B-Trees
(4.7 in Weiss)

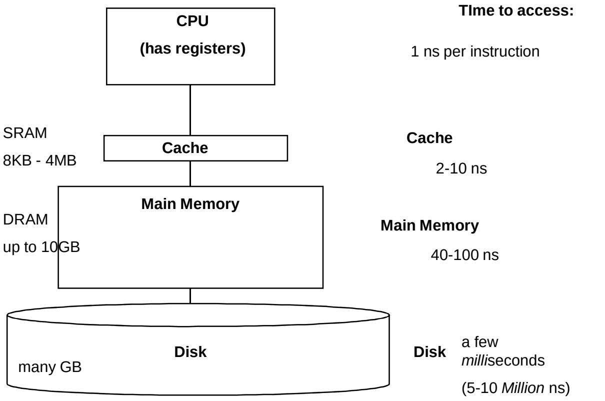
CSE 373
Data Structures & Algorithms
Ruth Anderson

Today's Outline

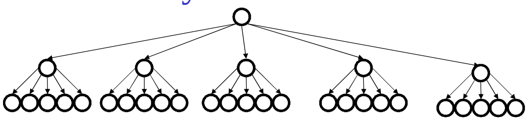
- **Admin**:
  - Final Exam – Tuesday December 11th, topic list posted soon
  - HW #6 – Sorting, due Thurs December 6 at 11pm
- **Sorting**
  - In-place and Stable Sorting
- **Dictionaries**
  - B-Trees

Trees so far

- BST
- AVL



M-ary Search Tree



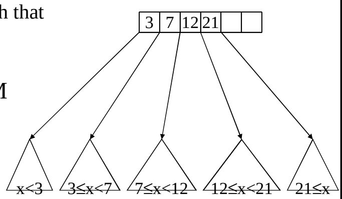
- Maximum branching factor of ***M***
- Complete tree has height =

\# disk accesses for *find*:

Runtime of *find*:

Solution: B-Trees

- specialized *M*-ary search trees
- Each **node** has (up to) M-1 keys:
  - subtree between two keys *x* and *y* contains leaves with *values* *v* such that $x \leq v < y$



- Pick branching factor M such that each node takes one full {*page*, *block*} of memory

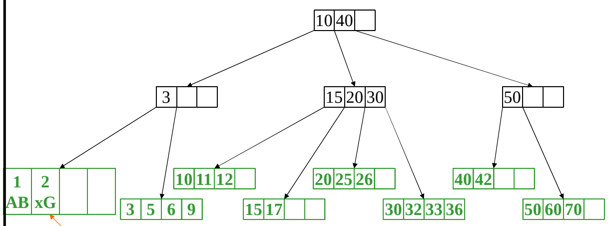
B-Trees

What makes them disk-friendly?

1. **Many keys stored in a node**
   - All brought to memory/cache in one access!
2. Internal nodes contain *only* keys;
   Only leaf nodes contain keys and actual data
   - The tree structure can be loaded into memory irrespective of data object size
   - Data actually resides in disk

B-Tree: Example

B-Tree with **`M = 4`** (# **pointers** in internal node)
and **`L = 4`** (# **data items** in Leaf)



Data objects, that I'll ignore in slides

Note: All leaves at the same depth!

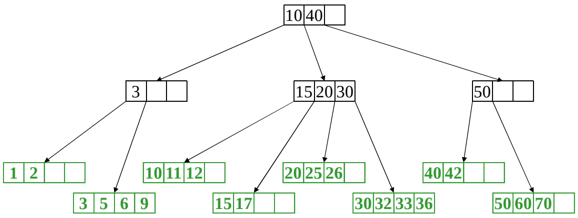
B-Tree Properties ‡

- Data is stored at the leaves
- All leaves are at the same depth and contain between $\lceil L/2 \rceil$ and L data items
- Internal nodes store up to M-1 keys
- Internal nodes have between $\lceil M/2 \rceil$ and M children
- Root (special case) has between 2 and M children (or root could be a leaf)

‡These are technically B^+-Trees

Example, Again

B-Tree with `M = 4`
and `L = 4`



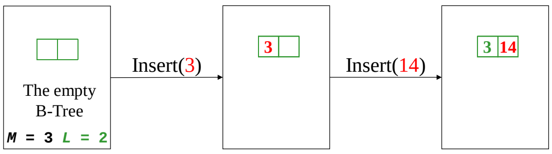
(Only showing keys, but leaves also have data!)

B-trees vs. AVL trees

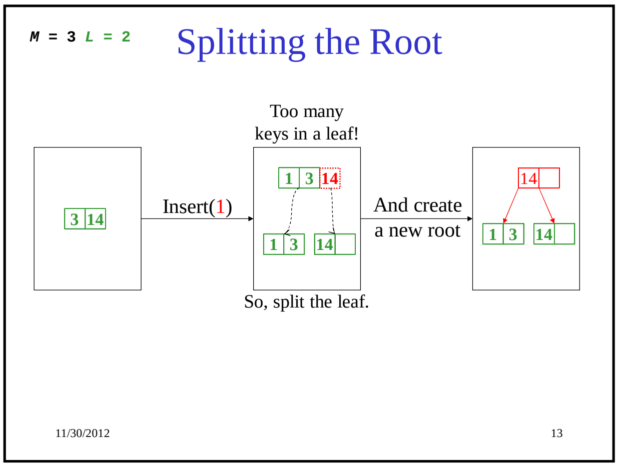
Suppose we have 100 million items (100,000,000):

- Depth of AVL Tree
- Depth of B+ Tree with M = 128, L = 64

Building a B-Tree



Now, Insert(1)?


M = 3 L = 2
Splitting the Root
Too many
keys in a leaf!
3 14
Insert(1)
1 3 14
1 3
14
And create
a new root
14
1 3
14
So, split the leaf.
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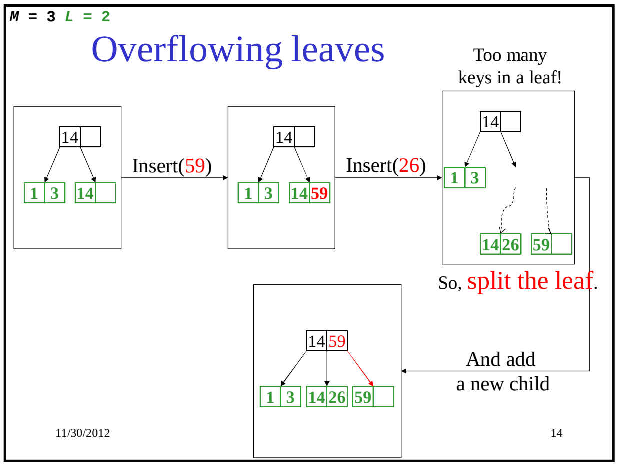
M = 3 L = 2
Overflowing leaves
Too many
keys in a leaf!
14
1 3
14
Insert(59)
14
1 3
14 59
Insert(26)
14
1 3
14 26
59
So, split the leaf.
14 59
1 3
14 26
59
And add
a new child
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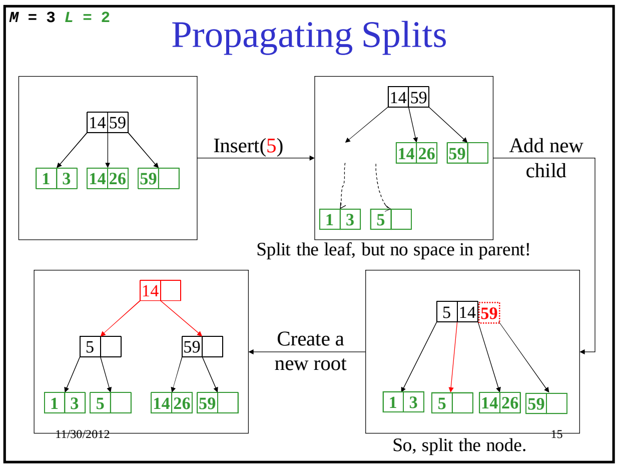
14


M = 3 L = 2
Propagating Splits
14 59
1 3
14 26
59
Insert(5)
14 59
14 26
59
1 3
5
Add new
child
Split the leaf, but no space in parent!
14
5
59
1 3
5
14 26
59
Create a
new root
5 14 59
1 3
5
14 26
59
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So, split the node.

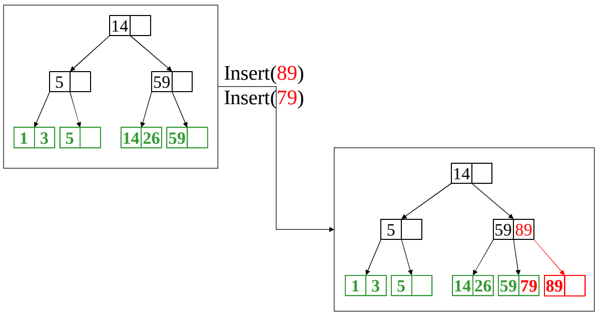
Insertion Algorithm

1. Insert the key in its leaf
2. If the **leaf** ends up with L+1 items, **overflow**!
   - **Split** the leaf into two nodes:
     - original with ⌈(L+1)/2⌉ items
     - new one with ⌊(L+1)/2⌋ items
   - Add the new child to the parent
   - If the parent ends up with **M+1** items, **overflow**!
3. If an **internal node** ends up with M+1 items, **overflow**!
   - **Split** the node into two nodes:
     - original with ⌈(M+1)/2⌉ items
     - new one with ⌊(M+1)/2⌋ items
   - Add the new child to the parent
   - If the parent ends up with **M+1** items, **overflow**!
4. **Split** an overflowed **root** in two and hang the new nodes under a new root

This makes the tree deeper!

M = 3 L = 2

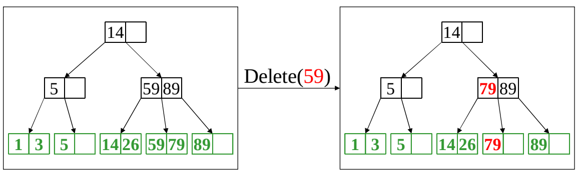
After More Routine Inserts



M = 3 L = 2

Deletion

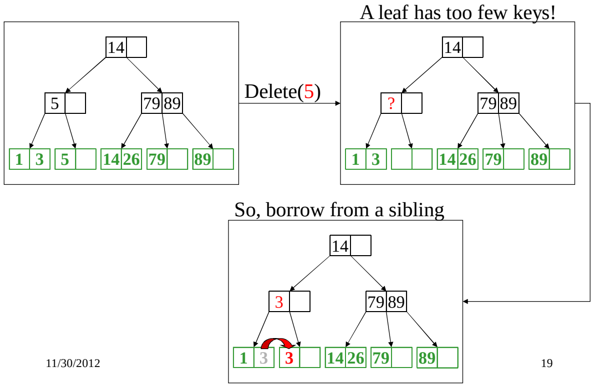
1. Delete item from leaf
2. Update keys of ancestors if necessary



What could go wrong?

$M = 3$ $L = 2$

Deletion and Adoption



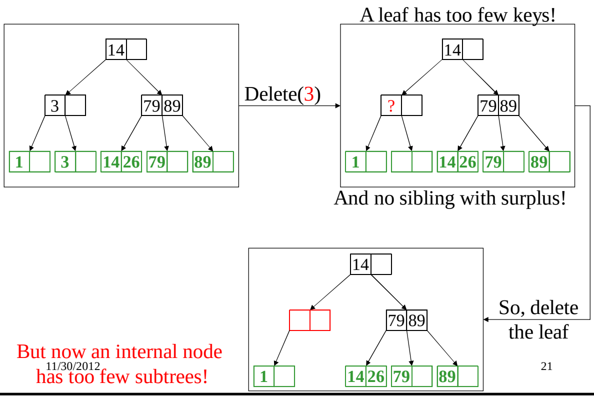
Does Adoption Always Work?

- What if the sibling doesn't have enough for you to borrow from?

  e.g. you have $\lceil L/2 \rceil$-1 and sibling has $\lceil L/2 \rceil$?

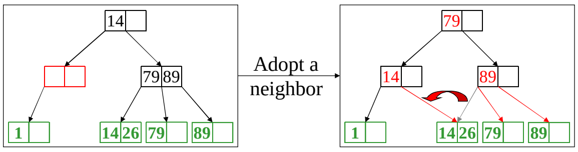
$M = 3$ $L = 2$

Deletion and Merging



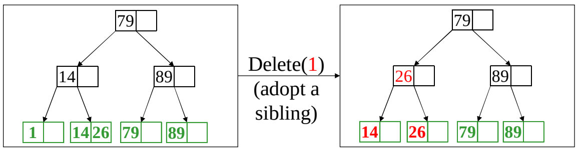
M = 3 L = 2

Deletion with Propagation (More Adoption)



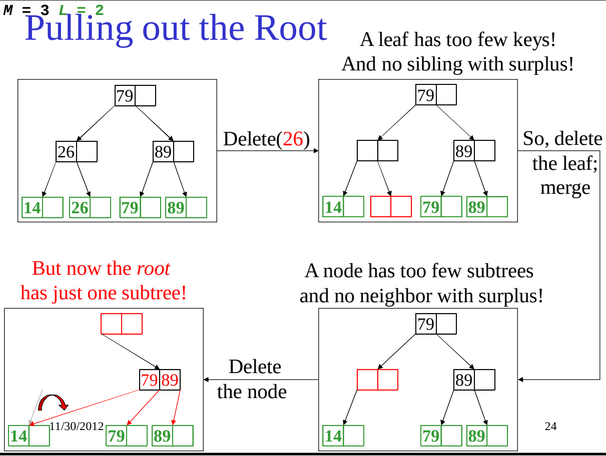
M = 3 L = 2

A Bit More Adoption



M = 3 L = 2

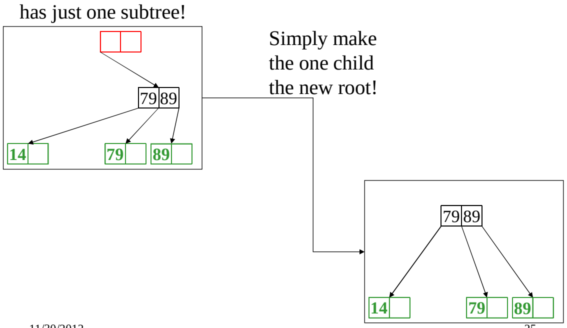
Pulling out the Root



$M = 3$ $L = 2$

Pulling out the Root (continued)

The *root* has just one subtree!



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Deletion Algorithm

1. Remove the key from its leaf

2. If the **leaf** ends up with fewer than $\lceil L/2 \rceil$ items, **underflow**!
   - **Adopt** data from a sibling; update the parent
   - If adopting won't work, delete node and **merge** with neighbor
   - If the parent ends up with fewer than $\lceil M/2 \rceil$ items, **underflow**!

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Deletion Slide Two

3. If an **internal** node ends up with fewer than $\lceil M/2 \rceil$ items, **underflow**!
   - **Adopt** from a neighbor; update the parent
   - If adoption won't work, **merge** with neighbor
   - If the parent ends up with fewer than $\lceil M/2 \rceil$ items, **underflow**!

4. If the **root** ends up with only one child, make the child the new root of the tree

This reduces the height of the tree!

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Thinking about B-Trees

- B-Tree **insertion** can cause (expensive) splitting and propagation
- B-Tree **deletion** can cause (cheap) adoption or (expensive) deletion, merging and propagation
- Propagation is rare if ***M*** and ***L*** are large *(Why?)*
- If ***M* = *L* = 128**, then a B-Tree of height 4 will store at least 30,000,000 items

Tree Names You Might Encounter

FYI:

- B-Trees with ***M* = 3**, ***L* = x** are called **2-3 trees**
  - Nodes can have 2 or 3 pointers
- B-Trees with ***M* = 4**, ***L* = x** are called **2-3-4 trees**
  - Nodes can have 2, 3, or 4 pointers

Determining M and L for a B-Tree

1 Page on disk = 1 KByte

Key = 8 bytes, Pointer = 4 bytes

Data = 256 bytes per record (includes key)

M =

L =

Student Activity