Hashing

Chapter 5 in Weiss

CSE 373

Data Structures and Algorithms

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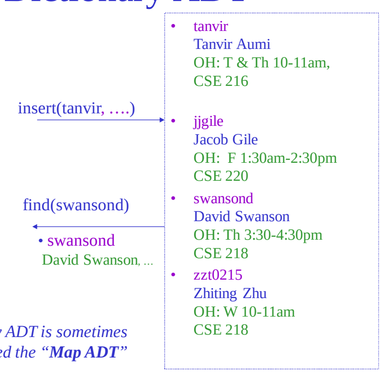
Today's Outline

- **Announcements**
  - **Homework #4 coming soon:**
    - **Java programming: disjoint sets and mazes**
    - **due Thurs, Nov 8th**
    - **partners allowed- MUST declare by 11pm Wed Oct 31st *at the latest* (email to Tanvir)**
  - **Midterm #2 – Fri, Nov 16**
- **Today's Topics:**
  - **Hashing**

The Dictionary ADT

- Data:
  - a set of (key, value) pairs
- Operations:
  - Insert (key, value)
  - Find (key)
  - Remove (key)



*The Dictionary ADT is sometimes called the "**Map ADT**"*

Dictionary Implementations

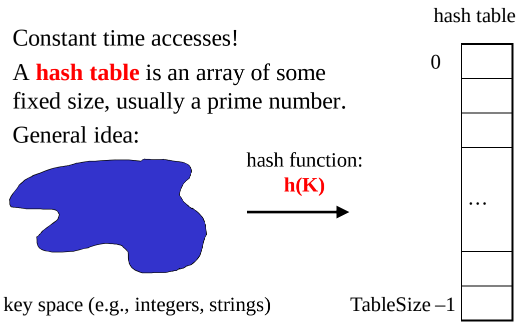
For dictionary with *n* key/value pairs

	`insert`	`find`	`delete`
• Unsorted linked-list	$O(1)$ *	$O(n)$	$O(n)$
• Unsorted array	$O(1)$ *	$O(n)$	$O(n)$
• Sorted linked list	$O(n)$	$O(n)$	$O(n)$
• Sorted array	$O(n)$	$O(\log n)$	$O(n)$
• BST			
• AVL Tree			

*Note: If we do not allow duplicates values to be inserted, we would need to do O(n) work (a find operation) to check for a key's existence before insertion

Hash Tables

- Constant time accesses!
- A **hash table** is an array of some fixed size, usually a prime number.
- General idea:



Hash Tables

Key space of size M, but we only want to store subset of size N, where N<<M.

- Keys are identifiers in programs. Compiler keeps track of them in a symbol table.
- Keys are student names. We want to look up student records quickly by name.
- Keys are chess configurations in a chess playing program.
- Keys are URLs in a database of web pages.

Example

- key space = integers
- TableSize = 10
- **h**(K) = K mod 10
- **Insert**: 7, 18, 41, 94

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Another Example

- key space = integers
- TableSize = 6
- **h**(K) = K mod 6
- **Insert**: 7, 18, 41, 34

0	
1	
2	
3	
4	
5	

Student Activity

Hash Functions

1. **simple/fast** to compute,
2. Avoid **collisions**
3. have keys distributed **evenly** among cells.

Perfect Hash function:

Sample Hash Functions:

- key space = strings
- $s = s_0\ s_1\ s_2\ \ldots\ s_{k-1}$

1. $h(s) = s_0$ mod TableSize
2. $h(s) = \left(\sum_{i=0}^{k-1} s_i\right)$ mod TableSize
3. $h(s) = \left(\sum_{i=0}^{k-1} s_i \cdot 37^i\right)$ mod TableSize

Designing a Hash Function for web URLs

$s = s_0\ s_1\ s_2\ \ldots\ s_{k-1}$

Issues to take into account:

h(s) =

Student Activity

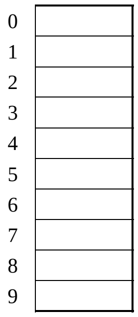
Collision Resolution

Collision: when two keys map to the same location in the hash table.

Two ways to resolve collisions:

1. Separate Chaining
2. Open Addressing (linear probing, quadratic probing, double hashing)

Separate Chaining



Insert:
10
22
107
12
42

- **Separate chaining**: All keys that map to the same hash value are kept in a list ("bucket").

Analysis of find

- The load factor, λ, of a hash table is the ratio:

$$\frac{N}{M} \quad \begin{array}{l} \leftarrow \text{no. of elements} \\ \leftarrow \text{table size} \end{array}$$

For separate chaining, λ = average # of elements in a bucket

- unsuccessful:

- successful:

How big should the hash table be?

- For Separate Chaining:

tableSize: Why Prime?

- Suppose
  - data stored in hash table: 7160, 493, 60, 55, 321, 900, 810
  - tableSize = 10
    data hashes to 0, 3, <u>0</u>, 5, 1, <u>0</u>, <u>0</u>
  - tableSize = 11
    data hashes to 10, 9, 5, 0, 2, <u>9</u>, 7

Real-life data tends to have a pattern

Being a multiple of 11 is usually *not* the pattern ☺

Open Addressing

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Insert:
38
19
8
109
10

- **Linear Probing**: after checking spot h(k), try spot h(k)+1, if that is full, try h(k)+2, then h(k)+3, etc.

Terminology Alert!

"**Open** Hashing"	"Closed Hashing"
equals	equals
"Separate Chaining"	"**Open** Addressing"

Weiss

Linear Probing

f(i) = i

- Probe sequence:

  0^{th} probe = h(k) mod TableSize

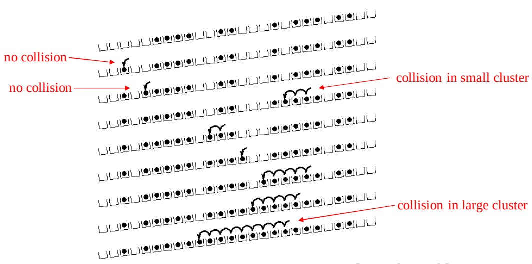
  1^{th} probe = (h(k) + 1) mod TableSize

  2^{th} probe = (h(k) + 2) mod TableSize

  . . .

  i^{th} probe = (h(k) + i) mod TableSize

Linear Probing – Clustering



[R. Sedgewick]

Load Factor in Linear Probing

- For *any* $\lambda < 1$, linear probing *will* find an empty slot
- Expected # of probes (for large table sizes)
  - successful search:

$$\frac{1}{2}\left(1+\frac{1}{(1-\lambda)}\right)$$

  - unsuccessful search:

$$\frac{1}{2}\left(1+\frac{1}{(1-\lambda)^2}\right)$$

- Linear probing suffers from ***primary clustering***
- Performance quickly degrades for $\lambda > 1/2$

Quadratic Probing

Less likely to encounter Primary Clustering

$f(i) = i^2$

- Probe sequence:

0^{th} probe = h(k) mod TableSize
1^{th} probe = (h(k) + 1) mod TableSize
2^{th} probe = (h(k) + 4) mod TableSize
3^{th} probe = (h(k) + 9) mod TableSize
. . .
i^{th} probe = (h(k) + i^2) mod TableSize

Quadratic Probing

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Insert:
89
18
49
58
79

Quadratic Probing:

- h(k) = k mod 7
- Perform these inserts:
  - Insert(65)
  - Insert(10)
  - Insert(47)

0	
1	
2	93
3	
4	
5	40
6	76

Quadratic Probing Example

insert(76) 76%7 = 6 | insert(40) 40%7 = 5 | insert(48) 48%7 = 6 | insert(5) 5%7 = 5 | insert(55) 55%7 = 6

But… insert(47) 47%7 = 5

0	
1	
2	
3	
4	
5	
6	76

Quadratic Probing: Success guarantee for λ < ½

- If size is prime and λ < ½, then quadratic probing will find an empty slot in size/2 probes or fewer.
  - show for all **0 ≤ i,j ≤ size/2** and **i ≠ j**
    (h(x) + i²) mod size ≠ (h(x) + j²) mod size
  - by contradiction: suppose that for some i ≠ j:
    (h(x) + i²) mod size = (h(x) + j²) mod size
    ⇒ i² mod size = j² mod size
    ⇒ (i² - j²) mod size = 0
    ⇒ [(i + j)(i - j)] mod size = 0

  BUT size does not divide **(i-j)** or **(i+j)**

Quadratic Probing: Properties

- For *any* λ < ½, quadratic probing will find an empty slot; for bigger λ, quadratic probing *may* find a slot
- Quadratic probing does not suffer from *primary* clustering: keys hashing to the same *area* are not bad
- But what about keys that hash to the same *spot*?
  - ***Secondary Clustering!***

Double Hashing

f(i) = i * g(k)
where g is a second hash function

- Probe sequence:
  - 0^{th} probe = h(k) mod TableSize
  - 1^{th} probe = (h(k) + g(k)) mod TableSize
  - 2^{th} probe = (h(k) + 2*g(k)) mod TableSize
  - 3^{th} probe = (h(k) + 3*g(k)) mod TableSize
  - . . .
  - i^{th} probe = (h(k) + i*g(k)) mod TableSize

Double Hashing Example

i^{th} probe = (h(k) + i*g(k)) mod TableSize

h(k) = k mod 7 and g(k) = 5 – (k mod 5)

	76	93	40	47	10	55
0						
1				47	47	47
2		93	93	93	93	93
3					10	10
4						55
5			40	40	40	40
6	76	76	76	76	76	76
Probes	1	1	1	2	1	2

Resolving Collisions with Double Hashing

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Hash Functions:

H(k) = k mod M

H_2(k) = 1 + ((k/M) mod (M-1))

M =

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13
28
33
147
43

Rehashing

Idea: When the table gets too full, create a bigger table (usually 2x as large) and hash all the items from the original table into the new table.

- When to rehash?
  - half full ($\lambda = 0.5$)
  - when an insertion fails
  - some other threshold
- Cost of rehashing?

Hashing Summary

- Hashing is one of the most important data structures.
- Hashing has many applications where operations are limited to find, insert, and delete.
- Dynamic hash tables have good amortized complexity.