



Complete Tree



Perfect Tree



Full Tree

ADTs Seen So Far

- Stack
  - Push
  - Pop
- Queue
  - Enqueue
  - Dequeue

The Dictionary ADT

- Data:
  - a set of (key, value) pairs
- Operations:
  - Insert (key, value)
  - Find (key)
  - Remove (key)

insert(tanvir, ….)

find(swansond)

• swansond
David Swanson, …

- tanvir
  Tanvir Aumi
  OH: T & Th 10-11am,
  CSE 216
- jjgile
  Jacob Gile
  OH: F 1:30am-2:30pm
  CSE 220
- swansond
  David Swanson
  OH: Th 3:30-4:30pm
  CSE 218
- zzt0215
  Zhiting Zhu
  OH: W 10-11am
  CSE 218

*The Dictionary ADT is sometimes called the "**Map ADT**"*

A Modest Few Uses

- Search : phone directories or other large data sets (genome maps, web pages)
- Networks : Router tables
- Operating systems : Page tables
- Compilers : Symbol tables

Probably the most widely used ADT!

Implementations

	insert	find	delete
• Unsorted Linked-list			
• Unsorted array			
• Sorted array			

Implementations

For dictionary with n key/value pairs

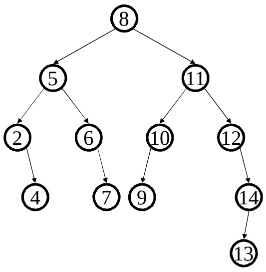
	`insert`	`find`	`delete`
• Unsorted linked-list	$O(1)$ *	$O(n)$	$O(n)$
• Unsorted array	$O(1)$ *	$O(n)$	$O(n)$
• Sorted linked list	$O(n)$	$O(n)$	$O(n)$
• Sorted array	$O(n)$	$O(\log n)$	$O(n)$

We'll see a Binary Search Tree (BST) probably does better,
but not in the worst case unless we keep it balanced

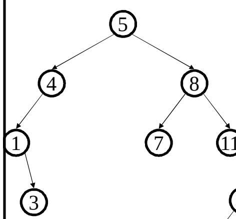
*Note: If we do not allow duplicates values to be inserted, we would need to do O(n) work (a find operation) to check for a key's existence before insertion

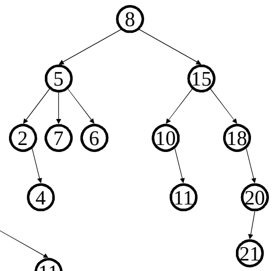
Binary Search Tree Data Structure

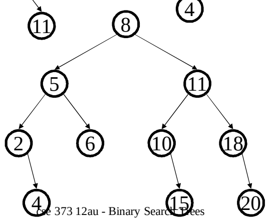
- Structural property
  - each node has ≤ 2 children
  - result:
    - storage is small
    - operations are simple
    - average depth is small
- Order property
  - all keys in left subtree smaller than root's key
  - all keys in right subtree larger than root's key
  - result: easy to find any given key
- What must I know about what I store?



Are these BSTs?

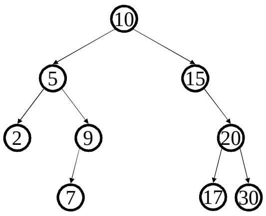






Activity

Find in BST, Recursive

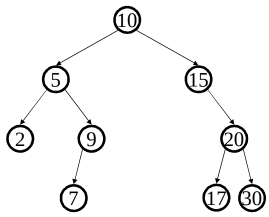


```
Node Find(Object key,
          Node root) {
  if (root == NULL)
    return NULL;

  if (key < root.key)
    return Find(key,
                root.left);
  else if (key > root.key)
    return Find(key,
                root.right);
  else
    return root;
}
```

Runtime:

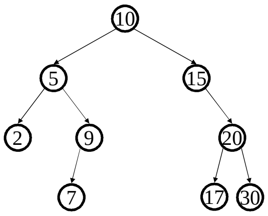
Find in BST, Iterative

```
Node Find(Object key,
          Node root) {

  while (root != NULL &&
         root.key != key) {
    if (key < root.key)
      root = root.left;
    else
      root = root.right;
  }

  return root;
}
```



Runtime:

Insert in BST



Insert(13)
Insert(8)
Insert(31)

Runtime:

BuildTree for BST

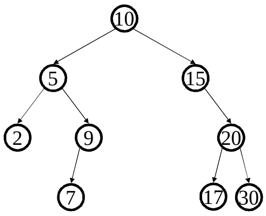
- Suppose keys 1, 2, 3, 4, 5, 6, 7, 8, 9 are inserted into an initially empty BST.

Runtime depends on the order!

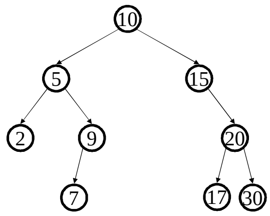
  - in given order
  - in reverse order
  - median first, then left median, right median, etc.

Bonus: FindMin/FindMax

- Find minimum
- Find maximum



Deletion in BST



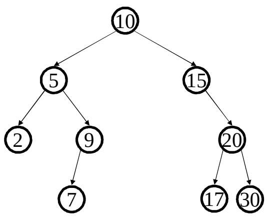
Why might deletion be harder than insertion?

Lazy Deletion

Instead of physically deleting nodes, just mark them as deleted

+ simpler
+ physical deletions done in batches
+ some adds just flip deleted flag

– extra memory for deleted flag
– many lazy deletions slow finds
– some operations may have to be modified (e.g., min and max)

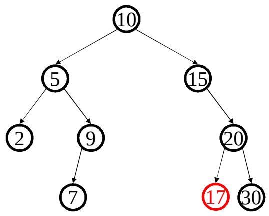


Non-lazy Deletion

- Removing an item disrupts the tree structure.
- Basic idea: find the node that is to be removed. Then “fix” the tree so that it is still a binary search tree.
- Three cases:
  - node has no children (leaf node)
  - node has one child
  - node has two children

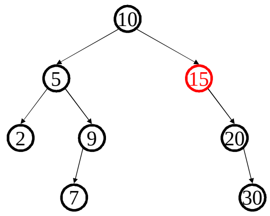
Non-lazy Deletion – The Leaf Case

Delete(17)



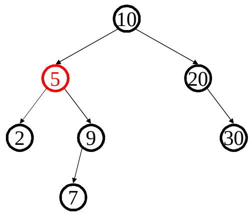
Deletion – The One Child Case

Delete(15)



Deletion – The Two Child Case

Delete(5)



What can we replace 5 with?

Deletion – The Two Child Case

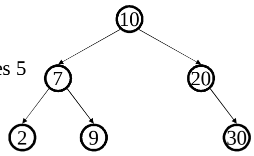
Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees!

Options:
- *succ* from right subtree: findMin(t.right)
- *pred* from left subtree : findMax(t.left)

Now delete the original node containing *succ* or *pred*
- Leaf or one child case – easy!

Finally…

7 replaces 5



Original node containing
7 gets deleted

Binary Tree: Some Numbers

Recall: height of a tree = longest path from root to leaf (count # of edges)

For binary tree of height *h*:
- max # of leaves:
- max # of nodes:
- min # of leaves:
- min # of nodes:

Balanced BST

Observation
- BST: the shallower the better!
- For a BST with *n* nodes
  - Average height is $\Theta(\log n)$
  - Worst case height is $\Theta(n)$
- Simple cases such as insert(1, 2, 3, ..., n) lead to the worst case scenario

Solution: Require a **Balance Condition** that
1. ensures depth is $\Theta(\log n)$ – strong enough!
2. is easy to maintain – not too strong!

Potential Balance Conditions

1. Left and right subtrees of the root have equal number of nodes

2. Left and right subtrees of the root have equal *height*

Potential Balance Conditions

3. Left and right subtrees of *every node* have equal number of nodes

4. Left and right subtrees of *every node* have equal *height*